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A REPORT ON THE COASTAL WATERS OF LABRADOR.

Based on Explorations of the "Chance" During the Summer of 1926.

By C. Iselin.

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A REPORT ON THE COASTAL WATERS OF LABRADOR,

Based on Explorations of the "Chance" During the Summer of 1926.

By C. Iselin.

Introduction.

The north westernmost corner of the Atlantic Ocean, because of its bottom conformations and closed current system, forms a favorable unit for oceanographic study. This body of water is bounded on the northeast by Greenland, on the northwest by Baffin Land, on the west by Labrador and Newfoundland, and is limited on the south by the Grand Banks. A study of the bottom contours of the region (Schott 1912) shows that this sea has the topography of a gulf, for a long, moderately shallow ridge extending in a southwesterly direction from Iceland limits the deepest (greater than 4000 meters) connection with the bottom waters of the North Atlantic to a comparatively narrow channel east of the Grand Banks. All other outlets of this basin, for example Hudson Strait and Kennedy Channel, are shallow and narrow. A counter-clockwise system of currents marks off even better than do the submarine topographic boundaries, the limits of this body of water. The icy Labrador Current forms 1600 miles of its western boundary. The much warmed North Atlantic Drift, sweeping in an arc from Flemish Cap towards Cape Farewell, may be taken as the southeastern boundary, and the West Greenland Current completes the circuit.

This region is an important one for oceanographic study because the Labrador Current provides a vast discharge of arctic water into the North Atlantic. Not only are great masses of icy water being continually transported by it to relatively low latitudes, but also quantities of arctic plankton are spread southward and finally left to die in the warmer waters south and east of the Grand Banks. Nevertheless, exact physical observations have been scanty for the waters of this region, and accordingly little attempt has been made to study its circulatory system in detail. Many ships sailing up into Baffin Bay have made surface observations and likewise much is known about the movement both of the bergs and of the pack ice, but subsurface observations, so necessary in modern oceanographic procedure, have been sadly lacking until very recently (Smith 1928, Riis-Cartensen, 1929).

Physical observations over the continental shelf and especially in the bays and inlets of the region are interesting from the biological point of view, because they provide the exact temperature and salinity of the water which swarms with so much life. Very few systematic plankton investigations have been made along the Labrador coast, especially north of Hamilton Inlet, and nothing at all is known about the chemistry of the water there. Often it is only through a knowledge of the current system of a region that the distribution of marine life can be accounted for.

It was with the view of making a beginning at the study of these phases of the northern sector of the Labrador Current, and its plankton, that the schooner "Chance" in the summer of 1926 undertook a cruise along the Newfoundland and Labrador coasts as far north as Cape Chidley. Since then (summer of 1928) the U. S. Coast Guard patrol boat "Marion" has made an extended physical survey of the circulation of the Labrador Sea, the results of which are now in the process of preparation (Smith 1928). And that same summer Commander E. Riis-Cartensen on the "Gothaab" made a general oceanographic survey of Davis Strait northward into Baffin Bay (Riis-Cartensen, 1929). The observations of the "Chance" on the Labrador Current therefore provide material which, when combined with the much more complete work of the "Marion," and "Gothaab" can be used for the study of annual variations. Furthermore, the "Chance" made some investigations of the conditions in the bays and fiords of the Newfoundland and Labrador coasts, where the "Marion," because of the extended nature of her cruise, could not stop. This report aims to present, as nearly as possible, the state of knowledge of the circulatory system of the Labrador Sea before the "Marion" began her work in 1928.

The work of the "Chance" included three cross sections of the Labrador Current, the first off Cape Race, Newfoundland, running out in a southeasterly direction as far as the shoal waters of the Grand Banks; the second off Sandwich Bay, Labrador (Lat. 54°), and the third off Nachvak Bay, Labrador (Lat. 59°). These last two extended over a hundred miles from the shore. Investigations were also made in the waters of White Bay, Newfoundland, and in Seglek, Nachvak, and Ryan's Bay, Labrador, these being chosen as representative of the different types of fiords which indent the northeast coast of subarctic America.

The schooner was equipped with a small winch and could collect subsurface water samples down to about 600 meters. Forty stations were occupied for temperature and salinity observations, while tow netting was carried on with #20 bolting silk net at eighteen surface stations. At a number of other points a much coarser and larger net was used at various depths, but this latter material has not yet been fully studied. On shore one of the members of the expedition made a fairly complete botanical collection which has already been reported on (Woodworth 1927).

The water bottles used were of the Ekman type. The thermometers were made by Negretti and Zambra of the latest design, equipped with auxiliary thermometers. The salinity of about half the water samples collected was found by the usual method of chlorine titration. That of the other half was calculated with the help of the new electrical apparatus recently developed for use on the Ice Patrol ships (U. S. Coast Guard, 1924). This was a considerable saving of time, not to mention the increase in accuracy.

In the work of preparing for the trip and in the writing of this report Dr. H. B. Bigelow has been a constant advisor and teacher

THE "CHANCE" PROFILES.

The Gully off Cape Race, Newfoundland. On the morning of July 20, 1926, after getting a sounding of 40 fathoms on the Newfoundland side of the Grand Banks, a station was made and the schooner then headed northwest for Cape Race, 40 miles away. Four stations were made about 10 miles apart, while crossing the gully. This profile was undertaken partly because of a report (Le Danois 1924) of abnormally low water temperatures to the westward of Cape Race (between Green Bank and St. Pierre Bank), and partly to give the crew practice in using the apparatus.

There exist several sets of previous observations on the waters of this gully. In 1914 the Ice Patrol (U. S. Coast Guard 1915, p. 54) made three stations between the Grand Banks and St. Johns, which, as they were occupied July 16–17, are directly comparable with the observations made by the "Chance" so far as the season of the year is concerned. A much more complete section, made by the Ice Patrol in April, 1924 (Smith, 1924) gives a picture of conditions earlier in the spring. Besides these, two stations were made by the "Scotia" (Matthews 1914) in the gully in 1913, and one by the "Michael Sars" (Murray and Hjort 1912) in July 1910. A comparison with these earlier records indicates that the temperatures and salinities encountered there by the "Chance" were in no way exceptional for the time of year.

Our profiles for temperature (Fig. 1) and for salinity (Fig. 2) show the gully filled with water which was apparently of Labrador Current origin, for below 40 meters all the water was colder than 0° C (minimum temperature $-1.36°$) and of a salinity between 32.5 and 33.5 parts per thousand. The surprisingly low temperatures ($-2°$ to $-4°$) found by Le Danois were in a region about 120 miles southwest of the "Chance's" stations. If his observations are correct it does not seem possible that the Cape Race gully is the path by which such icy water came from the north. For when the temperatures and salinities are translated into densities (Fig. 3) no gradient appears which could cause

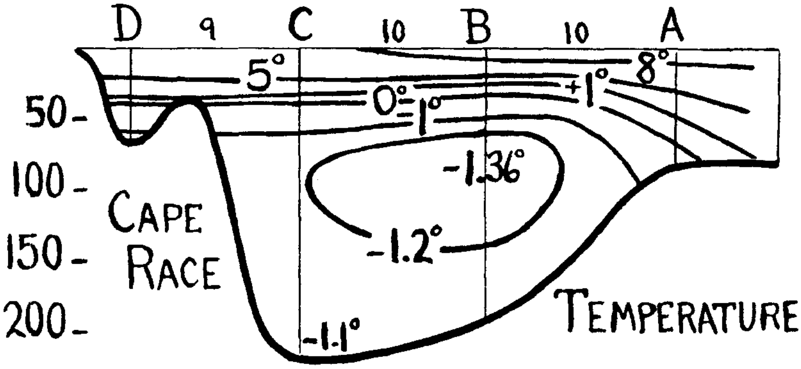
D
9
C
10
B
10
A
5°
0°
1°
+1°
8°
-1.36°
-1.2°
-1.1°
50-
100-
150-
200-
CAPE RACE
TEMPERATURE

FIGURE 1. Temperature profile across the gully off Cape Race, Newfoundland.

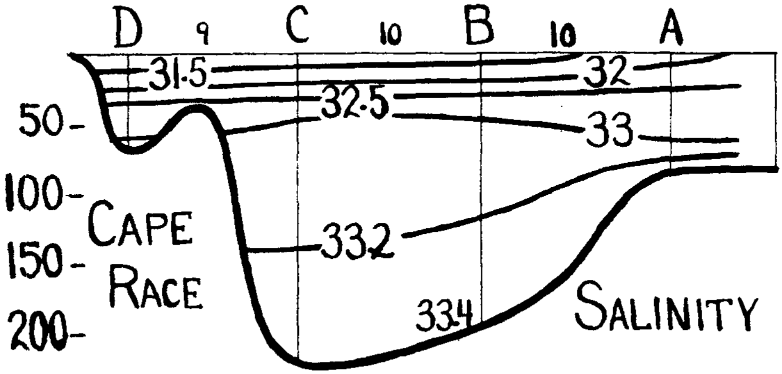
D
9
C
10
B
10
A
31.5
32
32.5
33
33.2
33.4
50-
100-
150-
200-
CAPE RACE
SALINITY

FIGURE 2. Salinity profile across the gully off Cape Race, Newfoundland.

water to flow in from the northeast. This conclusion is corroborated by the fact that dynamic calculations by the usual method (Smith 1926) based on these data fail to show any gradient for a current through the gully in either direction. It is true that various earlier observations have indicated a southwesterly drift through this gully. Each spring, for example, bergs are carried around Cape Race and up into Placentia Bay. Similarly, a series of current-meter readings made by W. Bell Dawson (1906), ten miles east of Cape Race, show a dominant southwesterly drift which he considered a branch of the Labrador Current. But it has also been reported (Dawson 1906, p. 23) that this movement of water from the Labrador Current to the southwestward past Cape Race is weak and easily reversed by a change in the wind (Matthews 1914, p. 32), and it is difficult to find dynamic warrant for such a current in any of the profiles at our disposal. Perhaps the solution to the matter is that in the early spring when land drainage is active, a narrow gradient-current of light, comparatively fresh, water develops along this part of the Newfoundland coast. Such a current would naturally be pressed close to the shore by the effect of the earth's rotation and might carry the bergs, of Labrador Current origin, around the cape and up into the bays on the south coast. In short, the arctic water found by the "Chance" in the gully during July was not under the influence of an active dynamic gradient; rather it represented a slow creep from the east, from the Labrador Current, which is constantly bringing icy water southward.

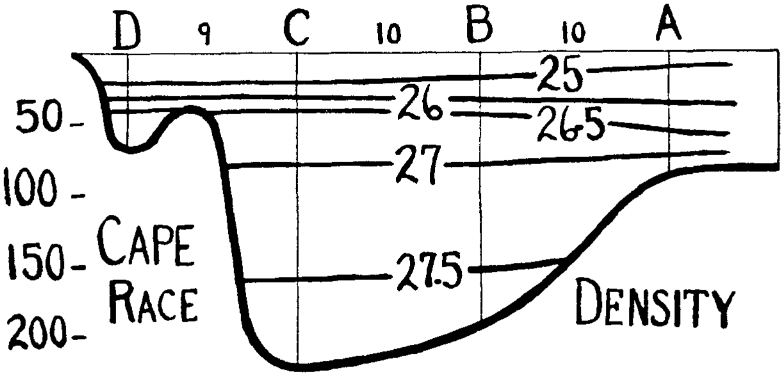

FIGURE 3. Density profile across the gully off Cape Race, Newfoundland.

Sandwich Bay Sector. The first of our two sections of the Labrador

Current, proper, ran out at approximately right angles to the general trend of the coast from a point twenty miles south of Hamilton Inlet. It was hoped that we would thereby obtain a cross section of the fresh coastal waters flowing southward along the shore, as well as of the Labrador Current itself, further at sea. After sailing eastward for 140 miles, a sounding showed no bottom at 400 fathoms, so knowing that we were then outside the edge of the continental shelf, and since the surface temperature of the water had risen from 2° C to 7° C, we headed back to get a cross section of the icy band on the continental shelf. The nine stations were all made while sailing on one course back towards the coast (July 28th to 30th); and since we secured a lucky land-fall at the Gannet Islands, which are accurately charted, and since a good sight was obtained on the second day as a further check, it is certain that our stations are accurately placed, although the weather was mostly cloudy.

It proved unfortunate that the section was not run several miles further north, since it crossed a deep gully, of which the few soundings on the chart gave little indication. These soundings are not sufficient to show whether this depression is an open channel or an inclosed bowl. But the fact that the water in the gully below 300 meters was found to be of the same temperature and salinity as the water at a corresponding depth outside the edge of the continental shelf, points to an open connection between this trough and the off shore waters.

A glance at the outer end of this temperature-profile (Fig. 4) shows, in the first place, a very abrupt transition from water of 0° to 4° between our two outer stations, that is, 120 to 135 miles from the coast. Fortunately the only other profile previously made in this locality (Matthews 1914), covers just this region, the "Scotia" having made, towards the end of June 1913, two stations on the continental shelf, and one other, over a hundred miles further out. Thus for the outer end of the "Chance" profile there is material for comparison. From his observations Matthews places the temperature wall within three miles of its location on our profile. Secondly, in both the "Chance" and "Scotia" sections the coldest water (−1°) extends outward as a thin tongue at a depth of between 40 and 100 meters, as is shown by the bulging outward at these depths of the isotherms which make up the temperature wall. Thirdly, both profiles show the water 2.5° warmer at the bottom of the gully than on a level with its rims.

For the inshore waters off this sector of the coast, the observations of the "Scotia" affords no comparisons, as she made no station within 50 miles of the shore. The "Chance" profile (Fig. 4) shows the coldest

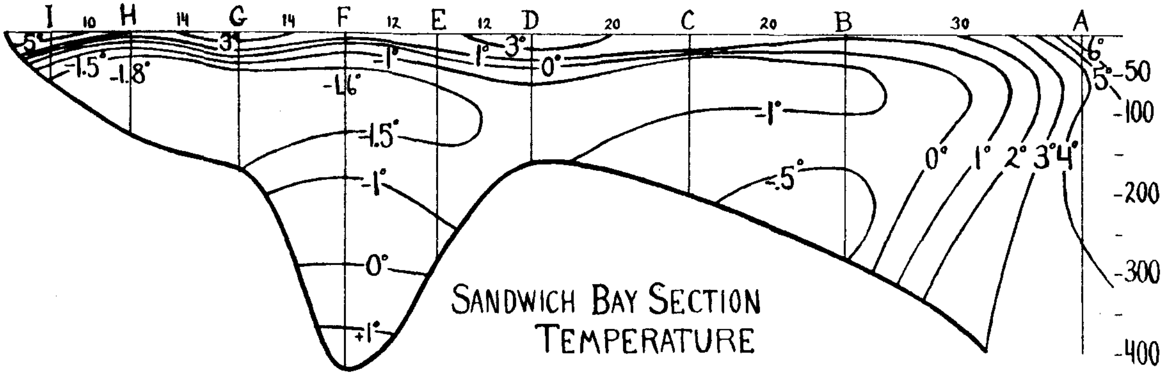

FIGURE 4. Temperature profile off Sandwich Bay, Labrador. The distances between stations are indicated in miles and the depths in meters.

water (−1.5° to −1.8°) approaching close to the coast and extending outward at intermediate depths to a point over the eastern edge of the gully. Another interesting fact is that inshore, where the water is less than 180 meters in depth, the profile shows no warming near the bottom. True Labrador Current water, being relatively light, spreads out on the surface and rarely extends in any of the available profiles more than 200 meters downward. Below this depth a certain amount of mixture with warmer, off shore water is always evident. The warmer surface water (3° to 8°), coming out of Hamilton Inlet, extended at this time to a point about 18 miles from the coast and to a depth of about 25 meters.

To sum up, there can be recognized in this temperature profile four different types of water: first, the warm, relatively fresh coastal stream, given its character by the outflow of river water from Hamilton Inlet; second, the icy intermediate band of the true Labrador Current, which had presumably come from some region far to the north of this profile; third, the warmer and more saline mass of the off shore, North Atlantic, water which is truly oceanic in character; and finally, the bottom water over the deeper parts of the continental shelf which is a mixture of these last two.

These conclusions are confirmed by the fact that the Sandwich Bay salinity-profile (Fig. 5) corresponds closely to the temperature profile. Thus the isohaline of 34 per thousand, marking the outer edge of the truly arctic water, corresponds in position to the isotherm of 0°.

The corresponding density profile (Fig. 6) of the superficial stratum shows comparatively regular density lines with their inshore ends all slanting downward. An instructive illustration of the dynamic slope is afforded by the curve for 27,* since it cuts the surface near station A, but lies 100 meters deep at its inshore end. If a density wall then existed, it was situated further off shore than this section extended. It is also significant that the density curves at the outer stations show a decided slant at even a depth of 300 meters. Thus the density profile indicates first, that all the water in this sector had an impulse for southerly movement; second, that this flow was fairly evenly distributed over the whole width of the continental shelf; and third, that the coastal stream of much fresher, warmer water was really a current within a current.

Dynamic analysis of the data for temperature and salinity corrob-

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* Densities are at temperature *in situ* and are expressed as specific gravity minus 1. times 1000.

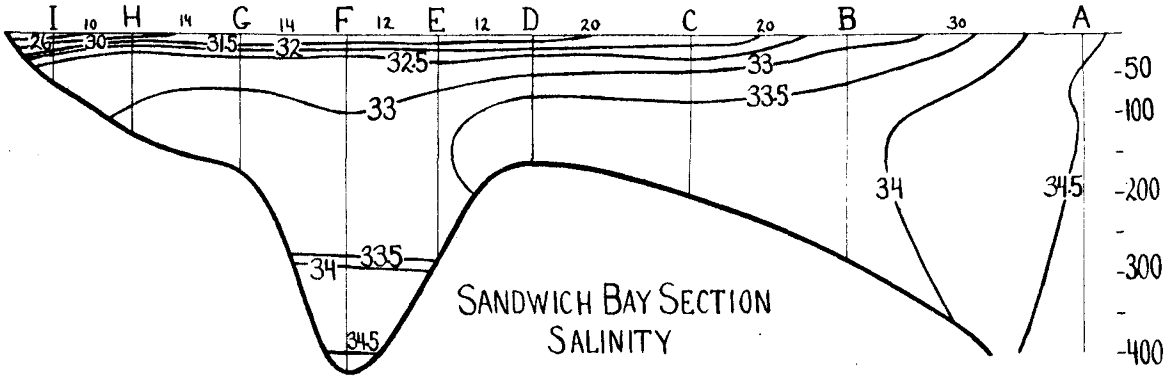

FIGURE 5. Salinity profile off Sandwich Bay, Labrador.

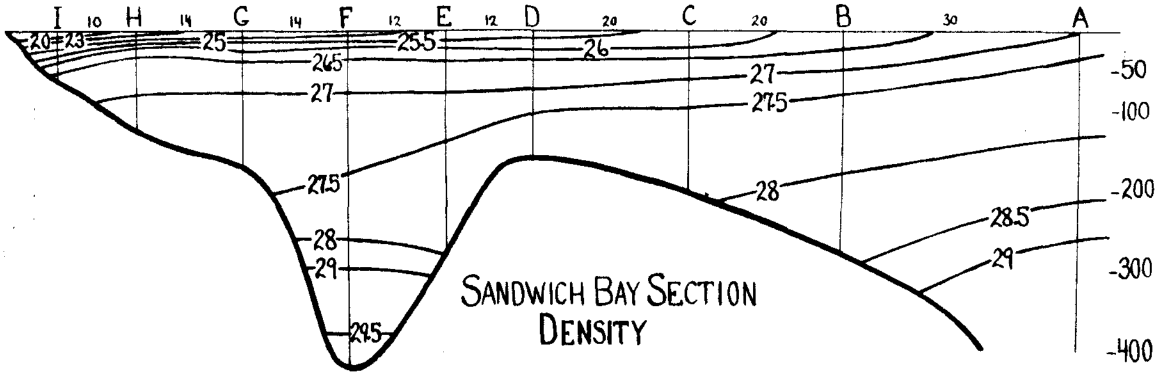

FIGURE 6. Density profile off Sandwich Bay, Labrador.

orates this interpretation of the profile, the calculated gradient calling for a comparatively uniform dynamic slope across the entire section, which is shown in horizontal projection in Figure 7. From station A to station E the gradient was 20.9 dynamic centimeters in a distance of 82 miles which points to an average current of about .3 knots. Calculations of the details of the velocity of the current off Sandwich Bay by the method developed by Jacobsen and Jensen (1926) in their investigations of the Fareo-Shetland Channel, is not satisfactory, first, because of the bottom conformation and second, because the profile shows no water that can be taken as a motionless, base plane. If the stations had been twice as many in number, and especially, if one other had been located between E and F, this method probably could have been applied with positive results.

Nachvak Sector. The relationship of the southern sector of the current to a more northern one, both as to the absolute distribution of temperature and salinity, and as to dynamic impulse toward a southerly flow, is brought out by comparing the above conditions off Sandwich Bay with our Nachvak profile, 311 miles further north.

On August 11 the "Chance" began a line of stations running off shore from Nachvak Fiord in a direction nearly at right angles to the general trend of the coast. The bottom was found there to slope gradually from 120 meters near the land to 300 meters at the edge of the continental shelf, 75 miles off shore. The current along the northern Labrador coast flows, therefore, over a narrower and shallower platform than off Sandwich Bay. Omitting mention of the fiord, which is discussed below (Page 24), it will be noticed (Fig. 8) that at the time of the "Chance's" observations the coldest water (below 0°) conforming to the relative width of the continental shelf, was about 45 miles narrower than off Sandwich Bay, water with its whole mass, surface to bottom, as warm as 4° being encountered on the Nachvak profile in the region of Station I, only 89 miles from the coast.

Our two sections of the Labrador Current agree in the following important respects. In neither case does the coldest water quite reach the coast line, being separated from the latter by a band of fresher, warmer, land-drainage water close in along the shore. In summer it is probable that this long-shore current runs inside of the arctic water along the whole length of the outer Labrador coast. In both profiles the coldest water extends off shore as a tongue with the lowest readings at a depth of from 30 to 100 meters, and in the region of sharp thermal transition, the water warms by about 4° at all depths in a distance of but 20 miles.

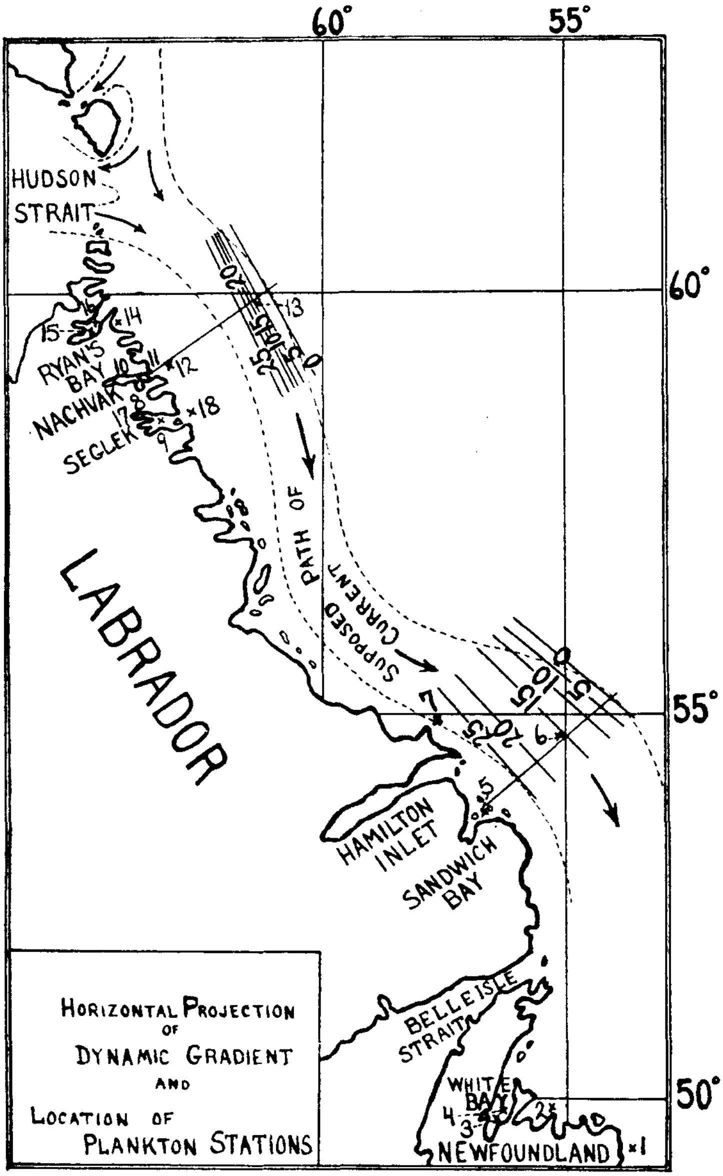
60°
55°
60°
55°
50°
HUDSON STRAIT
RYAN'S BAY
NACHVAK
SEGLEK
LABRADOR
PATH OF SUPPOSED CURRENT
HAMILTON INLET
SANDWICH BAY
BELLE ISLE STRAIT
WHITE BAY
NEWFOUNDLAND
20
25
15
10
5
0
0
5
10
15
20
25
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
HORIZONTAL PROJECTION OF DYNAMIC GRADIENT AND LOCATION OF PLANKTON STATIONS

FIGURE 7. Chart of the Labrador Coast showing the position of the Sandwich Bay and Nachvak section, the dynamic gradients (5 dynamic centimeter intervals), and the location (numbered) of the surface plankton hauls.

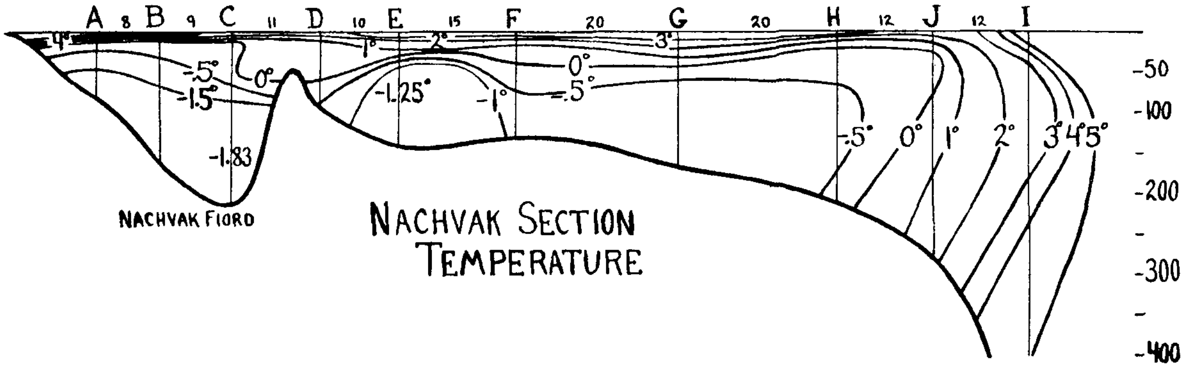

FIGURE 8. Temperature profile off Nachvak Bay, Labrador.

Although the agreement between the Sandwich Bay and Nachvak profiles is so close in a general way, there are several minor points of difference reflecting local causes. Thus the four inner stations on the northern section show no rise in temperature near the bottom. Had the continental shelf of Nachvak averaged as deep as it did off Sandwich Bay, it is probable that the lightness of the icy waters would have caused warmer waters to be drawn in underneath from off shore. Secondly, no water in the Nachvak section was quite as cold as the coldest water off Sandwich Bay, although lower temperatures are usually to be expected in higher latitudes. This apparent anamoly is to be explained by the fact that the Nachvak profile represents a considerably more advanced stage in summer warming. Thirdly, in the region of sharply slanting isotherms between the two outer stations, the strength of the current no doubt caused screwing movements and eddies, explaining the unusually low surface temperatures encountered there. Further inshore, the water layers were more stable, and convectional currents hindered by the stronger Archomedian forces. Therefore the surface shows more solar warming there because the sun's heat is but little communicated downward.

Finally, inshore, in the region of station D we see on the Nachvak profile water which is practically homogeneous from top to bottom. This last is best interpreted as reflecting the southward movement of water which has been well churned by the strong tides of Hudson Strait, as well as in passing over the broken line of shallow reefs that run off shore for 15 miles just north of this section. Any water which rounds Cape Chidley must pass over this reef and become well mixed by the time it reaches the mouth of Nachvak Fiord. Thus we have illustrated in this short section very striking thermal differences, giving a typical picture of the temperature conditions in a coastal current with the advance of summer in high latitudes.

The salinity profile (Fig. 9) off Nachvak shows as definite a correspondence between temperature and salinity as do the Sandwich Bay profiles though with the salinities somewhat lower especially near the bottom. This difference is the result of the shallower continental platform, for it is in small amounts only that water of greater salinity than 33 parts per thousand can overflow the continental shelf here. Probably the incorporatoin of Hudson Bay water, which rounds Cape Chidley just north of this sector, also lowers the salinity of this part of the current and may account for the fact that the isohaline of 32 per thousand lies at a greater depth than further south. At the outer edge of the current off Nachvak, the position of the isohaline of 34

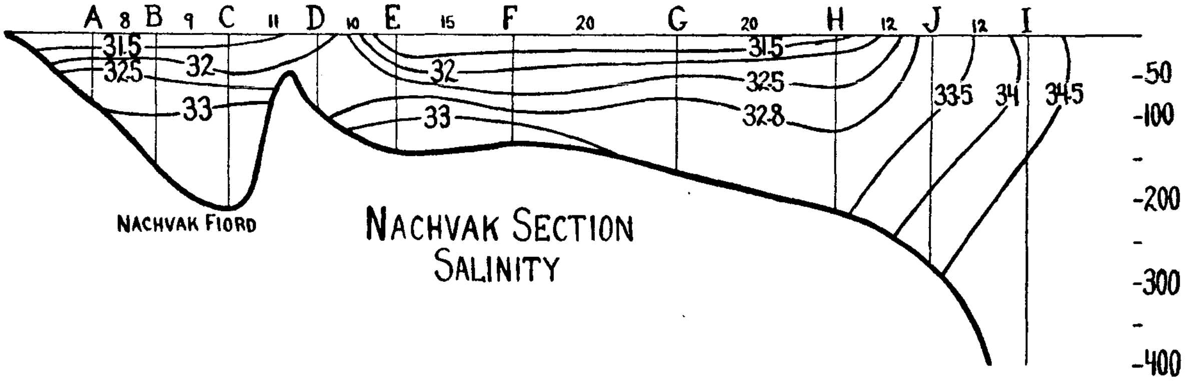

Figure 9. Salinity profile off Nachvak Bay, Labrador.

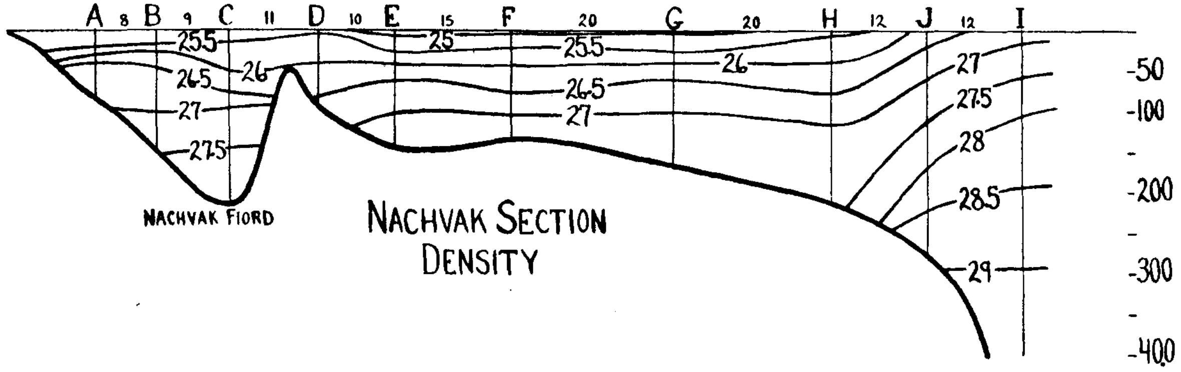

Figure 10. Density profile off Nachvak Bay, Labrador.

per thousand corresponds to the dividing line between water colder and warmer than 3°.

Although the distribution of salinity and temperature off Nachvak so much resembles that off Sandwich Bay, the combination of these factors in a density profile (Fig. 10) shows a different distribution of current-propelling forces. In this case all the curves are approximately horizontal as far out as station H, but show an abrupt tilting upward between stations H and I in the upper 250 meters of water. This indicates that the strength of the current occupied a band 25 miles wide, centering about 60 miles off shore.

As might be expected, on this profile the dynamic gradient all lay between the outer three stations, with a rise of 24.8 dynamic centimeters from station I to station H, but no appreciable gradient from station H in to the coast. At the surface between the two outer stations, the calculated average velocity is .67 knots. Between station J and H the average velocity is greater (.98 knots) and these figures can be considered reasonably accurate as at 300 meters the base plane between the two outer stations was dynamically level.

Horizontal projection of the dynamic gradients found from the Sandwich Bay and Nachvak profiles (Fig. 7) suggests a fanning out and slowing down of the current from north to south. Whether or not the narrowness and intensity of the current along the northern sector of the coast is a permanent phenomenon is an open question. The water coming out of Hudson Strait may be the cause.

Davis Strait Sector.

A set of observations made by Dr. Hjort in 1923 (Conseil International, 1925) across the narrows of Davis Strait show the arctic current just as it leaves Baffin Bay at a point 427 miles north of the "Chance's" most northern section. Six stations were taken, four of which were in line across the narrowest part of the Strait (Lat. 66° 31′ N). A fifth lay 120 miles further north and a sixth 30 miles south of the section. These last two can justly be interpolated since they are situated at about the same distance from the Greenland shore, and since the result fits well with the other four. This interpolated station is desirable because if four stations are used in the section there is left blank a distance of 86 miles between station 9, in the middle of the strait, and station 16 on the Greenland coastal bank. From the interpolated station, we can gain an idea of the bottom waters on the eastern side of the strait.

The temperature profile (Fig. 12) shows Arctic water (colder than 0°) filling the western half of the strait between the depths of 40 and 150 meters, and spreading out to the eastward in a thin tongue, just as was found further south by the "Chance." Pressed close to the Cumberland Island shore we see a core of −1.5° water overlying somewhat higher (+1°) bottom temperatures. At the surface the temperature warms from 1° on the western side to over 5° near the Greenland shore. The bottom temperatures are most striking, with a core of 4° water pressed close to the Greenland slope, while the Cumberland Island slope is bathed in water of a temperature between 0° and +.5°.

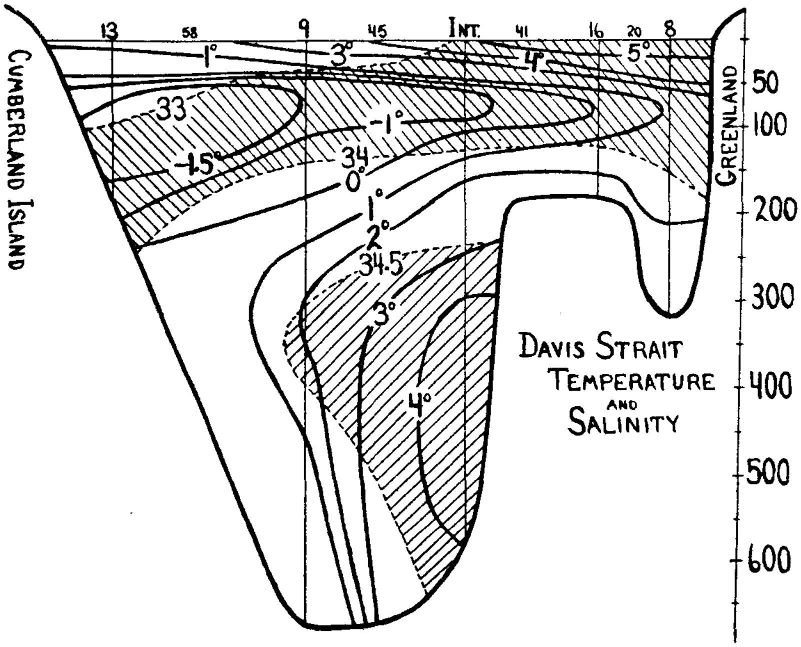

Figure 12. Profile of temperature and salinity across Davis Strait.

The salinity profile (Fig. 12) shows comparatively fresh water on both sides of the strait, notably on the western side, where the isohaline of 33 gives a very good picture of the limits of the strength of the south moving current. On the other hand the isohaline of 34.5

corresponds roughly to the isotherm of 3° and marks the limits of water of north Atlantic origin. This core of north Atlantic water, which would not have appeared at all in the section had the interpolated station been omitted, is a northern extension from the mass of oceanic water so noticeable outside the Larbrador Current in more southerly profiles. Its presence at an isolated station (# 14), 120 miles further north in Baffin Bay, proves that this core of comparatively warm water extends well up through the strait.

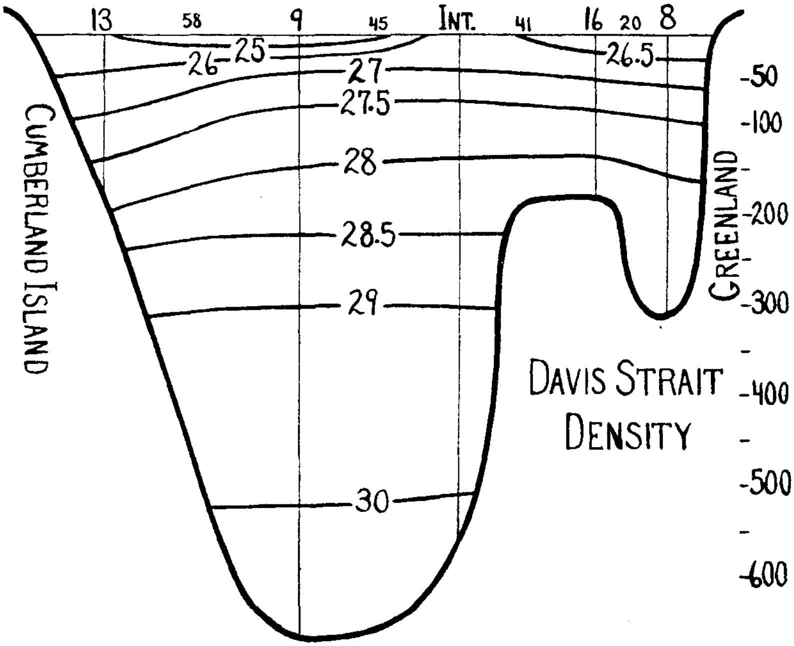

FIGURE 13. Density profile across Davis Strait derived from Figure 12.

The corresponding density profile (Fig. 13) shows that the water in the neighborhood of the interpolated station was without dynamic cause for movement at the time, for in this region the isopicnals are perfectly horizontal from the surface to the bottom. But west of this, the water of the upper 250 meters was being driven southward by a moderate dynamic gradient, with a rise of 14.9 dynamic centimeters in a distance of 58 miles between stations 9 and 13. At this latitude such a gradient is sufficient to give a velocity of about .2 miles per hour, or 5 miles a day, to the surface waters. But with the stations so far apart, and of very different depths, this calculation can be only an approximation.

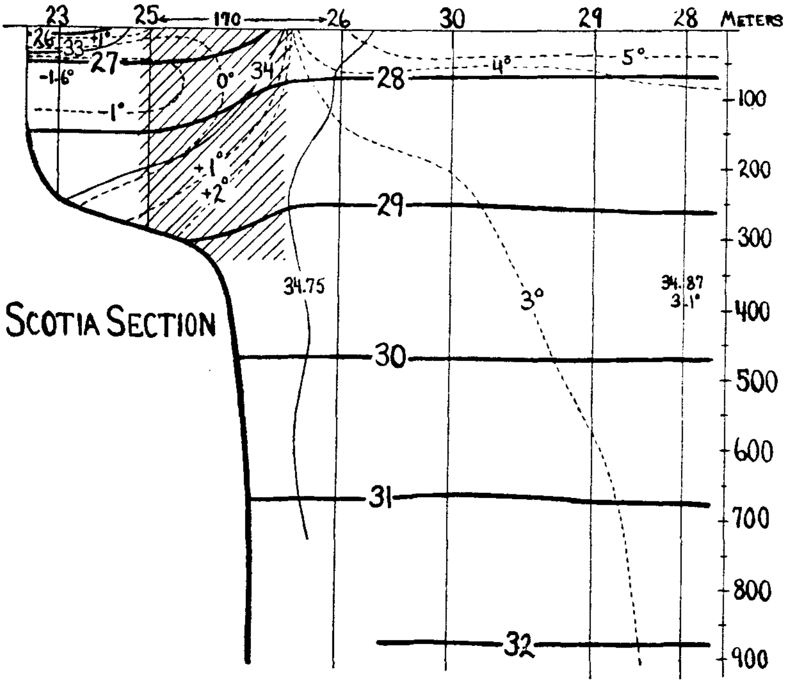
23
25
170
26
30
29
28
METERS
26
33
+1°
27
-1.6°
-1°
0°
34
28
4°
5°
100
200
+1°
+2°
29
300
34.75
3°
34.87
3.1°
400
SCOTIA SECTION
30
500
600
31
700
800
32
900

FIGURE 11. "Scotia" section across the lower Labrador Current showing temperature (broken curves), salinity (solid curves), and density (heavy curves).

In conclusion, this section shows that a southerly drift, to be interpreted as the chief source of the Labrador Current, is well developed even as far north as Davis Strait, and pressed closely against the western coast by the effect of the earth's rotation. It has at this northern point almost exactly the same temperatures and salinities as have been observed further south.

"SCOTIA" PROFILES.

All the hydrographic observations which have been discussed so far have been from stations situated in relatively shallow water on the continental shelf. Fortunately the "Scotia" made a line of stations which give us an indication of conditions prevailing near the center of the Labrador Sea. This section, crossing the southern part of the Labrador Current, and running out for 600 miles in a northeasterly direction from the mouth of White Bay, Newfoundland, is based on only six stations. Consequently it gives little more than a general picture of the waters of the region. But this condition is probably equally typical of the off shore waters further north where the "Chance" sections reached only the inner edge of the central mass of oceanic water which bounds the Labrador current on its outer side.

These temperature and salinity profiles, published in the report of the "Scotia" expedition (Matthews 1914), give an impression of great confusion, because of the bending and twisting of the isotherms and isohalines. But when the densities are calculated and plotted these unevennesses are found to counteract each other, and the density curves prove to be nearly horizontal both at their inshore and off-shore ends. Only in the narrow band of the Labrador Current itself does this profile show any dynamic cause for movement. The presence of the mass of motionless water inside the current is explained by the fact that this section was located where the current crosses the large, conspicuous indentation on the northeastern Newfoundland coast.

As in the "Chance" sections, the strength of the current on this "Scotia" profile lay just outside the edge of the continental shelf. Since stations 25 and 26 were 170 miles apart, they give no idea of the actual limits of the current, but we can safely say that it was narrower than this. The shaded area in Figure 11 shows the probable extent of the current at that time. The fact that there is no force indicated in this section to cause movement in the mass of the warmer, off-shore water deserves emphasis, for it indicates that this sector of the Labrador Current is not as wide as it has often been shown on current charts. Consequently, if any of the warmer water outside the edge of the continental shelf was moving southward, it was not being driven by any local dynamic force.

THE LABRADOR CURRENT.

The relative importance of the several possible sources of the Labrador Current must remain in doubt until more sub-surface observations have been made north of Davis Strait. Either it is an overflow of water from the Arctic Sea which comes south through Kennedy and the other northern channels, or it makes up in Baffin Bay, the necessary water being supplied by the indraft of comparatively warm, salt water along the west Greenland coast. Probably both sources work together to supply the Labrador Current with water, but the latter seems to be the more important because of the narrowness and shallowness of the northern channels.

The conditions in Baffin Bay seem highly favorable for the source of an icy current. We can suppose some such process as the following. Land drainage from the western shores and melting ice supply enough fresh water to account for the low salinity, while the cold winters and the protection of a covering of pack ice throughout the summer allow very little solar warming. Low salinity, in spite of low temperature, reduces the density of the water on the western side of the bay, while on the eastern side a higher density is maintained by the in-flow of North Atlantic water, *via* the West Greenland Current. This contrast is enough to set in operation dynamic forces to urge the western waters of the bay southward.

After the current has passed through Davis Strait, the observations at hand indicate that little change takes place in its volume or in its character until it reaches the Grand Banks, for we find that the cross section of the water colder than 0° measures 10.25 square miles as it passes through Davis Strait, 5.5 square miles off Nachvak, and 12.45 square miles off Sandwich Bay. At first sight the agreement does not seem to be particularly close, but we must remember that the Nachvak section was made three weeks later in the summer than the Sandwich Bay section. Since it was also 330 miles further north, we may add four weeks more to the time interval between the two sections, for at the rate of 12 miles a day it would take the water 28 days to move that distance south. Thus although we found more than twice as much water colder than 0° off Sandwich Bay than off Nachvak, the former represents a seven weeks' earlier stage in the life of the current, which will easily account for the difference, as we must expect a gradual dying off in volume and velocity as the summer advances. Again, it is probable that the current does not flow steadily, but in pulsations. Thus the Davis Strait section may have caught it at a maximum and the Nachvak section at a minimum. However this may be, the profiles prove that the Labrador Current is characterized for a distance of 750 miles, not only by a very similar temperature and salinity distribution, but also by a volume which remains of about the same order of magnitude from end to end.

After following the length of the Baffin Land coast, the current finds an opening to the right at Hudson Strait. The details of what happens within the latter are yet to be established. Some of the current urged by the rotational effect of the earth, undoubtedly flows in along the Baffin Land side and continues in a northwesterly direction some distance into Hudson Strait. Since the tidal range is great in this region (30 ft.), it is difficult to separate reports of current movements from those of tidal movements in the Straits. The five knot current reported on both sides of Resolution Island is surely chiefly tidal, for in the Labrador Current velocities never greatly exceed one knot for the summer months. But it is safe to say that the outflow from Hudson strait is considerable, since this is the first opening to the right for the counter-clockwise coastal current that has been reported from Husdon bay (Huntsman 1924, p. 275). Finally, it is probable that the circulation in Hudson Strait is dominated by two factors, first, the pumping action of the tides, and second, an actual difference in level between the surface of Hudson Bay and of the open sea off Hudson strait, caused by land drainage into the Bay. These forces result in a discharge into the Atlantic.

Continuing southward, the Labrador Current receives contributions along its inner edge from the streams flowing from the many lakes of Labrador. This addition, the summer temperature of which is between 5° and 8°, apparently supplies enough fresh water to the inside edge of the current to counteract the continual increase in salinity which must take place at the outer edge through mixture with the much more saline oceanic water beyond the continental shelf. A dynamic urge is thus supplied by streams along the whole length of the Labrador Coast. Off Sandwich Bay this coastal current is particularly noticeable, as at that point it has just been reinforced by the water coming out of Hamilton Inlet.

The characteristic temperature ($-1°$ to $-1.5°$) of the body of the current is maintained by two causes. First, the water layers are so stable vertically, as is shown by the density profiles, that solar warming is but little propagated downward by convection. Second, the many bergs and pack ice transported south from Baffin Bay are continually being melted, both by the stable surface film and by the deeper waters, a process that not only hinders the solar warming of the surface film, but also counteracts what little heat is transferred downward into the deeper waters by vertical currents. The shortness of the summer months and the fact that sea ice fringes the coast usually until July 1st, are also factors which aid in the thermal isolation of the heart of the current.

In the Straits of Belle Isle, as in Hudson Strait, the current suddenly finds an opening to the right. It has long been observed that each summer many bergs are carried in and grounded on the banks in the strait. Current measurements made in 1894 indicated little inflow of arctic water. But these were made at stations situated half way across the strait, and more recent work (Dawson 1907) revealed a dominant set inward on the north side of the strait and outward on the south side. These sets are weak, probably because of the narrowness of the strait. In all probability such currents would be better developed in Hudson Strait, the mouth of which is 60 miles across.

Having crossed the mouth of the Straits of Belle Isle, the bulk of the Labrador current sweeps on southward, following the edge of the continental shelf across White Bay and Notre Dame Bay, Newfoundland, until it reaches the Grand Banks. Here the greater volume follows the outer edge of the banks, only a small part creeping westward through the gully off Cape Race. Off the "tail" of the Banks, the Labrador Current water meets the much warmer and more saline mass of the Gulf Stream, where its fate has been so thoroughly traced by the work of the ice patrol ships as to need no discussion here.

Probably a fair average for the velocity of the Labrador Current throughout its whole length is 10 miles a day. Spring and summer observations tend to place this somewhat higher. By navigational reckoning the "Chance" found a drift of at least 20 miles a day in the narrow but strong current off Nachvak, and 14 miles per day off Sandwich Bay where the current was wider. The drift of the "Polaris" floe party during the early spring of 1873 averaged 14 miles a day from Cumberland Island to Hamilton Inlet. The early part of their drift through Davis Strait averaged slower than this (9 miles per day), probably owing to the congestion of the pack ice. But all these observations have been made after the spring thaw has given fresh impetus to the current, so that the average velocity for the whole year must be considerably less. In winter, with the dying out of the land drainage, the prevailing northwest wind probably does much to keep the current moving. As at that time of year the coastal strip is protected by a covering of pack ice, it would be interesting to know whether the strong winter winds also set some of the water outside the edge of the continental shelf moving southward.

In conclusion, the Labrador Current may be classed as a long narrow stream of coastal water, the character of which is maintained throughout its length by land drainage, favoring winds and melting ice. This coastal current is in sharp contrast with the warmer, more saline, water in contact with its outer edge. The latter is part of the homogeneous central core of the Labrador Sea, and is apparently affected by no physical factors strong enough to set up important dynamic movements.

BAYS AND FIORDS OF NEWFOUNDLAND AND LABRADOR.

During the cruise of the "Chance" several of the bays and fiords of the Newfoundland and Labrador coasts were visited. In some of these subsurface temperature and salinity observations were made, and their discussion is of interest because there exist no other data giving the exact physical conditions for these inlets, the deeper waters of which swarm with sea-life. The Labrador Current, sweeping along the coast, blocks off these bays from the effects of real oceanic water.

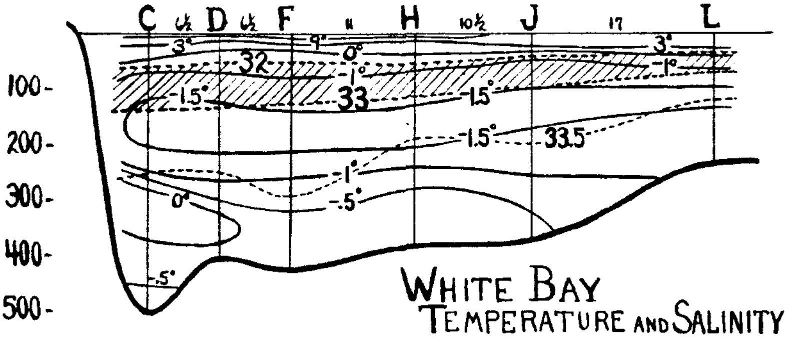

FIGURE 14. Profile of temperature (solid curves) and salinity (broken curves) running the length of White Bay, Newfoundland.

White Bay, Newfoundland (Fig. 7), about 45 miles long and averaging 9 miles in width, is a typical fiord, its mouth being about 210 meters deep while its head is in places as deep as 500 meters. The hydrographic profile (Fig. 14), made July 19 to 21, runs the length of the inlet and shows very much the same vertical distribution of temperature and salinity as existed outside in the Labrador Current. The isotherms and isopicnals, slanting towards the upper end of the bay, indicate that there the surface waters were slightly heaped up.

This probably was the result of the wind which blew steadily towards the head of the bay during the visit of the "Chance."

The stratum of coldest water (−1.5°), which is seen at a depth of from 100 to 200 meters, is an interesting feature of this inlet, for such a band of icy water is also found in the Gulf of St. Lawrence (Sandström 1919, p. 290) and everywhere over the Labrador continental shelf where the depth is great enough for its development. This coldest, intermediate water must have its origin during the winter, and is protected during the summer months from the sun by the exceedingly stable, fresher surface layer.

White Bay has a wide (14 mile) mouth, and there is every reason to expect that there is a certain amount of interchange between its waters and the Labrador Current outside. No dynamic evidence of this can be found from the "Chance" survey, but since there were several bergs well up the bay at the time of our visit, it seems fairly certain that the pumping action of the tides (Huntsman 1923) must effect a slow steady circulation of the upper water layers. Thus about the same temperature conditions are maintained inside the bay as out over the continental shelf.

Sandwich Bay in southern Labrador, is a broad shallow inlet with a shoal (6 meter) entrance, partly blocked by an island. There is little chance for salt water to enter the bay, while several good sized streams supply comparatively warm, fresh water in abundance during the summer months. As a result, the bay is practically a shallow lake and cannot be compared to the deep, fiord type of inlet.

Nachvak Bay in northern Labrador is a long, narrow, well developed fiord with steep mountainous shores. Here again the temperature—(Fig. 8) and salinity—(Fig. 9) distribution is very much the same as in the water outside over the continental shelf. On comparison with the White Bay profile, there is one noticeable difference, the lack of bottom warming in the northern fiord. The shallow, more contracted entrance of Nachvak probably accounts for this difference.

Seglek Bay is another deep fiord, which splits into three narrow branches about ten miles back from the coast. The "Chance" made two stations in this fiord, from which it is evident that conditions closely resemble those found in Nachvak. A more northern inlet, Ekortiarsuk, being mostly shallower (60 meters) contained no water colder than 1.1° C, or more saline than 32.3 parts per thousand.

A striking feature of all Labrador fiords is the surface film of nearly fresh water. During the summer there is much land drainage in this region and these waters flow out on the surface of the fiords. The result is that the surface is nearly barren of life and the beaches remind one much more of a lake than of a sea coast.

A comparison with the conditions found in the Norwegian fiords is instructive of the differences existing on the two sides of the Atlantic. The deeper parts of the Norwegian fiords are filled with much warmer (2°–6° C) and more saline (34 to 35 parts per thousand) water, and there exists no barrier such as the Labrador Current to cut them off from the oceanic waters, while the milder winters never greatly chill their depths (Nordgaard 1905).

PHYTOPLANKTON.

The surface plankton-hauls listed below (page 27) were made during July and August 1926 in the bays and coastal waters of the Labrador and Newfoundland coasts. The net was of # 20 silk with a diameter of half a meter at its mouth. The length of each tow was approximately one half hour. The richness of the hauls has been expressed as the number of cubic centimeters of living matter which settled in the jars after formalin had been added, and these figures are recorded along with the position of the stations on page 37.

With respect to the physical conditions of the water, the hauls can be classified into three groups: first, those made in the bays or fiords where the surface temperature varied from 6° to 8° and the salinity from about 25 parts per thousand to nearly fresh water; second, those made within about 10 miles of the coast where the temperature was still under the influence of the comparatively warm bay waters, but where the salinity was perhaps as high as 31 per thousand; and third, those made off shore in the Labrador Current proper where the surface temperatures were not over 3° and the salinity about 32 parts per thousand. But when the hauls themselves are examined they do not readily split up into these three divisions, showing that within a comparatively wide range, temperature and salinity do not control either the variety of the forms or their abundance in this region.

In the first four hauls made at the surface in or near White Bay, Newfoundland, we found almost no diatoms. The physical characteristics of the water at station 1 and 2 were apparently not very different from more southern stations where the diatoms were relatively abundant, so we can only suggest that these two hauls were made before the summer production occurred.

Three other hauls (8, 9, and 17) from Seglek Bay in northern Labrador, were also barren of diatoms. In this case the most reasonable explanation is that there the excessive land drainage flowing into the narrow fiord had produced such a stable surface layer that almost no vertical circulation could take place and thus few organisms were carried upward from the more saline deeper waters. It is probable that if our tows had been made at 10 meters, we would have escaped the too light surface films and secured a more reliable picture of the distribution of the diatoms along the Labrador coast.

The list of species identified in these 18 hauls is not complete, a few of the brakish water and bottom living forms having been omitted since they were observed only once or twice and were not typical of the coastal waters near the surface. The fact that the most northern fiords, near Cape Chidley, have a greater tidal range than the more southern bays, and are not so thoroughly blocked by the inshore current of low salinity, is reflected by the greater variety of their phytoplankton. For example, two oceanic forms, *Rhizosolenia styliformis* and *Coscinodiscus asteromphalus*, which occurred at Eclipse Harbor (14) and in Ecortiasuk Bay (15 and 16) are as good proof that offshore water comes into the northern inlets as are the higher salinities found there in the course of the hydrographic investigations. Three other oceanic forms were sparsely scattered well inshore, *Chaetoceras atlanticum*, *Ch. densum*, and *Thalassiothrix longissima*. Their presence near the land points to a continuous addition of oceanic water all along the outer edge of the Labrador Current and a thorough mixture of this salter water even in as far as the fresh coastal stream.

The "Chance" found 41 species of phytoplankton, and 3 infusorians fairly common in the Labrador Current waters. Of these, 24 species have been reported by Gran (1919) from the Gulf of St. Lawrence and 22 species are listed by Crawshay (1924) from the region of the Grand Banks. More recent work in Davis Strait (Gran 1929) shows only 22 of the "Chance" forms near the source of the current. An even more striking fact is that 30 of the "Chance's" list have been reported from the Gulf of Maine (Bigelow 1926) and 26 from the Norwegian fiords (Jorgensen 1905). All this shows that relatively few of the forms are definitely arctic. The fact that so many species common off Labrador are also common in the Gulf of Maine, and in the Norwegian fiords, is evidence that temperature and salinity are of minor importance in the distribution of these tiny organisms.

Although the "Chance" did not make quantitive measurements, it is evident from the volume of plankton obtained at each station, that nowhere was there an exceptional flowering of diatoms taking place. The average winter catch in the Gulf of Maine, before the great spring flowering, is in the neighborhood of 3 cc. Although the "Chance" used identical nets and towed for the same length of time (30 minutes) at each station, many of her catches did not greatly exceed this figure. In fact only at one station (5) could the Labrador diatoms be called really abundant. On the whole then, the surface microplankton of Labrador coastal waters was found to be relatively poor during July and August.

NOTES ON INDIVIDUAL SPECIES.

The species verified by Dr. Albert Mann are marked with asterisks.

DIATOMS.

1. *Acnanthes taeniata* GRUN. Occurred at four of our stations along the Labrador coast. [5, 6, 10, and 14].

2. **Bacterosira fragilis* GRAN. Was found at only two of the northern stations, 10 and 14.

3. **Biddulphia aurita* (LYNG.) BREB. Several examples of this form were observed in the sample from station 5, off Sandwich Bay.

4. **Chaetoceras atlanticum* CLEVE. Was found in moderate quantities in over half the samples examined (1, 2, 3, 6, 7, 11, 13, 14, and 16).

5. **Ch. boreale* BAILEY. Observed in our material in small numbers at only one station (6) 75 miles off Sandwich Bay.

6. **Ch. constrictum* GRAN. Found in considerable numbers at station 5.

7. **Ch. contortum* SCHUTT. Encountered once in abundance eight miles east of Cape Harrison (7) and in smaller numbers at 9 stations along the coast and in the most northern fiords (5, 6, 10, 11, 13, 14, 15, 16, and 18).

8. **Ch. criophilum* CAST. Abundant 14 miles off the mouth of Nachvak fiord with scattered specimens from 5 other coastal stations (5, 6, 13, 14, and 16).

9. **Ch. curvisitum* CLEVE. Dr. Mann lists this form as frequent from one station (5).

10. **Ch. debile* CLEVE. Although Gran (1929) lists this as "the commonest of all diatoms in August" in Davis Strait, it was only found by the "Chance" at the surface in three of the northern fiords, stations 11, 14, and 16.

11. **Ch. decipiens* CLEVE. Found abundantly 75 miles off the southern Labrador coast (no. 6) and again 14 miles east of Nachvak Fiord. It also occurred in small numbers at 9 other stations (1, 5, 7, 10, 11, 13, 14, 16, and 18).

12. **Ch. densum* CLEVE. Occurred in small numbers at 7 of the coastal stations (6, 7, 12, 13, 14, 16, and 18). This oceanic form can evidently only flourish where considerable oceanic water has been mixed with the fresher arctic stream.

13. **Ch. diadema* (EHR.) GRAN. Observed in abundance both off Sandwich

Bay (5) with endocysts and in Nachvak Fiord (11) and in small quantities at 6 other stations (7, 10, 13, 14, 16, and 18).

14. *Ch. *laciniosum* SCHUTT. Scarce at station 5 off Sandwich Bay.

15. *Ch. *sociale* LAUDER. Found in great numbers at station 16 (Ekortiasuk) and sparsely at four other stations (5, 11, 14, and 15).

16. *Ch. *teres* CLEVE. Scattered in small numbers through the catch of but four stations (5, 11, 13, and 16).

17. **Coscinodiscus asteromphalus* EHR. Identified by Dr. Mann in small numbers from station 16 (Ekortiasuk) and occurring even more sparsely at stations 14, 15 and 18).

18. **Detonula confervacea* (CLEVE) GRAN. A few examples found in the material from station 5 by Dr. Mann.

19. **Fragilaria islandica* GRUN. This form has not previously been reported from neighboring waters. It occurred commonly at station 5 off Sandwich Bay and more scarcely at stations 10, 11, 12, 13, 14, and 16.

20. **F. striatula* LYNG. In small numbers at stations 10, 12, 14.

21. **Lauderia glacialis* GRUN. Found sparsely scattered (with endocysts occasionally) at stations 5, 11, 12, 13, 14, 15, and 16.

22. **Melosira nummulus* MUEN. A few specimens were identified from three of our stations (6, 8, and 10).

23. **Navicula transitans* CLEVE. A few specimens found at station 16 by Dr. Mann.

24. *Nitzschia seriata* CLEVE. This form was very scarce being found in only 6 of our hauls (5, 7, 11, 13, 14, and 18).

25. **Rhizosolenia setigera* BRIGHT. Found, never in large numbers, at 9 of the stations (1, 2, 10, 11, 12, 13, 14, 16, and 18).

26. **Rh. styliformis* BRIGHT. A few specimens collected at three points (12, 14, and 16) along the northern Labrador coast.

27. **Sleletonema constatum* (GREV.) CLEVE. In great abundance at station 5, off Sandwich Bay, but not collected from any other locality.

28. *Synedra pulchella* KUTZ. A few specimens identified from station 16.

29. **Thalassiosira decipiens* (GRUN) JORG. Collected in very small numbers at stations 5, 6, and 14.

30. *Th. gravida* CLEVE. Observed occasionally at stations 11, 13, 14.

31. **Th. Nordenskjoldii* CLEVE. The dominant form at stations 11, 13 and 14 and occurring in small numbers at 6 other stations (5, 7, 10, 12, 16 and 18).

32. *Th. subtilis* (OST.) GRAN. Observed only just inside the mouth of Nachvak Fiord (11).

33. **Thalassiothrix longissima* CLEVE ET GRUN. This oceanic species occurred at 7 coastal stations (5, 6, 11, 12, 13, 14, and 15).

34. **Th. nitzschioides* GRUN. Common at three localities (5, 10, and 16).

SILICOFLAGELLATES.

35. *Distephanus speculum* (EHR). A few specimens were identified from four localities (9, 10, 11, 13).

PERIDINIANS.

36. *Ceratium arcticum* BAILEY. This arctic species was found in all the samples. It completely dominated the hauls from stations 3, 4, and 13; was abundant at five other localities (6, 8, 9, 12, and 18), and occurred in moderate numbers in the other samples. All the samples were carefully examined for *C. longipes*, but no specimen was seen which could not be called *C. arcticum*.

37. *Dinophysis norvegica* CLAP. et LACHM. Throughout the whole area at almost every station, but not common at any one locality.

38. *Peridinium depressum* BAILEY. Found abundantly at stations 6, 8, 9, and 17 and in smaller numbers at all but four of the remaining stations.

39. *P. ovatum* POUCHET. In moderate numbers in all but four of the hauls.

40. *P. pellucidum* (BERGH.) SCHUTT. Found in considerable numbers at station 1 and more sparsely at all but 4 of the other stations.

INFUSORIANS.

41. *Cyttarocylis denticulata* EHR. Collected at all but one station. At station 12, off Nachvak Bay, this form was abundant enough to dominate the catch.

42. *Ptychocylis urnula* CLAP. et LACHM. In small numbers at all but two of the northern stations (15 and 18).

43. *Tintinnopsis sp.* Possibly *Tintinnopsis urnula*. This form was dominant at station 10 and occurred at 4 other localities (8, 9, 11, and 15).

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LIST OF HYDROGRAPHIC STATIONS.

Cape Race Section.

Station	Latitude	Longitude	Depth	Temperature	Salinity	Density
A	46° 25′ 30″	52° 22′	0	8.05°	31.80	24.79
			10	8.0°	32.00	24.95
			40	3.3°	32.80	26.15
			75	−0.22°	33.16	26.65
B	46° 32′	52° 34′	0	8.1°	31.40	24.47
			20	7.2°	32.05	25.11
			45	−0.85°	33.00	26.56
			90	−1.36°	33.13	26.67
			200	−1.12°	33.40	26.88
C	46° 38′	52° 44′	0	7.5°	31.36	24.52
			20	7.6°	31.98	25.00
			50	−0.63°	33.01	26.55
			70	−1.18°	33.08	26.62
			125	−0.62°	33.15	26.67
			200	−1.14°	33.36	26.85
D	46° 45′	52° 54′	0	6.72°	31.28	24.57
			20	5.9°	31.80	25.17
			35	0.7°	32.60	26.16
			60	−1.08°	33.01	26.56

White Bay.

Station	Latitude	Longitude	Depth	Temperature	Salinity	Density
A	50° 13′ 30″	56° 7′ 30″	0	12.89°	—	—
			20	4.2°	29.94	23.77
			50	−0.67°	32.29	25.97
			100	−1.44°	33.03	26.59
			200	−1.32°	—	—
			300	−0.26°	33.78	27.17
B	50° 6′	56° 17′	0	9.44°	—	—
			20	0.28°	31.29	25.12
			50	−1.25°	32.56	26.20
			100	−1.38°	32.95	26.53
			200	−1.50°	33.24	26.77
			300	−0.42°	—	—
C	49° 47′	56° 42′	0	10.56°	28.06	21.54
			20	3.0°	30.28	24.32
			50	−0.05°	31.56	25.36
			100	−1.38°	32.79	26.40
			200	−1.36°	33.21	26.73
			280	0°	33.68	27.06
			480	−0.56°	33.91	27.28

Station	Latitude	Longitude	Depth	Temperature	Salinity	Density
D	49° 53′	56° 37′	0	9.89°	——	——
			20	2.5°	30.34	24.37
			50	−0.53°	31.92	25.65
			100	−0.62°	32.83	26.38
			200	−0.74°	33.21	26.68
			300	−0.37°	33.77	27.13
			400	−0.10°	33.78	27.15
E	49° 57′	56° 39′	0	10.17°	——	——
			20	2.57°	29.79	23.49
			50	−1.0°	32.36	26.04
			100	−1.30°	32.81	26.41
			200	−1.83°	33.19	26.73
			295	−0.41°	33.68	27.09
F	49° 59′	56° 32′	0	10.5°	——	——
			20	3.55°	29.92	23.83
			50	−0.54°	31.96	25.70
			100	−1.22°	32.88	26.46
			200	−1.58°	33.13	26.68
			300	−0.56°	——	——
			400	−0.27°	33.69	27.08
G	50° 0′	56° 35′	0	9.39°	——	——
			20	4.4°	30.07	23.68
			50	−0.63°	31.91	25.66
			100	−1.42°	32.79	26.40
			200	−1.54°	33.24	26.77
			300	−0.10°	33.62	27.02
H	50° 8′	56° 22′	0	9.83°	——	——
			20	3.7°	30.00	23.70
			60	−1.05°	——	——
			160	−1.5°	33.32	26.83
			260	−1.18°	33.79	27.20
			360	−0.1°	33.69	27.07
I	50° 10′	56° 27′	0	8.56°	——	——
			20	4.37°	30.32	23.89
			60	−1.14°	32.39	25.99
			140	−1.54°	33.28	26.79
			240	−1.0°	33.53	26.98
			340	−0.18°	33.75	27.13
J	50° 16′ 30″	56° 13′	0	7.95°	——	——
			20	5.1°	30.27	23.96
			60	−1.40°	32.68	26.31
			140	−1.55°	33.20	26.33
			240	−1.1°	33.68	27.11
			340	−0.10°	33.97	27.30

Station	Latitude	Longitude	Depth	Temperature	Salinity	Density
K	50° 19′ 30″	56° 19′ 30″	0	8.67°	———	———
			20	4.5°	30.43	24.13
			60	−0.92°	32.17	25.87
			120	−1.6°	33.08	26.64
			220	−1.2°	33.63	27.07
			320	−0.2°	33.98	27.32
L	50° 32′	55° 58′ 30″	0	8.83°	———	———
			10	6.44°	30.66	24.22
			20	3.57°	30.97	24.65
			50	−1.42°	32.91	26.49
			100	−1.7°	33.39	26.89
			200	−1.3°	33.74	27.16

SANDWICH BAY SECTION.

A	55° 33′	54° 12′	0	6.3°	34.32	27.00
			20	6.1°	34.29	27.01
			50	3.78°	34.67	27.57
			100	4.32°	34.52	27.43
			250	3.80°	34.76	27.62
			300	3.52°	34.85	27.74
B	55° 10′	54° 45′	0	1.0°	32.66	26.18
			20	0.46°	33.06	26.54
			50	−1.22°	33.37	26.77
			100	−1.0°	33.86	27.24
			200	−0.3°	33.84	27.20
			250	−0.4°	33.70	27.10
C	54° 54′	55° 8′	0	1.7°	31.94	25.57
			20	0.8°	32.10	25.77
			50	−1.8°	33.04	26.61
			100	−0.87°	33.57	27.01
			200	−0.58°	33.76	27.15
D	54° 38′	55° 30′ 30″	0	3.4°	31.96	25.46
			20	3.4°	31.97	25.47
			50	−0.3°	32.48	26.12
			100	−1.1°	33.82	27.23
			150	−1.0°	33.86	27.24
E	54° 28′ 30″	55° 43′	0	2.2°	31.46	25.27
			20	1.1°	31.95	25.67
			50	−1.3°	32.83	26.38
			100	−1.6°	33.15	26.64
			200	−1.4°	33.32	26.78
			250	−0.9°	———	———

Station	Latitude	Longitude	Depth	Temperature	Salinity	Density
F	54° 19′ 30″	55° 55′	0	2.9°	30.99	24.72
			20	−0.8°	32.28	25.97
			50	−1.6°	32.93	26.51
			150	−1.2°	——	——
			250	−0.3°	——	——
			300	0.1°	34.32	27.57
			400	1.42°	34.50	27.64
G	54° 8′	56° 11′	0	3.3°	30.87	24.60
			20	2.5°	31.41	25.09
			50	−1.6°	32.92	26.51
			100	−1.6°	33.07	26.63
			125	−1.56°	33.14	26.69
			150	−1.58°	33.24	26.76
H	53° 58′	56° 27′	0	5.4°	29.45	23.32
			20	−1.0°	32.40	26.07
			50	−1.8°	32.96	26.55
			125	−1.65°	33.00	26.58
I	53° 55′	56° 43′	0	8.2°	25.12	19.55
			10	5.0°	——	——
			20	3.2°	28.19	22.48
			55	−1.55°	32.81	26.41
				NACHVAK SECTION.		
A	59° 4′	63° 56′ 30″	0	6.6°	——	——
			10	−0.3°	31.37	25.22
			20	−0.95°	31.67	25.48
			50	−1.6°	32.96	26.55
			70	−1.6°	32.94	26.52
B	59° 1′ 15″	63° 42′	0	4.8°	——	——
			10	−0.48°	31.37	25.23
			25	−0.9°	32.33	26.01
			75	−1.63°	32.80	26.40
			150	−1.8°	33.20	26.72
C	59° 4′	63° 26′	0	4.1°	——	——
			10	−0.3°	31.52	25.34
			20	−0.21°	31.76	25.53
			50	−0.02°	32.02	25.73
			100	−1.83°	33.12	26.67
			200	−1.8°	33.20	25.73
D	59° 11′ 30″	63° 13′	0	2.4°	31.86	25.46
			10	0.8°	31.89	25.57
			20	0.84°	32.01	25.67
			50	0.2°	——	——
			100	−0.45°	32.41	26.06

Station	Latitude	Longitude	Depth	Temperature	Salinity	Density
E	59° 17′ 30″	62° 57′ 30″	0	4.0°	31.30	24.88
			20	1.7°	31.45	25.18
			50	−1.25°	32.41	26.09
			100	−1.2°	33.02	26.58
			125	−1.2°	33.07	26.61
F	59° 25′	62° 32′ 30″	0	4.5°	31.32	24.85
			20	0.6°	31.54	25.30
			50	−0.2°	32.11	25.80
			125	−0.88°	33.21	26.72
G	59° 36′	61° 58′	0	3.45°	31.35	24.97
			20	2.38	31.50	25.16
			50	−0.1°	32.54	26.14
			100	−0.8°	32.96	26.51
			150	−0.8°	32.99	26.54
H	59° 46′ 30″	61° 25′	0	3.3°	31.35	24.97
			20	−0.1°	32.00	25.71
			50	−0.4°	32.24	25.92
			100	−0.68°	32.71	26.31
			125	−0.6°	32.82	26.39
			150	−0.6°	32.90	26.46
			200	−0.45°	32.98	26.52
J	59° 53′	61° 6′	0	2.3°	32.89	26.29
			20	0.1°	——	——
			50	−0.1°	33.22	26.70
			100	0.15°	33.50	26.91
			150	0.7°	33.78	27.11
			200	1.3°	34.07	27.30
			250	1.8°	34.30	27.50
I	59° 59′	60° 45′	0	5.3°	34.10	26.95
			20	5.15°	34.02	26.92
			50	2.18°	34.04	27.21
			100	2.37°	34.42	27.49
			150	2.6°	34.50	27.54
			200	3.5°	34.67	27.59
			250	4.2°	34.74	27.57
			300	4.8°	34.78	27.55
			400	4.9°	34.83	27.56
			FIORDS.			
	59° 54′	64° 23′	0	1.9°	31.58	25.27
			10	1.5°	31.80	25.47
	EKORTIARSUK		20	1.1°	32.08	25.72
			50	1.1°	32.27	25.87
	SEGLEK, North Arm		0	6.6°	——	——
			40	−1.3°	31.76	25.56

LIST OF SURFACE PLANKTON HAULS

Station	Latitude	Longitude	Date	Volume	Locality
1	49° 30′	53° 15′	July 16	5	14 miles SSW of Funk Island
2	49° 55′	54° 50′	July 16	18	Off Notre Dame Bay
3	49° 53′	56° 37′	July 19	21	Station D, White Bay
4	49° 59′	56° 32′	July 20	30	Station F, White Bay
5	53° 53′	56° 40′	July 25	105	Just off Sandwich Bay
6	54° 50′	55° 10′	July 27	37	75 miles east of Sandwich Bay
7	55° 2′	57° 50′	August 3	7	8 miles east of Cape Harrison
8	58° 33′	63° 48′	August 11	22	North arm of Seglek Bay
9	58° 33′	63° 36′	August 11	3	Seglek Bay
10	59° 3′	63° 52′	August 18	3	Nachvak Bay
11	59° 1′	63° 42′	August 18	30	Station B, Nachvak Bay
12	59° 14′	63° 0′	August 18	12	14 miles east of Nachvak Bay
13	59° 37′	61° 15′	August 20	15	70 miles east of Nachvak Bay
14	59° 51′	64° 9	August 27	18	Off Eclipse Harbor
15	59° 45′	64° 46′	August 29	7	Near head of Ekortiarsuk Bay
16	59° 53′	64° 20′	August 31	45	Station A, Ekortiarsuk Bay
17	58° 33′	63° 48′	August 10	22	North Arm of Seglek Bay
18	58° 40′	62° 38′	September 3	18	5 miles east of Seglek Bay

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THE NEW WORLD SPECIES OF THE GENUS SOLENOPSIS (HYMENOP. FORMICIDAE).

By William S. Creighton.

THE NEW WORLD SPECIES OF THE GENUS SOLENOPSIS (HYMENOP. FORMICIDAE).

BY WILLIAM S. CREIGHTON.[1]

PART 1.

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Carlo Emery once characterized the genus Solenopsis as the *crux myrmecologorum*. That the term is apt no one who has experienced the difficulties of the group will deny, least of all the author who, at the end of three years of study, still finds the "cross" a heavy burden. At the inception of the work there was little thought that it would assume its present proportions. Its growth has necessitated constant restriction but, even after lopping off all the Old World species, the bulk of the remainder is extremely unwieldy. It therefore seemed advisable, since the monograph can be divided into two parts, to publish these separately. The present paper presents an analysis of the species in the subgenera *Solenopsis*, *Euophthalma*, *Diagyne* and *Oedaleocerus*. The subgenus *Diplorhoptrum* will be dealt with in a subsequent publication.

The author is glad to avail himself of the opportunity to acknowledge here the help which he has received. It is impossible to thank Dr. W. M. Wheeler in adequate measure for his support. His generosity in allowing me access to his collection, his counsel and most of all the inspiration which has been derived from association with him have made the present work possible. I also wish to express my sincere thanks to Dr. F. Santschi of Kairouan, Tunis, who generously sent me many specimens of South American forms, without which this work would have been seriously handicapped. My thanks are also due Dr. W. M. Mann and Dr. M. R. Smith for the gift of specimens. I wish also to express my gratitude to my friends Dr. C. T. Van Aller and Mr. H. P. Löding for material sent me and further for their coöperation while I was engaged in field work near Mobile, Ala.

The genus Solenopsis is at present represented by approximately one hundred species, two-thirds of which occur in the New World. In the remaining third, six species are known from the Ethiopian region, eleven from the lands adjacent to the Mediterranean, eight are Asiatic, five Polynesian, while a single species occurs in Australia and another in Tasmania. The Asiatic species, particularly those from China, show a strong relationship to European forms and it is probable that they represent variants of species whose range extends entirely across Europe and Asia.

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[1] Contributions from the Entomological Laboratory of Harvard University, No. 326.

The distribution of the species in the New World gives a great preponderance of South American forms. Of the sixty-odd species approximately twenty per cent are peculiar to North America, twenty per cent are indeterminate occurring either in the Antilles or Central America or ranging over both continents, and sixty per cent are indigenous to South America. It may be argued that the numerical superiority of the South American forms proves that continent to be the center of distribution of the genus. This view is, however, open to question. When we consider the amount of each continent which is included in the tropics the difference in the number of forms becomes more explicable. In addition the two species, *geminata* and *globularia*, whose range includes the Antilles, North and South America are unquestionably of Antillean origin. Furthermore the existence of a number of "couples" of closely related species one member of which occurs in North America while the other is native to South America points to an Antillean origin for the group.

The genus Solenopsis has always been extraordinarily intractable in the matter of subdivision. The older workers were satisfied with a very loose split into two groups, one of which was designated the "*geminata* group" while the other was usually referred to simply as the "small species." The first subgenus to be set up was *Synsolenopsis*. This was erected by Forel in 1918 on the basis of a single, aberrant female. This insect, to which he gave the specific name, *bruchi*, differed principally in the presence of spines on the epinotum and was thought by Forel to be a parasitic form. In 1923 Santschi described the worker of an ant which he called *Synsolenopsis photophila*. This insect also had spines on the epinotum and this character was the basis for its inclusion in the subgenus *Synsolenopsis*. In the same paper Santschi changed the name of *bruchi* to *bruchiella*, since the former name was preoccupied. In the last analysis the confirmation of Santschi's supposition depends upon the discovery of the female of *photophila*, but the presence of epinotal spines in both forms certainly appears to indicate relationship. Forel's description of *bruchiella* is rather meager, but in the case of *photophila* Santschi has not only given a very complete account of the worker but has figured it as well. A study of these has convinced me that the differences shown by this insect are more than sufficient for generic status. In addition to the presence of epinotal spines the pronotum and mesonotum are finely striate and the eyes are very large and reniform. There is no species in the genus Solenopsis which shows either of the first two characters and only one, *S. macrops,* in which the eyes are at all comparable to those of *photophila.* The eyes of *macrops,* however, although large and elongate, are suboval and borne far forward with the anterior border very close to the insertion of the mandibles and well toward the ventral surface of the head. In *photophila* the eyes are further removed from the base of the mandibles and more dorsal in position. I therefore propose to regard Synsolenopsis as a separate genus.

In 1923 Santschi erected the subgenus *Diagyne* for *S. succinea.* It had long been known that the very small females of this insect possess ten-jointed antennae in distinction to those of all the other species in the genus in which the antennae are of eleven joints. In addition the clypeus of both the female and worker of *succinea* is unique in that it possesses neither carinae nor teeth and is quite different in general configuration from that of any other member of the genus. This condition is approached by several species, perhaps most closely by *S. xyloni* subsp. *amblychila,* but in such cases the carinae are always present although the carinal teeth may have disappeared. The subgenus *Diagyne* is therefore quite distinct although it undoubtedly belongs in the genus Solenopsis to which, as Santschi has pointed out, it bears much the same relation as does the subgenus *Decacrema* to Crematogaster.

Neither of the divisions proposed above helped to reduce the species in the genus Solenopsis to groups of a more convenient size. The first definite step in this direction was made by Santschi in 1924. I give below a translation of the note which appeared in his publication of that date:

"The strongly dimorphic Solenopsis and generally those of large size belong to the subgenus *Solenopsis* s. str. while the small monomorphic species ought to be united in the subgenus *Diplorhoptrum,* Mayr."

While I heartily commend the spirit of Santschi's proposal I cannot agree to the proposition itself since it is morphologically unsound. In the first place it is impossible to draw any hard and fast line on the basis of polymorphism alone since the members of the genus, taken collectively, present a very complete series of stages from well-developed polymorphism to monomorphism. Secondly, there are certain constant structural features which are in many cases entirely independent of size, that permit the establishment of clearly delimited subgenera within the group. These differences may be most readily presented in the form of a key after which the subgenera will be more fully characterized.

1. Female with ten-jointed antennae; the clypeus in both female and worker without teeth or carinae....................subgen. *Diagyne*
   Female with eleven-jointed antennae; the clypeus in both female and worker aways with carinae and usually with teeth................2
2. The second and usually the third funicular joint of the worker at least one and one-half times as long as broad........subgen. *Solenopsis*
   Second and third funicular joints of the worker at most very slightly longer than broad, usually much broader than long...........3
3. Eyes of the worker large, usually composed of twenty or more facets, or if fewer are present (never less than twelve) the postpetiole is greatly dilated............................subgen. *Euophthalma*
   Eyes of the worker small, sometimes absent, composed at most of fifteen facets, usually not more than ten.........................4
4. Antennal scapes of the female greatly thickened and sharply bent; the postpetiole about two and one half times as wide as the node of the petiole....................................subgen. *Oedaleocerus*
   Antennal scapes of the female not unduly thickened and gradually curved; the postpetiole not more than one and one-half times as wide as the node of the petiole................subgen. *Diplorhoptrum*

SUBGENUS DIAGYNE SANTSCHI.

The characteristics of this subgenus have been given on the preceding page. It is represented by a single species, *S. succinea* Emery.

SUBGENUS SOLENOPSIS (FABR.).

Strongly polymorphic species of large size. Eyes large, of thirty or more facets. The second and third funicular joints of the major worker always twice as long as broad, usually less elongate in the minor but in this caste the second joint never less than one and one-half times as long as broad and the third joint at least one and one-quarter times as long as broad. Female at least 6.0 mm. in length, usually more. Its thorax bulky only slightly narrower than the head. Type of the subgenus: *Atta geminata* Fabr.

The subgenus is made up of the following species: *bondari*, *gayi*, *geminata*, *tridens*, *saevissima* and *xyloni*.

SUBGENUS EUOPHTHALMA, SUBGEN. NOV.

Feebly polymorphic or monomorphic species not exceeding 2.7 mm. in length. The head of the worker usually quadrate. Eyes ordinarily of twenty or more facets. Second and third funicular joints very slightly longer than broad or as broad as long. Thorax usually unimpressed at the mesoepinotal suture, the suture sometimes deeply incised. Type of the subgenus: *Myrmica globularia* Fred. Smith.

It is unfortunate that it is necessary to designate *globularia* as the type of the subgenus since in some respects it is far from typical. Its extraordinary postpetiole is quite unique and moreover two of its variants are exceptional in that the eyes of the smaller workers have only twelve facets. In other regards, however, it is fairly representative. So little is known of the females of this group that no generalization concerning this caste can be made. The following species are included in the subgenus *Euophthalma: andina, globularia, huachucana, macrops, metanotalis, occulata, nigella, picquarti, schilleri* and *silvestrii.*

SUBGENUS OEDALEOCERUS, SUBGEN. NOV.

The antennal scape of the female greatly thickened, its greatest diameter at the basal third, abruptly bent at that point. Postpetiole very wide, approximately two and one-half times as wide as the node of the petiole. Worker with the same characteristics as that of the following subgenus. Type of the subgenus: *Solenopsis angulata* Emery.

The antennal modifications of the female in *S. angulata* are of such an extraordinary nature that it stands quite alone in the genus and should have subgeneric rank. It is unfortunate that these characters do not occur in the worker since their absence in that caste necessitates the examination of the female for certain subgeneric recognition.

SUBGENUS DIPLORHOPTRUM (MAYR).

Species of variable size, monomorphic to highly polymorphic. Eyes small, never more than fifteen facets present and usually less, rarely without eyes. Second and third funicular joints as broad as long or broader than long. Impression of the dorsum of the thorax at the mesoepinotal suture variable. Size and character of the female variable. Type of the subgenus: *Formica fugax* Latr.

Despite the heterogeneous nature of the species composing this subgenus the group is exceedingly compact. Although the extremes differ greatly in size there is such a complete series of transitional forms that further subgeneric division appears to be impossible. The species may, however, be separated into a number of groups in which the characteristics are fairly consistent. The means of separation cannot be given here but will be presented in a future publication. The groups and the species which compose them are as follows:

Westwoodi Group.

gallardoi, germaini, iheringi, krockowi, orestes, patagonica, pilosula, salina, tetracantha, wasmanni, westwoodi.

Laeviceps Group.

albidula, brevipes, goeldi, laeviceps, leptanilloides, manni, (sp. nov.), *pergandei, pygmaea.*

Molesta Group.

castor, conjurata, corticalis, hammari, helena, molesta, pollux, subtilis texana, truncorum.

Azteca Group.

azteca, brevicornis.

Basalis-tenuis Group.

altinodis, basalis, decipiens, franki, clytemnestra, hayemi, hermione, latastei, picea, picta, schmalzi, spei, stricta, subadpressa, tenuis.

The above list contains several forms hitherto considered subspecies or varieties. There are also a number of previously described species missing, which have either been synonymized or dropped because of inadequate description. The reasons for these changes will be fully discussed in the section dealing with the subgenus *Diplorhoptrum.*

SUBGENUS SOLENOPSIS (FABRICIUS).

The characteristics of the members of this subgenus have been given on the preceding page. Since there has been a considerable amount of alteration within the group it seems advisable to begin the account of the subgenus with the arrangement of forms followed in this work:

S. (S.) *bondari* Santschi

S. (S.) *gayi* (Spinola)
    var. *fazi* Santschi
    subsp. *bruesi,* subsp. nov.

S. (S.) *geminata* (Fabr.)
= *geminata* var. *nigra* Forel
subsp. *eduardi* (Forel).
= *eduardi* var. *perversa* Santschi
subsp. *galapageia* Wheeler
subsp. *medusa* Mann.
= *eduardi* var. *bahiaensis* Santschi
subsp. *rufa* (Jerdon)
= *geminata* var. *diabola* Wheeler

S. (S.) *tridens* Forel
var. *substituta* Santschi

S. (S.) *saevissima* (Fred. Smith)
= *moelleri* var. *gracilior* Forel
var. *moelleri* (Forel)
= var. *incrassata* Forel
= var. *morosa* Santschi
= var. *pylades* Forel (Santschi emend.)
var. *perfida* Santschi
var. *richteri* Forel
= var. *tricuspis* Forel
var. *quinquecuspis* Forel
subsp. *interrupta* Santschi
= var. *macdonaghi* Santschi
subsp. *electra* Forel
var. *wagneri* Santschi

S. (S.) *xyloni* (MacCook)
var. *maniosa* (Wheeler)
subsp. *aurea* (Wheeler)
subsp. *amblychila* (Wheeler)

The variety *lehmann-nitschei* Santschi, has been dropped as impossible of certain determination.

Key to the species in the subgenus Solenopsis

1. Mandibles of the majors and larger medias abruptly curved, the teeth aborted or absent . . . . *geminata*
Mandibles in all the worker castes gradually curved with three or four well-developed teeth 2
2. The antennal scape of the minor worker surpassing the occipital border 3

The antennal scape of the minor worker failing to reach the occipital border by a distance about equal to its greatest thickness..........5

3. Pronotum with strongly-marked humeral angles; two blunt, keel-like projections present at the junction of the basal and declivious faces of the epinotum..*tridens*

Pronotum without well-marked humeral angles; no projections at the junction of the basal and declivious faces of the epinotum..........4

4. Postpetiole in all the worker castes globose when seen from above..*bondari*

Postpetiole in the majors and medias transverse when seen from above..*saevissima*

5. Major workers with cordate heads and 3-toothed mandibles (southern U. S. and Mex.)..*xyloni*

Major workers with quadrate heads and 4-toothed mandibles (Chile and Peru)..*gayi*

S. (Solenopsis) bondari Santschi.

S. bondari, Santschi, Bull. Soc. Ent. Belg. Vol. 65, p. 236 (1925) ☿.

Worker: Length 3.0–3.5 mm. (Plate IV, figs. 7, 8, 9.)

Head, exclusive of the mandibles, as broad as long, the sides moderately convex, rather strongly narrowed at the mandibles, the occipital angles much rounded, the occiput with a narrow and feeble median impression. Clypeus strongly projecting, the clypeal teeth very divergent, each terminating in a very short tooth which is scarcely more than a blunt angle, lateral denticles reduced to feeble sinuousities on the clypeal margin, a very short and exceedingly blunt median tooth is present between the carinal teeth. Mandibles rather slender the external border feebly convex, the masticatory border armed with four large teeth. The antennal scape in repose surpasses the occipital border by an amount equal to its greatest thickness. First funicular joint somewhat longer than the following two together, joints 2–7 all longer than broad; club stout, the terminal joint approximately twice as long as the penultimate. Eyes oval, composed of thirty facets, separated from the insertion of the mandible by a distance twice as great as their maximum diameter.

Pronotum seen from above with the humeral angles much rounded. Seen in profile the promesonotum is feebly and evenly convex, the mesoepinotal suture is broad and very deep. Epinotum slightly lower than the adjacent portion of the mesonotum, its basal face virtually straight, not sharply separated from the very short declivious face. Node of the petiole in profile low, thick and obtuse, scarcely higher than thick and slightly inclined backward, the peduncle as long or a little longer than the base of the node, its ventral tooth reduced to a tiny denticle or absent. Postpetiole in profile slightly thicker than high, obtusely rounded above, with a prominent, angular ventral projection. Seen from above the node of the petiole is four-fifths as wide as the globose postpetiole. Anterior edge of the first gastric segment truncate.

Color clear golden yellow, the abdominal segments bordered with yellowish brown. The integument has a peculiar, translucent appearance which causes the punctures to appear in sharp contrast, although they are not very large and only moderately numerous. Hairs long, erect and moderately abundant. Mandibles feebly striate, meso- and metapleurae and the declivious face of the epinotum with very fine striae, the rest smooth and shining.

Redescribed from two cotypes sent me through the kindness of Dr. Santschi. The sexual forms are unknown.

Localities: Brazil, Bahia. (Type loc.) (Bondar.)
British Guiana, Kartabo. (W. M. Wheeler.)

Bondari is very closely related to *saevissima* but may be easily distinguished by its much smaller eyes and globose postpetiole. It is also much less polymorphic. I was at first inclined to doubt that the fifteen specimens upon which this species was founded showed the entire size range of the species. However, field notes given me by Dr. Wheeler indicate that this is the case. A colony consisting of about one hundred workers was found by him in a rotten log in heavy forest in British Guiana. Although the insects varied slightly in size the difference is negligible when compared with that shown by the workers of *saevissima*. The situation of the nest was also at variance with the usual nesting sites of *saevissima*, since *saevissima* ordinarily nests in open, sunny places. The Kartabo material is slightly darker in coloration than the two cotypes which I examined and the clypeal teeth appear to be a little longer. It may represent a separate variety but, until there is more material available for comparison, I prefer to include it with the typical form.

S. GAYI (SPINOLA).

Introduction.

S. gayi is the only representative of the subgenus Solenopsis which occurs in the southern portions of the Andean coastal plain. The older records, without exception, come from an area in central Chile roughly six hundred miles in length, of which Valpariso may be taken as the center. More recent investigations, however, have extended the range of *gayi* to Peru and future explorations in that country and northern Chile will undoubtedly add much to our knowledge of the species.

The taxonomic history of *gayi* demands only a brief explanation. The species was originally described as *Myrmica gayi* by Spinola in 1851. Seventeen years later Mayr transferred it to the genus Pogonomyrmex. Why he did so is rather difficult to see since he himself set up the genus Pogonomyrmex and could hardly have failed to note the differences of thoracic structure and sculpture mentioned in the original description of *gayi*. In 1886 he regarded the insect as identical with *S. geminata* but nine years later Emery showed that it must at least be considered of subspecific rank. Finally in 1909 Forel gave *gayi* its present position as a separate species in the genus Solenopsis. Santschi in 1923 described the variety *fazi* and a new subspecies, *bruesi* is described in the present paper.

Key to the forms of S. gayi.

1. Head of the major worker widest just behind the eyes which are composed of approximately 45 facets............................2
   Head of the major worker widest between the eyes and the insertion of the mandibles, notably narrowed behind, the eyes composed of 60–65 facets............................subsp. *bruesi, subsp. nov.*
2. Color sordid yellow to golden yellow........................*gayi*
   Color piceous brown...var. *fazi*

S. gayi (Spinola).

S. gayi (Spinola),in Gay, Hist. fis. Chile, Zool., Vol. 6, p. 242 (1851). ☿ ♀ ♂. (Myrmica.)
S. gayi, Forel, Deutsche Ent. Zeitschr, p. 269 (1909).
Pogonomyrmex gayi, Mayr, Annuar. Soc. Nat. Modena, Vol. 3, p. 170 (1868).
S. geminata, Mayr. Verh. Zool-bot. Ges. Wein, Vol. 36, p. 365, 460 (1886).
S. geminata subsp. *gayi*, Emery, Act. Soc. Sci. Chili, Vol. 5, p. 11 (1895). ☿ ♀.

Worker major: Length 4.3 mm. (Plate IV, fig. 6.)

Head exclusive of the mandibles very slightly broader than long (approximately one-tenth), sides moderately convex, widest just behind the eyes, occipital angles much rounded, the occiput broadly and feebly concave. Clypeus scarcely projecting, the anterior edge virtually straight except at the median lobe which is rather sharply marked off from the rest of the clypeus although it projects very little beyond it. Clypeal carinae divergent, very thick and rather irregular above and terminating in two long but powerful teeth; lateral denticles reduced to mere lobes at the base of the carinal teeth. At the point where the extreme lateral portion of the clypeus meets and fuses with the anterior border of the gena there is a small more or less semicircular notch. Mandibles rather short and stout, the outer border rather strongly curved but not bent as in *geminata*, masticatory border with three large, very coarse teeth and a much smaller and more acute innermost tooth. Eyes oval, of forty-five facets. The antennal scape in repose fails to reach the occipital border by a distance just equal to the length of the first funicular joint. This joint very slightly shorter than the following two together. Joints two and three twice as long as broad; the remaining small joints all somewhat shorter. Club slender, five-sixths as long as the remainder of the funiculus, the terminal joint one and three-quarter times as long as the penultimate.

Promesonotum seen from above pyriform, the humeral angles very much rounded. The almost obliterated promesonotal suture forms a chevron-shaped impression on the dorsum of the thorax. The sides are strongly constricted at the mesoepinotal suture. Seen in profile the pronotum appears rather angular. There is a short declivious anterior face and a much longer very feebly convex posterior face. The mesonotum is straight throughout except at the posterior end where it passes, by means of a very short and steep declivity, to the prominent mesoepinotal suture. Basal face of the epinotum only slightly lower than the adjacent portion of the mesonotum, not sharply separated from the much shorter declivious face. Node of the petiole in profile only slightly higher than the length of its base, the anterior face straight and descending at an angle of about seventy degrees, the summit rather narrowed, the posterior face moderately and evenly convex from crest to base, peduncle stout, as long as the base of the node, without ventral lamella or tooth. Postpetiole in profile three-fourths as high as the node of the petiole but of about equal thickness, as thick at the summit as at the base, the summit evenly convex, the anterior and posterior faces short and virtually perpendicular, the ventral face of the node rather long and sinuate. Seen from above the postpetiole is very slightly wider than the node of the petiole (approximately one-fifteenth), both are transverse, the petiole roughly trapezoidal, the postpetiole suboval in outline. Base of the first gastric segment truncate and feebly concave.

Color sordid yellow to rich golden yellow, the mandibles, anterior border of the head, a patch on the vertex, antennae and legs lighter yellow. Mandibular teeth piceous brown. Posterior borders of the gastric segments tinged with brown. Hairs moderately abundant, erect, golden, very unequal in length on the head, thorax and petiolar nodes, of more nearly uniform length on the abdomen. Punctures small but moderately numerous. Mandibles covered with very coarse, irregular, rounded striae. Striations on the genae just behind the insertion of the mandible much reduced, not reaching the anterior border of the eye. Metapleurae very feebly striate. Posterior articulations of the petiole and postpetiole striato-rugulose. The rest smooth and shining.

Worker minor: Length 2.3 mm.

Head exclusive of the mandibles one-ninth longer than broad, the sides virtually straight, the occiput flat. The antennal scape in repose fails to reach the occipital border by a distance equal to its greatest thickness. First funicular joint as long as the following two together, second joint one and one-half times as long as broad, third joint only slightly longer than broad, the remaining small joints all approximately as broad as long. Eyes of forty facets. Promesonotal suture absent. Seen from above the node of the petiole is transversely oval and the postpetiole subcircular in outline. In other regards as in the major worker.

I have seen no females of the typical *gayi*. For a description of this caste the reader is referred to the variety *fazi*.

The male of *gayi* appears to be known only from Spinola's original description a translation of which is given below:

"Male: Length 4.2 mm.

"Antenna filliform, without appearing to have a scape. First joint short, not reaching the level of the eyes. Head small, one-third narrower than the thorax, very convex and almost round. Eyes strongly projecting, coarsely granulate and occupying more than a third of the side of the head. Ocelli as in the female. Mesothorax proportionally larger and more convex. Epinotum shorter and more steeply inclined towards the back. Abdomen, wings and legs as in the female. General color of the body bright black. Joints of the antennae grey. Wings hyaline, the veins testaceous."

Localities: Chile, Coquimbo. (Type loc.) (No collector.)
Santa Rosa. (Type loc.) (No collector.)
Valpariso. (Kinberg.)

Santiago. (Silvestri.)
Talca. (Silvestri.)
Vina del Mar. (Silvestri.)
Temuco. (Silvestri.)
Peru, Lima. (C. T. Brues.)

The color of a series of specimens collected by Brues at Lima, Peru is a rich golden yellow while in the material taken by Silvestri at Talca, Chile the color is a somewhat paler yellow and less brilliant in appearance. While it is possible that the Peruvian specimens may represent a distinct variety in view of the highly variable coloration of some of the related species I consider it advisable to make no separation until more material from northern Chile is available for examination.

Solenopsis (Solenopsis) gayi var. **fazi** Santschi.

S. gayi var. *fazi*, Santschi, Rev. Suisse Zool., Vol. 30, p. 261 (1923).

This variety differs principally from the typical *gayi* in its much darker coloration. The two cotypes which I have examined are of a piceous brown with the front of the head, a median spot on the vertex, the base of the first gastric segment, the tarsi and the funicular joints yellowish brown. In the original description of *fazi* Santschi stresses the absence of the frontal furrow as an additional means of distinguishing this form. However, since the frontal furrow in the typical *gayi* is at best obsolete and often entirely absent, the distinction is without much value. The decidedly darker color of *fazi* appears to me sufficient in itself to give it varietal status.

Female: Length, 6 mm. (Plate IV, fig. 3.)

Head exclusive of the mandibles slightly broader than long, the sides strongly convex, narrowed more in the anterior than in the posterior half, the occiput flat and rather narrow because of the excessive rounding of the occipital angles. Clypeus as in the major worker (see *gayi*) but with the carinae and the carinal teeth greatly thickened. The notch in the genae behind the insertion of the mandible is less deep than in the major worker but still prominent. Mandibles as in the major worker. The antennal scapes in repose reach, or very slightly surpass, the occipital border. First funicular joint slightly longer than the following two together. Second joint one and one-half times as long as broad, the remaining joints gradually decreasing in length and increasing in width, the seventh joint slightly broader than long. Club slender, approximately five-sixths as long as the remainder of the funiculus, the terminal joint one and one-half times as long as the penultimate. Eyes of moderate size, oval, their anterior border separated from the insertion of the mandible by a distance slightly less than half their greatest diameter, their posterior border approximately at the middle of the side of the head.

Thorax elliptical, twice as long as broad, at its widest point very slightly narrower than the head. In profile the scutum slightly overhangs the pronotum. Epinotum obtusely angular, the basal face longer than the declivious. Node of the petiole seen in profile narrowly triangular, almost twice as high as it is thick at the base, the summit rather acute, peduncle short, thick and tapering. Postpetiole as in the major worker. Seen from above both nodes are strongly transverse, the postpetiole is one and one-fourth times as wide as the node of the petiole, the latter very narrowly elliptical, postpetiole twice as thick and sub-oval in outline. Base of the first gastric segment truncate.

Color reddish brown, the anterior fourth of the head, front and vertex yellow. The scutum bears two narrow yellow bands which converge posteriorly to form a broad median band. Scutellum entirely yellow. Base of the first gastric segment yellow, the remainder of the abdomen piceous. Punctures larger and more prominent than in the major worker. Hairs moderately abundant, of irregular length, erect, golden. Upper portions of the epinotum and petiolar nodes covered with extremely fine and rather irregular striae which give those parts a somewhat duller appearance. Striae on the genae extending back to the anterior border of the eye, the base of the petiolar nodes striato-rugulose. The rest smooth and shining.

Localities: Chile, Santiago. (Type loc.) (Faz.)
Valpariso. (Hoffman.)

Solenopsis (Solenopsis) gayi subsp. **bruesi**, subsp. nov.

The major worker of this subspecies differs from that of the typical *gayi* as follows:

The head is widest between the eyes and the insertion of the mandibles, the sides are more convex, the head notably narrowed posteriorly. The occiput is unimpressed or at most very feebly impressed. The eyes are composed of 60–65 facets. The antennal scape in repose fails to reach the occipital border by a distance slightly less than the length of the first funicular joint. This joint as long as the following two together. The club approximately two-thirds as long as the remainder of the funiculus. Node of the petiole with an evenly rounded, obtuse summit, the posterior face straight and perpendicular, the anterior face with a very slight forward slope. Postpetiole considerably thicker than the node of the petiole, both it and the petiole lower than in the typical *gayi* and the sculpturing at their bases is much more feeble. Seen from above the nodes are narrower and less angular than in the type. Color sordid yellow, the abdominal segments tinged with piceous brown. (Plate IV, figs. 4, 5.)

I have not seen any of the very small workers of this subspecies but those measuring 3.2 mm. show much the same characters as the major worker. The sexual phases are also unknown to me and it is largely for this reason that I have retained *bruesi* as a subspecies of *gayi*. Although the differences shown by the worker of the new subspecies might be considered sufficient to entitle it to specific status, in the last analysis the question turns upon the character of the sexual forms and until these are discovered it is better to class it with *gayi* to which it is closely related.

Described from a number of specimens taken by Prof. C. T. Brues in Chosica Canyon (2000 ft.) Peru.

SOLENOPSIS GEMINATA (FABR.).

Introduction.

Since its recognition in 1804 *S. geminata*, the type of the genus, has been described a surprising number of times. There are no less than eleven names for this species which have been abandoned on the synonymic junk-heap. It is inevitable that such a widely distributed and abundant ant should repeatedly appear in the literature under new names and it is remarkable that, despite this plethora of descriptions, the taxonomy of *geminata* has largely escaped confusion. At various times related species have been regarded as variants of *geminata* but, during the past sixty years there has been little doubt as to what constitutes the typical form. Truth compels one to add that this unusual situation is more a result of the characteristic cephalic structure of the major worker than an outcome of taxonomic acumen.

There is no necessity for anything more than a brief outline of the history of the species. Originally described as *Atta geminata* by Fabricius in 1804, it was given the present generic name nearly half a century later when Westwood set up *Solenopsis mandibularis* as a new species. The insect subsequently fell into the hands of Frederick

Smith and Buckley who between them managed to assign it to four different genera and eight separate species before it was rescued by Mayr in 1867. Since that time there has been practically no confusion in regard to the true *geminata* but subsequent interpretations of subspecies and varieties must often be taken with a grain of salt.

The distribution of *geminata* is Antillean, that is to say it occurs in the greatest abundance throughout the islands of the West Indies with fringes in those portions of the Americas which border the Gulf of Mexico and the Carribean Sea. In addition the insect is found in the northern part of the narrow, western coastal plain of South America and there is a subspecies known from the Galapagos. Wherever it occurs it is one of the dominant insects and consequently much material has come into collections and a number of variants have been described. We shall disregard for the moment the single aberrant subspecies known from the Galapagos Islands and confine our attention to the forms which occur in the West Indies and continental America.

The first variant of *geminata* to be described was the subspecies *rufa*. As nearly as can be determined the "*Atta rufa*," described by Jerdon in 1851 from specimens taken in India, was identical with the subspecies of *geminata* which at present bears that name. Forel, in 1902, first placed the subspecies on a recognizable basis when he pointed out the presence of the mesosternal spine which characterizes it. For many years this variant was known only from the Old World tropics and it was assumed that *rufa* was indigenous to the East Indies and southern Asia. Its occurrence there was thought to constitute the sole exception to the strictly New World distribution of the so called "*geminata* group." During the past twenty-five years, however, specimens of *rufa* from Mexico and the portions of the United States which border the Gulf of Mexico have been accumulating in collections, and the place of origin of this form has become highly questionable.

The subspecies *diabola* was next described by Wheeler in 1908 from material taken in Texas. It was separated from the typical *geminata* through its smaller cephalic punctures and lighter color. This led Forel to set up what he characterized as the "other extreme," the dark variety *nigra*, based upon specimens from Zent, Costa Rica. In addition to its piceous color the variety was said to show a more pronounced punctuation than the typical form. Finally Mann, in 1916, described the subspecies *medusa* from northeastern Brazil.

The largest workers of this variant have the genae strikingly dilated just behind the insertion of the mandible. The color of the type specimens is even darker than the specification given for *nigra* and the punctuation is unusually strong.

In the course of the present investigation it became necessary to assign several hundred specimens to their proper place among the forms just mentioned. Since the material was unusually abundant and distributed over the entire known range of the insect it seemed an excellent opportunity to attempt the correlation of structural variation with geographical distribution. It was at once apparent that the insular specimens can be classified largely by locality, since there appears to be but one form, the typical *geminata*, throughout the islands of the West Indies. Not so the continental specimens which vary *ad infinitum*. As long as previously described forms were kept in mind there seemed to be little meaning to distributional facts and less to most of the criteria upon which the variants were established. Color characteristics, unless accompanied by some definite structural difference, were found to be wholly without value as a means of separation. Not only was it possible to find light and dark specimens in practically every locality but frequently these differences were shown by individuals coming from the same nest. Variation in punctuation was equally unsatisfactory for the same reason. Under such conditions it is impossible to maintain the validity of varieties based wholly upon differences in color and punctuation, consequently both *nigra* and *diabola* become of no significance. There is no need to refer again to *nigra*, since it must be considered a synonym of the typical *geminata*. *Diabola*, however has a slightly different relationship which is discussed in a subsequent paragraph. After the breakdown of these two varieties the entire preëxisting scheme was thrown out and a series of specimens whose localities ranged from Florida to eastern Brazil was assembled. A study of this series led to very interesting results. By disregarding variations in punctuation and color it was possible to divide the material into two groups, each showing a structural difference which could be correlated with geographical distribution. The range of the first group extends northward from Peru through Ecuador and western Colombia into Central America and thence through Mexico to the southern United States. That of the second group begins in northeastern Colombia and passes through Venezuela, the Guianas and into eastern Brazil apparently terminating about the latitude of Bahia. Colombia, as the meeting place of the two forms, becomes, therefore, of peculiar significance. A study of the topography of that country shows that it is crossed, roughly from north to south, by three ranges of mountains whose altitude is often in excess of thirteen thousand feet. This natural barrier divides Colombia into two separate faunal regions and there is every reason to suppose that there would be differences in the forms on the opposite sides of the mountains.

Except for their greater variation in color and sculpture the members of the first group agree closely in structure with the insular *geminata.* In the major worker the sides of the head are approximately parallel to the point where the occipital angles begin. There is a shallow and rather narrow impression of the sides just in front of the eyes but the genae are no wider in front of this impression than is the rest of the head. On the other hand we find that in the second group the major workers have the sides of the head gradually narrowed from the eyes to the occipital angles, the impression in front of the eyes is reduced or absent and the genae are expanded which gives that portion of the head a greater width than elsewhere. The condition just described may be recognized as that which characterizes the subspecies *medusa.* In *medusa,* however, we have a very extreme form which shows these characters in the highest degree. The specimens from Colombia, while they are distinct from the typical *geminata* are, nevertheless, intermediate between that form and *medusa.* The two groups, therefore cannot be considered as representatives of separate species.

Each of the groups can be further subdivided but the subdivisions are apparently without geographical significance. In the material coming from the north of Panama a separation can be made on the presence or absence of the mesosternal spine. Specimens which possess this character are invariably light in color, the exception noted above to the usual high degree of color variation. I was surprised to find the mesosternal spine present in the cotypes of the subspecies *diabola* for its supposed absence constitutes the means of separation of *diabola* and *rufa.* I believe that Wheeler may have been misled by the unstable character of this structure which shows a startling amount of variation even within a series from the same nest. There may be from one to four needle-like spines, or a rather obtuse triangular point or points, or at times a broadly truncate projection. Such fluctuations would, if at all constant, provide excellent means for separation. Unfortunately they are entirely without value unless taken collectively. An attempt to find other characters by means of which *diabola* and *rufa* might be separated only led me to believe more firmly in their identity.

The subdivision of the second group has been already suggested, since the Colombian material can scarcely be included with the typical *medusa.* However, an entirely unexpected outcome resulted from the study of the Colombian specimens. I have been fortunate in having at my disposal a large number of specimens collected by Dr. George Salt and Mr. P. J. Darlington in the state of Magdalena, Colombia. Many of these were taken at Rio Frio. In going over the material from that locality an attempt was made to recognize the species *eduardi.* As may be recalled Forel described this species from a single minor taken at Rio Frio. The only outstanding characteristic peculiar to the species lies in the structure of the epinotum, which was described as follows:

"The whole epinotum hollowed from one end to the other by a broad, central, longitudinal groove and bordered on its two faces as in *tridens* but rounded and not subdentate or tuberculate between the two as in that species."

Aside from the fact that it is always extremely inadvisable to base a new species upon the minor worker of a polymorphic form, Forel should have realized that the characteristics given above are regularly shown by the smaller medias of the typical *geminata* and that it is not impossible to find minors which approximate this condition. In the case of the material from Rio Frio, of which I have examined several complete series of workers, the minors all show the epinotal structure characteristic of *eduardi.* Furthermore the sides of the epinotum and mesonotum have the dense, rugose-reticulate sculpture mentioned by Forel in his description of that form. I therefore have no hesitancy in identifying the insect described by Forel as *eduardi* with the Colombian form of *geminata.* However, as has already been noted, the major workers of this form are transitional as regards the dilation of the genae and the minor workers are virtually unique in the shape and sculpture of the epinotum, since similar conditions occur but rarely in the minors of the typical *geminata* or the subspecies *medusa.* We may, therefore, retain *eduardi* as a subspecies of *geminata.* The two variants of this form described by Santschi must, on the other hand, be synonomized. One, the variety *bahiaensis,* of which I have seen cotypes, is obviously a synonym of the subspecies *medusa.* The other, the variety *perversa,* since it differs from *eduardi* only through its darker coloration will have to be considered identical with that form. As has already been explained the coloration of the continental *geminata* is too variable to permit the establishment of varieties upon this character alone.

A summary of the facts presented above may be stated as follows: The typical *geminata* occurs uniformly throughout the islands of the West Indies, in the extreme southern portions of the United States from Florida to Texas, in Mexico, in Central America and in the Pacific coastal plain through Colombia, Ecuador and Peru. From Florida to Panama the range of the typical form is overlapped by that of the subspecies *rufa.* The typical *geminata* is replaced in the Atlantic coastal region of Colombia, in Venezuela, the Guianas and north eastern Brazil by the subspecies *eduardi.* This form grades into the subspecies *medusa* which occurs in the more easterly portions of Brazil.

An interesting outcome of this explanation lies in the strong evidence which it presents for the Neotropical origin of the subspecies *rufa.* While it is by no means impossible that an introduced ant might achieve an even distribution over an area as great as that occupied by *rufa,* the number of records from localities well inland and often at a considerable altitude make such an explanation seem improbable.

The following key embodies in condensed form the morphological differences just discussed and in addition includes diagnostics for the separation of the subspecies *galapageia.*

1. Major worker 4.5 mm., female 6.6 mm. in length....subsp. *galapageia*
   Major worker 6.0 mm., female 7.5–8.0 mm. in length................2
2. Epinotum of the smallest workers with a median longitudinal groove, bordered at the edges; sides of the epinotum and mesonotum densely rugose-reticulate................................subsp. *eduardi*
   Epinotum of the smallest workers without a median longitudinal groove, the edges not bordered; sides of the epinotum and mesonotum feebly rugose and somewhat shining....................................3
3. Head of the largest worker with approximately parallel sides, the genae not dilated behind the insertion of the mandible............4
   Head of the largest worker tapering from the eyes to the occipital angles, the genae dilated just behind the insertion of the mandibles. subsp. *medusa*
4. Thorax bearing a mesosternal spine or projection..........subsp. *rufa*
   Mesosternum of the thorax without spine or projection........*geminata*

In view of the large number of descriptions of *geminata* already in the literature another account may seem entirely superfluous. However aside from the fact that its absence would occasion a rather conspicuous hiatus in a monograph of the genus, there are additional considerations which make its inclusion advisable. Chief among these is the fact that *geminata* is undergoing a rapid dissemination to new localities through the agency of modern commerce. There is always the temptation to give such immigrants varietal status and already two forms of questionable validity have been described from tropical Africa. This practice is in any case to be deplored, particularly when the species in question is as variable as *geminata*. It is hoped that the inclusion of a full description of the typical form may lead investigators to a more conservative view in the establishment of new varieties on the basis of "tramp" specimens.

Solenopsis (Solenopsis) geminata (Fabr.)

S. geminata (Fabricius), Syst. Piez., p. 423 (1804). ♀ (*Atta*).
Formica geminata, Roger, Berl. Ent. Zeitschr., Vol. 6, p. 289 (1862). ☿ ♀ ♂
S. geminata, Forel, Mitt. Munchen, Ent. Ver., Vol. 5, p. 10 (1881). ☿ ♂
Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 24, p. 424 (1908). ☿
Forel, Deutsche Ent. Zeitschr., p. 268 (1909).
S. geminata var. *nigra*, Forel, Bull. Soc. Vaud. Sci. Nat., Vol. 44, p. 45, (1908) ☿
Forel, Deutsche Ent. Zeitschr., p. 268 (1909).
Forel, Bull. Soc. Vaud. Sci. Nat., Vol. 49, p. 23 (1913). ☿
S. mandibularis, Westwood, Ann. Mag. Nat .Hist., Vol. 6, p.87 (1841). ☿
S. saevissima, Mayr, Verh. Zool.-bot. Ges. Wien., Vol. 12, p.751 (1862). ☿
Myrmica virulens, Fred. Smith, Cat. Hym. Brit. Mus., Vol. 6, p. 132 (1858).
Atta clypeata, Fred. Smith, *Ibid.*, Vol. 6, p. 169 (1858). ♀ ♂.
Diplorhoptrum drewseni, Mayr, Europ. Formicid., p. 71 (1861). ☿
S. drewseni, Ern. Andre, Rev. Mag. Zool. (3), Vol. 2, p. 200 (1874). ☿
Myrmica glaber, Fred. Smith, Trans. Ent. Soc. Lond. (3), Vol. 1, p. 34 (1862). ☿
Myrmica polita, Fred. Smith, *Ibid.*, Vol. 1, p. 34 (1862). ☿
Myrmica (*Monomorium*) *saxicola*, Buckley, Proc. Ent. Soc. Phil., Vol. 6, p. 341 (1866). ☿
Atta lincecumi, Buckley, *Ibid.*, Vol. 6, p. 344 (1866). ☿ ♀
Atta brazoënsis, Buckley, *Ibid.*, Vol. 6, p. 345, (1866). ☿
Atta coloradensis (part), Buckley, *Ibid.*, Vol. 6, p. 346, (1866). ☿

Worker major: Length 6 mm. (Plate I, fig. 1, 11.)

Head, exclusive of the mandibles, very slightly broader than long, the sides parallel from the insertion of the mandibles to the point where the occipital angles begin except for a slight impression just in front of the eyes. Occiput divided into two lobes by a profound median impression. At the bottom of this impression lies a prominent groove which extends across the frontal area to a point just behind the frontal lobes. Clypeus not projecting, its anterior edge slightly concave with a shallow, concave, impression between the carinal teeth. Carinal teeth very stout and moderately divergent. Lateral denticles much smaller and set close to the side of the carinal teeth. Mandibles very thick and abruptly bent so that the apical half lies at right angles to the basal. In some specimens the apex of the mandible is pointed and the inner margin bears two poorly defined teeth. Usually, however, the mandibles are toothless and in many cases the inner ends are rounded off to such an extent that the mandibles fail to meet when closed. The antennal scapes in repose reach two-thirds of the distance to the occiput; first funicular joint as long as the following two together, second funicular joint twice as long as broad, the remaining joints all longer than broad but gradually decreasing in length and increasing in thickness; club slender, about three-fourths as long as the remainder of the funiculus, the terminal joint very slightly less than twice as long as the penultimate. Eyes rather flat, composed of one hundred or more facets, their outline irregularly oval, their border delimited by a shallow groove. They are separated from the insertion of the mandibles by a distance twice as great as their maximum diameter.

Thorax with a well-defined pro-mesonotal suture. In profile the pronotum slopes sharply forward from the suture in a straight line, the mesonotum is much longer and feebly convex throughout except at the posterior end where it passes by means of a short, rather steep declivity to the deep, slot-like mesoepinotal suture. The epinotum in profile is very angular with basal face somewhat longer than the declivious.

Node of the petiole in profile high and thin, the summit narrow, the posterior face virtually perpendicular, the anterior face sharply sloping forward and meeting the peduncle at a marked angle, the peduncle as long or slightly longer than the base of the node, rather thick with a ventral lamella but no ventral tooth. Post petiole in profile approximately four-fifths as high as the node of the petiole;

as long as high with the summit evenly convex and the anterior and posterior faces perpendicular. Seen from above both petiole and postpetiole are strongly transverse. The postpetiole is slightly wider and irregularly oval in outline. The base of the abdomen is truncate and bears a concave impression at its point of junction with the postpetiole.

Punctures coarse and numerous, those on the head, particularly on the occiput and mandibles, often elongate. Hairs golden, stout and erect; short on the frontal area, occiput and antennal scapes, longer on the mandibles, thorax, petiolar nodes and abdomen, very long on the anterior edge of the clypeus. Mandibles and summits of the petiolar nodes with a few coarse, scattered striae. Area between the eye and the insertion of the mandibles, pleurae and the entire epinotum with numerous fine irregular striae. Base of the petiolar nodes rugulose. Other parts of the insect strongly shining.

The color is highly variable. The only constant feature is the piceous coloration of the mandibles and the anterior border of the head. In other regards the insect shows a range of color which extends from brownish yellow specimens in which the gastric segments are bordered with light brown bands to individuals having a sooty, brownish coloration with the abdomen and occiput piceous.

Worker minor: Length 2.4 mm. (Plate I, figs. 6, 10, 12.)

Head, exclusive of the mandibles, slightly longer than broad, the sides feebly convex and gradually narrowing from the eyes to the occiput, the latter flat. Clypeus moderately projecting, the carinal teeth relatively longer and thinner than the major worker, the lateral denticles relatively larger and less closely approximated to the carinal teeth. Mandibles gradually curved, their masticatory border with four large teeth. The antennal scape in repose surpasses the occipital border by less than its own thickness. First funicular joint as long as the following two together, second joint slightly less than twice as long as broad, the remaining joints gradually increasing in thickness but not decreasing in length; club relatively stouter than in the major worker, the terminal joint a trifle more than twice as long as the penultimate. Eyes oval, of about thirty facets, separated from the insertion of the mandible by a distance one and one-half times as great as their maximum diameter.

Promesonotal suture absent. Promesonotum in profile evenly convex throughout, descending to the mesoepinotal suture through a short and abrupt declivity. Mesoepinotal impression relatively feebler than in the major worker but still strong. Basal and declivious faces of the epinotum not sharply separated, the profile appearing as a somewhat irregular curve.

Node of the petiole in profile relatively lower, thicker and blunter than the major worker, the posterior face slightly inclined forward, the angle at which the anterior face meets the peduncle less clearly defined, the peduncle relatively longer and thinner and without ventral lamella or tooth. Postpetiole in profile approximately three-fourths as high as the node of the petiole, its anterior face slightly inclined backwards, the summit strongly convex, the posterior face strongly inclined forward. Seen from above the nodes are much less transverse than in the major worker. The postpetiole is scarcely wider than the node of the petiole and oval in outline. The first gastric segment is truncate at the base but lacks the concave impression at the base of the postpetiole.

Punctures fine and rather sparse, the long, erect, yellow hairs which they bear of equal length on head, thorax and abdomen, those on the appendages shorter and somewhat more appressed. Mandibles longitudinally striate, pleurae and base of the epinotum striato-rugulose, base of the petiolar nodes rugulose, for the rest smooth and shining. The color varies from a clear yellowish brown with the abdomen and often the petiolar nodes piceous brown to a uniform piceous brown. The appendages and mandibles are in all cases of a lighter color than the rest of the insect.

Female: Length 7.5–8 mm. (Plate I, fig. 5.)

Head, exclusive of the mandibles, one-sixth broader than long, quadrate, a little wider behind the eyes than in front of them, the sides very feebly convex from the eyes to the occipital angles, straight or nearly straight in front of the eyes and meeting the anterior border of the head at a sharp angle. Occipital angles well-marked, the occiput flat with a narrow and shallow median impression, occipital furrow clearly defined, frontal furrow short, clearly marked only for about half the distance from the median ocellus to the base of the frontal lobes, thereafter becoming shallow and indistinct. Ocelli large and prominent. Clypeus feebly projecting, carinal teeth very stout and rather blunt, the edge of the clypeus between them with a shallow concave impression; lateral denticles small, often poorly defined and in some cases represented only by a sinuousity in the edge of the clypeus. Mandibles strongly bent but less so than in the major worker, the masticatory border with three large teeth and usually the rudiment of a fourth. Eyes large, strongly convex, irregularly oval in outline, their posterior border reaching a point half way between the occiput and the anterior border of the head. The antennal scape in repose just reaches the lateral ocellus. Funicular joints and club as in the major worker.

Thorax robust, elliptical, its maximum width three-fifths of its length, only slightly narrower than the head (the eyes excluded). Seen in profile the mesonotum shows a straight posterior half and a convex anterior portion which overhangs the pronotum. Scutellum as high as the mesonotum, slightly convex with a short, perpendicular posterior face. Angle of the epinotum well-defined but very obtuse, the basal and declivious faces of about equal length. Mesosternum large and subglobose beneath.

Petiolar nodes very similar to those of the major worker except that the peduncle is thicker, the node of the petiole slightly lower and the postpetiole bears on either side an obtuse, somewhat conical, ventral projection with a small opening at its summit (this condition is sometimes found in the major worker but in that caste the conical projection is usually absent and the opening occurs as a small tubercle on the side of the node). Seen from above the nodes are very strongly transverse and of approximately equal width. Abdomen as in the major worker. Wings hyaline with yellow veins.

Punctures smaller and less numerous than in the major worker. Somewhat larger on the head than on the thorax and abdomen. Body hairs long, golden and erect, somewhat longer on the head than elsewhere, longest on the anterior edge of the clypeus. Mandibles with a few coarse, indistinct striae, epinotum almost completely covered with fine wavy striae, petiolar nodes, except their summits which are shining, striato-rugulose. For the rest smooth and shining. The color varies from a clear yellowish brown with the front of the head, the mesosternum and the appendages paler and the mandibles and the posterior half of the abdomen castaneous, to a deep castaneous brown with only the extreme anterior portion of the head yellowish brown.

Male: Length 5.8 mm. (Plate I, fig. 4.)

Head trapezoidal, its maximum width (including the eyes) approximately one-fourth greater than its length. Eyes very large, strongly convex and oval in outline, occupying more than one-half the side of the head, their anterior border reaching the insertion of the mandible. Ocelli very large and prominent, the lateral ocelli which mark the boundary of the occiput with a shallow concave impression between them. Anterior edge of the clypeus approximately straight; seen from the side the clypeus shows a blunt, beak-like central lobe. Mandibles small, linear, bidentate. Antennal scape about one and one-half times as long as broad, roughly cylindrical; first funicular joint subglobose, broader than the scape or the following joint; second funicular joint more than twice as long as broad, third joint one and one-half times as long as broad, the remaining joints all more than twice as long as broad and progressively decreasing in width.

Thorax bulky, elliptical, its greatest width two-thirds of its length, only slightly less than twice as wide as the head (eyes included). Seen from the side the anterior part of the mesonotum is greatly swollen and overhangs the pronotum which is so much displaced that the head of the insect appears to be attached to the ventral surface of the thorax. Epinotum rather rounded, the basal face strongly convex transversely and slightly convex longitudinally, declivious face flat and virtually perpendicular. Node of the petiole in profile low but with an acute summit, the anterior face not sharply separated from the thick peduncle, the posterior face perpendicular. Seen from behind the summit of the node shows a broad, shallow median impression. Postpetiole in profile as high as the node of the petiole, about one and one-half times as high as long with a long, backward-sloping anterior face, a rounded summit and a very declivious posterior face. The conical lateral projections are even stronger than in the female. Seen from above both nodes are very transverse, the postpetiole is approximately three times as broad as long and one-sixth wider than the node of the petiole. First gastric segment truncate at the base but not impressed. Wings hyaline, the veins clear yellow.

Punctures fine and fairly numerous, the hairs which they bear long, thin, golden, erect or suberect and of uniform length over the body, those on the legs shorter and stiffer; antennae without long hairs but clothed with a dense short pubescence. Base of the epinotum, area between the eye and the insertion of the antenna and the area between the ocelli striato-granulate. Base of the petiolar nodes granulate. For the rest smooth and shining. Color yellowish brown to piceous brown, the antennae and legs pale yellow.

To present a list of the localities in which *geminata* has been taken would require several pages. Its distribution may be summarized as uniform throughout the West Indies and on the Continent from Florida to Costa Rica. In the eastern Gulf States its range extends inland only about a hundred to a hundred and fifty miles and this also appears to be true for the greater part of Texas. From Mexico southward, however, the distribution is from coast to coast.

The habits of this species have been so often described that only the briefest comment is necessary here. In general *geminata* prefers to nest in open fields or sunny glades, avoiding the shade of deep woods. The nests are usually irregular, sandy craters of loose construction but sometimes rotten stumps are utilized as nesting sites. The ferocity of this ant is proverbial, for the activity of the workers when disturbed never fails to attract attention, however callous the observer. For a good description of the habits of *geminata* the reader is referred to Forel's account published in 1881 (see synonymy) or to the observations of H. H. Smith quoted by Forel in his *Formicides de l'Antille St. Vincent* (Trans. Ent. Soc. Lond. p. 396, 1893).

Solenopsis (Solenopsis) geminata subsp. **galapageia** Wheeler.

S. geminata var. *galapageia*, Wheeler, Proc. Cal. Acad. of Sci., Vol. II, p. 272 (1919). ☿ ♀

Aside from its smaller size there is little to distinguish the worker of *galapageia* from that of the typical *geminata*. Such a distinction in the case of a polymorphic ant is quite valueless unless it is accompanied by a similar difference in the female. Not only is the female of *galapageia* decidedly smaller (6–6.5 mm.) but it also shows certain structural variations which lead me to believe that this form should have subspecific rank. The frontal furrow is much more prominent and is clearly marked to the base of the frontal lobes. In the typical *geminata* this furrow is poorly defined, in some specimens virtually absent, in the anterior half of the head. The summit of the petiole in *galapageia*, when seen from behind, shows a very distinct, obtuse, median notch. I have been able to find a transition to this condition in two *geminata* females from Jamaica but in both of these the notch is poorly indicated. In all other females which I examined, both of the typical form and also of the subspecies *medusa* and *rufa*, the summit of the petiolar node is entire and slightly convex. The color of the female of *galapageia* is a clear castaneous brown with the front of the head, the antennae, tibiae and tarsi yellow. The mandibles are castaneous with piceous teeth and the femora are castaneous with their bases and apices tipped with yellow. Male unknown. The subspecies is known only from type material. Type locality: Charles Island, Galapagos Is. (F. X. Williams.)

Solenopsis (Solenopsis) geminata subsp. **rufa** (Jerdon.)

(Plate I, figs. 7, 8.)

S. geminata subsp. *rufa* (Jerdon) Madras, Jour. Litt. Sci., Vol. 17, p. 106 (1851). ☿ (Atta.)

S. geminata, Mayr, Tijdschr, v. Ent. Vol. 10, p. 109 (1867). ☿ ♀ ♂

S. geminata, Emery, Bull. Soc. Ent. Ital., Vol. 23, p. 166 (1892).

S. geminata st. *rufa*, Forel, Jour. Bombay Nat. Hist. Soc., Vol. 14, p. 689 (1902). ☿

S. geminata var. *rufa*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 23, p. 272 (1907). ☿

S. geminata var. *rufa*, Forel, Deutsche Ent. Zeitschr., p. 268 (1909).

S. geminata var. *rufa*, Bingham, Fauna Brit. India, Hym, Vol. 2, p. 158 (1903). ☿ ♀ ♂

S. geminata var. *diabola*, Wheeler, Bull. Amer. Nat. Hist. Vol. 24, p. 424 (1908). ☿

S. geminata var. *diabola*, Forel, Deutsche Ent. Zeitschr. p. 268 (1909).

S. cephalotes, F. Smith, Jour. Proc. Linn. Soc. Lond. Zool., Vol. 3,p. 149, (1858). ☿

Crematogaster laboriosus, F. Smith, *ibid.*, Vol. 4, suppl., p. 109, (1860). ☿

As has been pointed out in the introduction the worker of the subspecies *rufa* is characterized by the presence of a mesosternal spine or projection. In addition they can generally be distinguished by their lighter color which is a clear, reddish yellow with the abdominal segments cleanly and narrowly bordered with brown. The mandibles are usually darker, particularly in the major workers, where they are often piceous red. In some specimens the occiput and pronotum are tinged with brown.

Aside from its lighter coloration, which is like that of the worker, I can see no difference between the female of *rufa* and that of the typical *geminata*. The males are indistinguishable.

In view of the close similarity of the sexual forms it would seem that *rufa* is scarcely more than a variety of *geminata*. However, in order to eliminate the possibility of confusion and to reduce taxonomic juggling to a minimum it seems preferable to let matters stand as they are. My reasons for synonymizing *diabola* with *rufa* have been fully discussed in the introduction and need not be repeated here. The following list comprises the New World locality records of *rufa*.

United States: Florida, Miami. (W. M. Wheeler.)
Biscayne Bay. (No collector.)
Alabama, Mobile. (H. P. Löding.)
Georgia, Spring Creek. (Bradley.)
Okeefinokee Swamp. (Bradley.)
Mississippi, Tupelo. (M. R. Smith.)
Louisiana, Lake Charles. (La. Exp. Sta.)
Cameron, (La. Exp. Sta.)
Texas, Huntsville. (W. M. Wheeler.)
Austin. (W. M. Wheeler.)
Dallas. (W. D. Hunter.)
Granite Mts. (W. H. Long.)
Montopolis. (W. M. Wheeler).
Langtry. (W. M. Wheeler.)
Paris. (Rucker.)
New Mexico, Almagordo. (von Krockow.)
Mexico: Vera Cruz, San Francisco. (Petrunkewitch.)
Jalapa. (No collector.)
Cordoba. (Mann & Skewes.)
Morelos, Cuernavaca. (No collector.)
Jalisco, Guadalara. (Mc. Clendon.)
Yucatan, Merida. (W. M. Mann.)
British Honduras: Manatee, (J. D. Johnson.)
Guatemala: Quirigua. (W. M. Wheeler.)
Puertos Barrios. (W. M. Wheeler.)
Zacapa. (W. M. Wheeler.)

Solenopsis (Solenopsis) geminata subsp. **eduardi** (Forel.)

(Pl. I, figs. 2, 9.)

S. eduardi, Forel, Mem. Soc. Ent. Belg., Vol. 20, p. 12 (1912).
Santschi, Ann. Soc. Ent. Belg., Vol. 64, p. 13, (1924).
S. eduardi var. *perversa*, Santschi, Ann. Soc. Ent. Belg., Vol. 64, p. 13 (1924).

The subspecies *eduardi* may readily be distinguished from all the other forms of *geminata* by the longitudinally grooved and bordered epinotum of the minor worker. The sculpture of the sides of the epinotum and mesonotum is unusually strong, rugose and densely reticulate. The transverse rugae are well marked on both basal and declivious faces. The bordering is rather thick and, when seen from above, appears to diverge posteriorly. In the majors the epinotum is deeply grooved and densely sculptured but as this is also true to a lesser degree in the majors of the typical *geminata* the difference is not particularly striking in this caste. The shape of the head in the largest workers of *eduardi* is intermediate between that of the typical *geminata* and the subspecies *medusa.* The sides are only slightly narrowed toward the occiput and the genae, while considerably more expanded than in the typical *geminata,* are less so than in the subspecies *medusa.* The color of *eduardi* is extraordinarily variable. A number of the specimens collected by Dr. Salt have a coloration as light as that of *rufa,* while another series from a locality only a few miles distant are deep piceous brown. It is because of this fluctuation in color that I cannot see the justification for Santschi's variety *perversa,* which was established principally because of the black color of the head of the worker. I have not seen the female of *eduardi* but the insect described by Santschi seems very similar to that of the subspecies *medusa.* The antennal scapes are shorter than in the female of the typical *geminata.* The thorax is said to be higher than that of *geminata* with the mesonotum overhanging the pronotum, a character not shown by the other forms. The male of *eduardi* is unknown.

There is no need to repeat here my reasons for reducing *eduardi* to a subspecies of *geminata.* A full explanation may be found on page 57 of the introduction to *geminata.*

Localities: Colombia, Magdalena, Rio Frio. (Type loc.) (A. Forel, George Salt, P. J. Darlington.)
Venezuela, Frines. (No collector.)
Brazil, Pernambuco, Tapera. (R. P. Wasermann.)

Solenopsis (Solenopsis) geminata subsp. **medusa** Mann.

(Plate I, fig. 3.)

S. geminata subsp. *medusa,* Mann. Bull. Mus. Comp. Zool. Harvard, Vol. 60, p. 447, pl. 4, fig. 31 (1916). ☿
S. eduardi var. *bahiaensis,* Santschi, Bull. Soc. Ent. Belg., Vol. 65, p. 236 (1925). ☿

In the major worker of the subspecies *medusa* the narrowing of the posterior portion of the head and the expansion of the genae reach extreme conditions. The head is one and one-third times as wide at the genae as at the occiput. This permits an easy separation from the typical *geminata* and there is usually little difficulty in distinguishing the majors of *medusa* from those of the subspecies *eduardi* although in this case the difference is less pronounced. However since the minor worker of *medusa* does not have the grooved and sculptured epinotum characteristic of *eduardi* the two forms may be separated more readily by this character.

The female of *medusa* is very similar in structure to that of the typical *geminata*. It does not have the expanded genae or narrowed occiput of the major worker. There are, however, two characters which permit accurate identification. The antennal scape of the female of *medusa* is shorter than that of *geminata*, failing to reach the lateral ocellus by a distance equal to its greatest thickness. The transverse rugae on the basal face of the epinotum are very strong. In the typical *geminata* they are feeble and often entirely obliterated in the anterior half of the basal face The male of *medusa* is unknown.

The color of *medusa* is less variable than in some of the other forms of *geminata*. As yet I have seen no completely light colored individuals although some of the specimens in the type series have a reddish brown area on the front of the head, the pronotum and the base of the gaster. The majority of the specimens I have examined are uniform piceous black in color. As has already been noted Santschi's *eduardi* var. *bahiaensis* must be synonomized with *medusa*. A comparison of the cotypes of this variety with those of *medusa* failed to show any differences by which the two could be separated.

Localities: Brazil, Ceara, Maranguape Mts. (Type loc.) (W. M. Mann.)
Bahia. (Bondar.)

SOLENOPSIS SAEVISSIMA (FRED. SMITH).

Introduction.

In approaching the subject of *S. saevissima* one has the unpleasant feeling that he is entering a battle-field strewn with unexploded missiles and that there is a strong probability that one of these taxonomic duds may, through tampering, bring the investigator to grief. It therefore behoves him to proceed with as much care and circumspection as he is able to command. Perhaps it is this feeling of restraint which has prevented work on the species, certainly it has been desperately needed. Santschi's revision published in 1916 is the sole effort in this direction and, while it marks a great advance, there still remains much to be done. It would be difficult to find a more vicious complex among the Formicidae than that of the forms of *saevissima*. Its seemingly endless variability is enough of a problem in itself and when to this one adds misconstrued relationships, contradictions and disputes concerning questions of priority, the situation becomes so hopelessly involved that one despairs of any adequate solution. Under the circumstances it may seem paradoxical to hold that relief may result from the inclusion of forms hitherto considered representatives of another species, but such is unquestionably the case. There are no reasons why Forel should have given *moelleri* specific rank and its inclusion with *saevissima* clears up much of the confusion.

After studying a large amount of material comprising cotypes or homeotypes of practically all the described forms, I am of the opinion that the following arrangement best expresses the relationship of the various forms of *saevissima:*

saevissima (Fred. Smith)
        = *pylades* Forel
        = var. *gracilior* Forel
    var. *moelleri* (Forel)
        = var. *incrassata* Forel
        = var. *morosa* Santschi
        = var. *pylades* Forel (Santschi *emend*)
    var. *perfida* Santschi
    var. *quinquecuspis* Forel
    var. *richteri* Forel
        = var. *tricuspis* Forel
  subsp. *interrupta* Santschi
        = var. *macdonaghi* Santschi
  subsp. *electra* Forel
    var. *wagneri* Santschi

The variety *lehmann-nitschei,* Santschi does not appear in this list. I shall subsequently give reasons why this form should be dropped. The explanation for the arrangement just proposed is intimately connected with the taxonomic history of the species and can be most easily explained by a discussion of the circumstances under which the various forms were originally recognized. *Saevissima* was first described by Frederick Smith in 1855 and placed by him in the genus Myrmica. Seven years later Mayr corrected the mistake as to the genus but made an equally unfortunate error by identifying the form with *S. geminata* subsp. *rufa.* It is doubtless due to this that during the next forty years the literature contains no references to *saevis-*

sima. It is inconceivable that specimens of this widely distributed insect should have failed to reach European taxonomists over so long a period and we must, therefore, suppose that these were grouped with *rufa.* In 1904 Forel described a subspecies of *geminata* which he called *pylades,* from a female taken in Mexico. He was doubtful as to the exact status of this form and, since he had no workers, preferred to group it with *geminata* to which it showed obvious relationship. Five years later he was able to identify a complete series of material from Sao Paulo, Brazil with *pylades* and separated it as a distinct species since the largest soldier caste characteristic of *geminata* was absent. In 1915 Wheeler called attention to the long neglected *saevissima* and pointed out that Smith's description of this form exactly corresponded to *pylades.* Wheeler therefore placed *pylades* in the synonomy of *saevissima* and reduced it again to a subspecies of *geminata.* Forel bitterly opposed this view, defending both the validity of the name *pylades* and the specificity of the form thus described. He was finally forced to concede the priority of the name *saevissima* when Wheeler sent specimens to Donisthorpe for comparison with Smith's types. Forel's opinion of the specific rank of this form was, however, correct and since 1916 *saevissima* has been recognized as a separate species. Santschi in his key to the forms of *saevissima* ('16) retains *pylades* as a distinct variety but this appears to me more a sop to Cerberus than a taxonomic reality. It will be subsequently shown that the variety *pylades* must be regarded as a synonym of *saevissima* var. *moelleri.*

Before leaving the subject of the typical form it seems advisable to discuss a fact which may, if neglected, cause trouble in the future. As has already been noted Forel's original description of *pylades* was based upon a female taken in Mexico. His subsequent recognition of the worker of the species was supposed to have eliminated any doubts concerning the relationships of the female first described. Unfortunately for Forel's supposition the range of *saevissima* extends very little to the north of the Guianas. On the other hand we know that two forms (*maniosa* and *aurea*) of the North American species *xyloni* occur in Mexico. The female of this insect is very similar to that of *saevissima* and there can be little doubt that it is to some form of *xyloni* that Forel's original description of *pylades* applies. However, in view of the previous difficulties connected with the name *pylades,* it would be extremely inadvisable to replace either *maniosa* or *aurea* with the older name. I feel certain that the most ardent adherent to the law of priority would willingly yield the point in this case for the sake of the more important consideration of clarity.

We may now take up Forel's treatment of *moelleri*. This leads to a very complicated discussion involving five described forms, four of which appear to be identical. *S. moelleri* was described in 1904 from material taken at Blumenau, Brazil. In the same paper Forel established the variety *gracilior* from specimens secured at Ceara, Brazil. Since *gracilior* has only an indirect bearing on what is to follow the discussion of this form has been deferred to a subsequent paragraph. As may be noted the date of the description of *moelleri* precedes by five years Forel's recognition of the worker of *saevissima*. There is nothing in the account of the worker of *moelleri* to distinguish it from *saevissima*, but the characters given for the sexual phases are, to say the least, most peculiar. In the first place the length of the female is said to be 4.0 mm. Since *moelleri* is admittedly of close relationship to *saevissima* one would scarcely expect to find such a striking exception in the size of the female. The females of the species in the subgenus *Solenopsis* are all 6.0 mm. or more in length. However, even if we assume that *moelleri* may constitute an exception to this rule there still remain other discrepancies. In the females of the subgenus *Solenopsis* the thorax is invariably wide, almost equalling the head in width. Forel's description of the female of *moelleri* stresses the very narrow thorax, which is said to be much narrower than the head. Finally the clypeal teeth of the female of *moelleri* are described as feebler than those of the worker, a condition which does not occur in any member of the genus known to me. The male slso shows comparable irregularities of size and structure. The above considerations lead me to believe that the female and male described as the sexual forms of *moelleri* really belong to a widely different species related to *picta* or *corticalis* in the subgenus *Diplorhoptrum*. We must, therefore, consider only the worker of *moelleri* in determining its relationships.

For seven years after its description Forel made no mention of *moelleri* but in a publication of 1911 he again referred to it. I give here a translation of his statement:

"The worker of *S. pylades* (= *saevissima*) is very closely related to *moelleri* Forel, it is, however, more thickset (funicular joints somewhat thicker than long, longer than thick in *moelleri*) with thicker nodes."

This is a most extraordinary observation since there is not a single form of *saevissima* in which the funicular joints are thicker than long. Moreover, while Forel never adequately described the typical *saevissima* he did give a very complete account of the variety *richteri* in which he distinctly specifies that the funicular joints are all much longer than thick. The distinction between *moelleri* and *saevissima* would therefore seem to be largely a matter of preconceived ideas which break down upon close examination.

It is now necessary to give a brief discussion of the history of the variety *incrassata*. This insect was first noted as a form of *geminata* by Forel in 1908. The original description is quite worthless since it deals entirely with a comparison of the shape of the petiole to that of *geminata* and makes distinctions which are shown by every form belonging to *saevissima*. The following year Forel, having seen the worker of *saevissima*, rectified his error and made *incrassata* a variety of that species. This he did in a synoptic table giving no characters by which the two might be separated. Thus we have the unusual situation of the erection of a variety which, for all practical purposes was never described. As might be expected this has led to regrettable results.

It is my opinion that Forel's *moelleri* is identical with the insect which he described four years later as *incrassata*. Unfortunately I have not seen a cotype of *moelleri* but I am led to this conclusion by the following facts:

I have examined a cotype of *incrassata* and find that it exactly agrees with the description of the worker of *moelleri;* there is in the collection of Dr. W. M. Wheeler a large series of specimens collected by Witte at Blumenau (the type locality of *moelleri*) which differ in no way from the cotype of *incrassata;* finally except for the typical *saevissima* this insect appears to be the only form belonging to this species which has been taken up to the present in southern Minas Geraes, Sao Paulo and Parana.

There are two remaining synonyms in this complex, Santschi's variety *morosa* and the variety *pylades*, a form nominally under the authorship of Forel but actually set up by Santschi in 1916. *Morosa* was characterized as follows:

"Entirely brown; the legs, antennae and the anterior third of the head more or less reddish. Clypeus bidentate. Blumenau, Brazil, type locality."

Santschi in his original description of *morosa* included some specimens from Missiones, Argentina but he later corrected this, assigning the Argentinian material to the variety *richteri*. The color of the variety *pylades* was given by Santschi ('16) as deep yellow with the bordering of the gastric segments not sharply delimited. He subsequently ('23) notes that the form may be distinguished from *incrassata* by its feeble clypeal teeth. In this last observation he was entirely incorrect and we must therefore depend solely upon color as a means of separating *pylades*. Unfortunately the color of *pylades* is less distinctive than Santschi indicates. In the homeotype which he sent me the posterior two-thirds of the head and the entire thorax are dirty brownish yellow, the abdomen slightly darker, the anterior third of the head and the appendages light yellow. As may be seen the difference between this condition and that of *morosa* is slight. I have already mentioned Witte's material from Blumenau and given reasons for considering it identical with *moelleri*. In one series of workers from a single nest (No. 128) the variation in color is sufficient to include both *morosa* and *pylades*. It is consequently impossible to maintain the validity of either of these varieties since both represent insignificant fluctuations of a single variable form.

The discussion just given may be summed up as follows:

In the southeastern part of Brazil from southern Minas Geraes to Santa Catharina there is a form of *saevissima* whose color varies from dirty brownish yellow to sordid brown with the abdomen somewhat darker and the anterior third of the head and the appendages somewhat lighter than the rest of the body. This form was first described as *moelleri* and this name therefore takes precedence over the synonyms *incrassata*, *morosa* and *pylades*.

We may now consider the variety *gracilior*. I have been fortunate in having for examination a single cotype of this insect. As the name implies *gracilior* was distinguished by its slender structure. In addition the first node of the petiole was said to be thinner and with a longer peduncle than that of *moelleri*. I have compared the cotype mentioned above with *moelleri* and find that it differs in the characters specified. Unfortunately for *gracilior* exactly the same differences are shown by the typical *saevissima*. A comparison of *gracilior* and *saevissima* convinces me of their identity. I cannot attach much significance to Forel's statement that in *gracilior* the dorsum of the epinotum bears a rather distinct concavity. The epinotum of the cotype is rather badly crushed in at one side but hardly enough to obliterate a concavity had one been present. It seems to me that Forel must have intended the declivious rather than the basal face in his description since the latter is usually more or less concave.

Another described form which must be synonomized is Forel's variety *tricuspis*. This was originally supposed to differ from *richteri* in the presence of a well-developed median denticle on the anterior edge of the clypeus. The clypeus in *richteri* usually shows a low and very broad projection between the carinal teeth but this is frequently produced into a median tooth. The variation is often within a series from the same nest, hence the character is of no significance for varietal determination. The same criticism applies to the clypeal teeth of *quinquecuspis* but this form appears to be distinct since the major worker lacks the reddish yellow band at the base of the first gastric segment, which is very constant in *richteri*.

There remain for discussion the forms *interrupta, mac-donaghi, wagneri* and *electra*. The first three of these were originally described as varieties of *saevissima* while the last was given subspecific status. As indicated in the table at the beginning of the introduction I consider *interrupta* and *mac-donaghi* identical and of subspecific rank while *wagneri* must apparently be regarded as a variety of the subspecies *electra*. My reasons for synonomizing *interrupta* and *mac-donaghi* may be very briefly stated. The only difference between the two forms is the degree of infuscation of the abdomen of the minor worker. In *mac-donaghi* the gaster of the smallest worker is completely infuscate while in *interrupta* it is virtually clear yellow with only a trace of infuscation at the edges of the segments. It is obviously impossible to maintain the validity of a variety based upon a character so prone to fluctuation. Furthermore I have examined specimens of *interrupta* and *mac-donaghi* sent me by Dr. Santschi and am unable to find any differences in the two which will permit separation. The major workers of both show an expanded condition in the posterior portion of the head the significance of which will be subsequently discussed. Both forms were described in the same paper and as a result we must utilize pagination as the only means of determining priority. Curiously enough they are both described on the same page but, since *interrupta* precedes, *mac-donaghi* must go into the synonomy.

The case of *electra* and *wagneri* is less clear. Although *wagneri* was set up as a variety of *saevissima* all the characteristics in the original description were derived from a comparison to *electra*. *Wagneri* was said to differ from *electra* in its greater size and more uniform and somewhat lighter color. Unfortunately I have not seen any specimens of this form, nor is it possible to deduce its relationships from its distribution since the type locality is in a region covered by the ranges of both *richteri* and *electra.* In size and coloration it appears to be transitional between *interrupta* and *electra* and until we are better acquainted with the form it may be provisionally assigned as a variety of the latter subspecies.

A comparison of the typical *saevissima, moelleri, richteri, interrupta* and *electra* shows a marked difference in the size of the major worker. In *interrupta* these reach a length of 6.8 mm. in *richteri* 6.0 mm., in the typical *saevissima* 5.3 mm. and in *moelleri* and *electra* 4.7 mm. If this size variation were the only difference the case would be relatively simple. Actually, however, it seems at first sight that the size of the workers is of little importance since there are structural similarities apparently independent of size. In the typical *saevissima* and *moelleri* the head of the largest worker is widest just behind the eyes and, since the amount of narrowing anterior and posterior to that point is approximately equal, the head is sub-oval in outline. In *interrupta, richteri* and *electra,* on the other hand, the head is widest a short distance behind the eyes and hence of a somewhat cordate shape. This is most pronounced in *interrupta,* but in *electra* and *richteri* the head, while less cordate than in *interrupta* is more so than in *saevissima.* The explanation of these differences may, I believe, be derived from a comparison of the size of the worker and female. This is most easily presented in tabular form:

	Length of major	Length of female	Actual dif.	Percentage dif.	Shape of head in major
interrupta	6.8 mm.	8.0 mm.	1.2 mm.	15%	strongly cordate
richteri	6.0 mm.	7.7 mm.	1.9 mm.	25%	mod. cordate
saevissima	5.3 mm.	8.2 mm.	2.9 mm.	37%	oval
moelleri	4.7 mm.	7.7 mm.	3.0 mm.	39%	oval
electra	4.7 mm.	6.4 mm.	1.7 mm.	27%	mod. cordate

As may be seen from the above the relative difference in size is least in the case of *interrupta,* most in the case of *saevissima* and *moelleri* and transitional in *richteri* and *electra.* This would seem to indicate that the expansion of the posterior part of the head is a function of the degree in which the size of the worker approaches that of the female and quite independent of the actual size of the worker. Such a fact can be best explained by assuming that two distinct developmental tendencies are operating in the case of *saevissima*. One leads to the elimination of the largest caste of workers but does not effect the size of the female. The other reduces the size of both female and worker but does not eliminate the largest caste relatively speaking. If this is true we must regard *interrupta* as most nearly approximating the ancestral condition since the size of the female is undiminished and the largest caste of workers is still present. *Saevissima* represents a form produced by the operation of the first factor, the largest workers are no longer present but the size of the female is unaffected. *Electra* and to a lesser extent *richteri*, show the effect of the second factor, the size of the female is decreasing but relatively large workers are still produced. In *moelleri* both factors have been at work causing a reduction in the size of the female and the dropping out of the largest caste of workers.

Regardless of what value may be attached to the above considerations it is obvious that in the forms of *saevissima* we have an unusually clear case of incipient production of new species. The absence of some of the transitional forms would make it difficult to include the extremes in the same species. Indeed it is quite thinkable that subsequent investigations may so narrow the lines by which a species is delimited that all the forms of *saevissima* will be given specific status. At present, however, it is more consistent to regard them as members of a single highly variable species.

Before approaching the subject of geographical distribution it is necessary to discuss the status of the variety *lehmann-nitschei*. This variety has had a singularly unfortunate history and the treatment about to be proposed will be a fitting termination to the career of this taxonomic step-child. *Lehmann-nitschei* was first described by Santschi in 1916 as a variety of *tridens*. In 1923 he rectified his mistake and made the form a variety of *electra*. While Santschi was undoubtedly correct in removing *lehmann-nitschei* from *tridens*, the advisability of assigning it to *electra* is open to question, since the type material of the form is composed only of minors and small medias. The extraordinary degree of structural similarity between the small workers of the forms of *saevissima* forces one to utilize color characters for their separation. Santschi evidently realized this when he transferred *lehmann-nitschei*, but the distinctions of color which he makes are far from satisfactory. I have examined specimens of *lehmann-nitschei* sent me by Dr. Santschi and can see no reason why they might not be considered minors of *interrupta*.

On the other hand they seem to fit in equally well with the description of the variety *wagneri*. Although the locality in which the types were secured favors the first interpretation the question is impossible of certain settlement because of the initial absence of major workers and must always remain so in the event that any synonomy is attempted. Since synonomy is out of the question and since the form does not appear to me sufficiently distinct to justify varietal status I can see no reason for its retention and suggest that it be dropped as impossible of exact determination.

In the foregoing discussion the considerations have been in large part morphological. In addition it is possible to show that the arrangement of the forms of *saevissima* proposed at the beginning of the introduction is closely correlated with geographical distribution. Heretofore the distribution of the variants of *saevissima* has appeared far too hap-hazard to permit satisfactory explanation and, considering the tangled taxonomy of the species, this is hardly surprising. In view of the ever increasing importance of distributional facts in determining the value of varietal characters it is very gratifying to find an agreement between morphological features and geographical ranges.

The distribution of the typical *saevissima* is considerably more extensive than that of any one of its variants. As has been already noted the earlier records of this ant from Central America and Mexico must be regarded as belonging to *geminata* subspecies *rufa* or one of the forms of *xyloni*. It is not until we reach the Guianas that we find *saevissima* in abundance, although it must certainly occur in Venezuela and probably gets into the more eastern portions of Colombia. In the Guianas the typical form is widespread in the drier and more open portions of the country. The same is true of its occurrence in the Amazon Basin. Bates, in his classic account of this insect remarks,

"It is found only on sandy soils in open places and seems to thrive most in the neighborhood of houses and weedy villages—, it does not occur at all in the shades of the forest."

With the exception of the variety *perfida* Sants., which is a rare and local variant known only from Piricicabo, the typical *saevissima* is the only form which occurs throughout the tremendous area in Brazil extending from the boundary of the Guianas to the southern part of the state of Minas Geraes. Here it meets the variety *moelleri* and the two then extend together through the small states of Espirito

Santo and Rio de Janeiro into Sao Paulo. The range of the typical *saevissima* apparently terminates here but that of *moelleri* extends into Parana, Santa Catharina and Rio Grande do Sul. It is known from Missiones, Argentina, but I believe I am correct in stating that it does not extend across northern Argentina as formerly supposed (*vide* page 86). The variety *richteri* is first met with in Uruguay but most of the range of this insect lies in Argentina where it extends on the northwest almost to Bolivia, on the west to the Andes and southward as far as the northern boundary of the state of Rio Negro. Three other forms overlap portions of this range. The variety *quinquecuspis* occurs throughout the eastern portion of the state of Buenos Aires as far south as Bahia Blanca. The subspecies *interrupta* whose range extends into the state of Rio Negro on the south, reaches Mendoza on the west and the state of Entre Rios on the northeast. The subspecies *electra* and its variety *wagneri* are more northern in distribution, extending from Uruguay northwestward into Bolivia.

The following key presents characters for the separation of the various forms of *saevissima:*

1. Dorsum of the thorax in the majors and larger medias rectilinear or sub-rectilinear in profile, the epinotum long and low with the short declivious face completely covered with transverse striae, color reddish brown to sordid brown 2
   Dorsum of the thorax in the majors and larger medias arcuate or broken in profile, the epinotum only slightly longer than high, the declivious face striate only at the lower portion or if completely striate the color is a deep piceous brown 3
2. Gaster of the majors and large medias with a basal band of sordid yellow .... subsp. *electra*
   Gaster in all castes uniform piceous brown .... var. *wagneri*
3. Head of the majors strongly cordate, color bright yellow . . subsp. *interrupta*
   Head of the majors oval, or if moderately cordate the color is piceous brown 4
4. Head and thorax pale yellow to golden yellow, the abdominal segments more or less tinged with brown .... *saevissima* s .str.
   Head and thorax sordid brownish yellow to deep piceous brown, the abdominal segments more or less piceous 5
5. Head and thorax sordid brownish yellow to dirty brown the gastric segments edged with piceous brown .... var. *moelleri*
   Head and thorax deep piceous brown, the gaster wholly piceous or piceous except for a roseate band at the base of the first segment ... 6
6. Base of the first gastric segment in the larger workers with a broad transverse band of reddish yellow .... var. *richteri*
   Gaster in all castes uniform piceous brown .... var. *quinquecuspis*

The variety *perfida* is not included in the key. I have not seen the form and the characters given in the description are so similar to those of the variety *moelleri* that its inclusion would certainly lead to difficulties. The reader is referred to the translation of Santschi's description on page 86.

Solenopsis (Solenopsis) saevissima (F. Smith).

S. saevissima, F. Smith, Trans. Ent. Soc. Lond. (2), Vol. 3, p. 166, pl. 13, fig. 18, 1855. (*Myrmica.*) ☿

S. saevissima, Santschi, Physis Buenos Aires, Vol. 2, p. 378–380 (1916). ☿

S. pylades, Forel, Bull. Soc. Vaud. Sci. Nat. Vol. 51, p. 723 (1917); Deutsche Ent. Zeit., p. 279 (1911). ☿ ♀

S. geminata var. *pylades*, Forel, Ann. Soc. Ent. Belg., Vol. 48, p. 172 (1904). ♀. Forel, Deutsche Ent. Zeitschr. p. 268 (1909).

S. geminata subsp. *saevissima*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 395 (1915). Wheeler, Psyche, Vol. 23, p. 142 (1916). Wheeler, Proc. Cal. Acad. Sci., Vol. 2, p. 272, (1919).

S. moelleri var. *gracilior*, Forel, Ann. Soc. Ent. Belg. ,Vol. 48, p. 174 (1904). ☿

Worker major: Length, 5.3 mm. (Plate II, figs. 1, 2.)

Head, exclusive of the mandibles, as broad as long, the sides moderately convex, the occipital angles strongly rounded, the occiput with a shallow and rather narrow median concave impression, a feeble groove extends from the front to the occiput. Clypeus feebly projecting, the carinae prominent, strongly divergent, angular above and terminating in rather short but very stout teeth; lateral denticles smaller and closely appressed to the carinal teeth, a low, feebly convex median lobe lies on the anterior margin of the clypeus between the carinal teeth. Mandibles with the external border moderately and evenly convex, the masticatory border very oblique and armed with four teeth of which the innermost is considerably smaller but quite distinct. The antennal scapes in repose fail to reach the occipital border by a distance slightly exceeding their greatest thickness. First funicular joint four times as long as broad, as long as the following three together. Second funicular joint twice as long as broad, the remaining small joints gradually decreasing in length and increasing in thickness, the seventh joint approximately one and one-half times as long as thick. Club slender, the terminal joint twice as long as the penultimate. Eyes oval, of about ninety facets, separated from the insertion of the mandibles by a distance approximately one and one-half times their maximum diameter.

Thorax seen from above with the humeral angles much rounded, the pronotal suture obsolete but its position indicated by a shallow groove, the constriction of the sides at the mesoepinotal suture only moderately strong. Seen in profile the promesonotum is moderately convex throughout except at the extreme posterior end where it descends to the mesoepinotal suture by means of a short, almost perpendicular declivity. Mesoepinotal suture deep and narrow Epinotum in profile notably lower than the adjacent portion of the mesonotum, the basal face flat or very feebly convex and at least one-third longer than the declivious face. Node of the petiole in profile obtusely triangular, about one-fourth higher than the length of its base, the anterior face virtually straight, the posterior face slightly convex, the summit bluntly rounded; peduncle thick, scarcely longer than the base of the node and without a ventral tooth. Postpetiole in profile very slightly higher than thick, the summit broadly and feebly convex, the ventral face with an obtusely rounded anterior projection. Seen from above the node of the petiole is sub-triangular in outline and slightly narrower than the transversely oval postpetiole. Anterior border of the first gastric segment narrowly truncate.

Color pale yellow to reddish yellow, the abdominal segments margined with brown. In some specimens this brownish coloration may cover most of the segment but it is usually confined to a rather distinct band at the posterior edge of the segment. Mandibles completely covered with coarse, irregular striae. The area between the anterior margin of the eye and the insertion of the mandible feebly striate. Meso- and metapleurae and the lower portion of the declivious face of the epinotum covered with very fine striae, base of the postpetiole finely reticulate. Punctures small and moderately numerous. The rest of the insect smooth and shining. Hairs long, erect, golden on the body; those on the appendages somewhat shorter but stout and erect except on the antennal club where they are very short, fine and subappressed.

Worker minor: Length 2.8 mm. (Plate II, figs. 9, 11.)

Head exclusive of the mandibles one-eighth longer than broad, the sides strongly narrowed from the eyes to the occiput, the latter flat. Clypeus moderately projecting, the carinal teeth small and acute, median and lateral denticles reduced to sinuousities in the anterior edge of the clypeus. The antennal scape surpasses the occipital border by slightly less than its greatest thickness. First funicular joint slightly shorter than the following two together. Second joint about one and one-half times as long as broad, third joint slightly longer than broad, the remaining small joints approximately as broad as long, club as in the major worker. Eyes of forty to forty-five facets, separated from the insertion of the mandible by a distance only slightly exceeding their maximum diameter.

Dorsum of the thorax without a trace of the promesonotal suture, the pronotum feebly convex in profile, the mesoepinotal suture with a moderately strong impression, the epinotum angular, about as high as long with its basal face lower than the mesonotum. Node of the petiole in profile approximately as high as it is thick at the base. Postpetiole in profile slightly thicker than high. Seen from above the postpetiole is subglobular and about one-fourth wider than the node of the petiole. Area between the eye and the insertion of the mandible and the declivious face of the epinotum smooth. Otherwise as in the major worker.

The conditions in the medias vary between the extremes described.

Female: Length 8.2 mm. (Plate II, fig. 3.)

Head exclusive of the mandibles as broad as long, markedly quadrate, the occiput with a narrow, concave, median impression. Clypeal carinae very strong, less divergent than in the worker and more rounded above, their terminal teeth very short and stout but quite sharp. Lateral and median denticles reduced to almost imperceptible flattened lobes. The antennal scapes in repose slightly surpass the lateral ocelli. Funicular joints as in the major worker. Eyes large, strongly convex, oval, a line drawn through the middle of the side of the head would pass through their posterior third. Other cephalic characters as in the major worker.

Thorax slightly narrower than the head, oval, slightly less than twice as long as broad. Basal face of the epinotum meeting the declivious face at a very wide angle, the two virtually forming a continuous slightly arcuate slope. Node of the petiole in profile high and very acute, both anterior and posterior faces straight, the summit narrow and sharp, peduncle short and thick and without a ventral tooth. Postpetiole in profile much higher than long, the summit obtusely rounded, the ventral face very short. Seen from above both nodes are strongly transverse, the postpetiole about one-sixth wider than the node of the petiole. Wings hyaline the veins pale yellow.

In sculpture, pilosity and color the female is essentially the same as the major worker.

I have not seen the male of the typical *saevissima*. For a description of this caste the reader is referred to the variety *moelleri* the male of which appears to be identical with that of *saevissima* except for its somewhat smaller size. The male of *saevissima* measures 6.2 mm. in length.

It is exceedingly difficult to compile a list of localities for *saevissima* since many of the older records were assigned to *geminata*. The following list represents those for which there is little or no possibility of error.

Localities: British Guiana, Georgetown. (Crampton, H.O. Lang.)
Kartabo. (W. M. Wheeler, Wm. Beebe.)
Brazil, Para, Obidos. (No collector.)
Itacoatiara. (W. M. Mann.)
Manaos. (W. M. Mann.)
Tapajos River. (Type loc.)
Ceara, Ceara. (Diaz da Rocha.)
Minas Geraes, Diamante. (T. J. Borgmeier.)
Matto Grosso, Cuyaba. (No collector.)
Sao Paulo, Santos. (Ris.)
Bahnhof. (Luderwald & von Ihering.)
Riaz da Sierra. (Luderwald & von Ihering.)
Ypiranga. (Luderwald & von Ihering.)

Solenopsis (Solenopsis) saevissima var. **moelleri** (Forel).

S. saevissima var. *moelleri* (Forel) Ann. Soc. Ent. Belg., Vol. 48, p. 173 (1904). ☿ (*S. moelleri.*)

S. geminata var. *incrassata* Forel, Verh. Zool-bot. Ges. Wien, Vol. 58, p. 362 (1908). ☿

S. pylades var. *incrassata*, Forel, Deutsche Ent. Zeitschr., p. 269 (1909).

S. geminata subsp. *saevissima* var. *incrassata*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 397 (1915).

S. saevissima var. *incrassata*, Santschi, Physis. Buenos-Aires, Vol. 2, p. 380 (1916). ☿

S. saevissima var. *morosa*, Santschi, Ibid., Vol. 2, p. 381, (1916). ☿. Rev. Suisse Zool., Vol. 30, p. 265 (1923).

S. saevissima var. *pylades*, Santschi, Physis Buenos-Aires, Vol. 2, p. 380 (1916) ☿ Rev. Suisse Zool. Vol. 30, p. 267 (1923) ☿ (*nec pylades*, Forel, 1904 etc.)

Aside from its darker color and smaller size the workers of the variety *moelleri* differ from those of the typical *saevissima* in a number of characters. In the minors and smaller medias the thorax is distinctly stouter, the ratio of the length (neck of the pronotum included) to the width is 4.5: 2, (4.5 : 1.8 in *saevissima*). The constriction at the mesoepinotal suture is more pronounced, the node of the petiole in profile is thicker and, seen from above, both nodes are notably broader in *moelleri* than the typical *saevissima*. These differences are not shown by the larger medias and majors which are virtually identical with *saevissima* workers of corresponding size except for their darker color.

While the color of the variety *moelleri* is variable in regard to shade, the pattern is quite constant. Over most of the body the color varies from a dirty, brownish yellow to a sordid, blackish brown. The anterior third of the head, mandibles and appendages are of a lighter shade, their color varying from a light yellow to yellowish brown. In the more darkly colored individuals the difference is not striking and is usually less pronounced in the medias and not shown by the minors whose coloration is uniform. In all castes the edges of the gastric segments are infuscated.

Female: Length 7.7 mm.

Identical with the female of *saevissima* except for its smaller size and darker color. General color of the insect rich castaneous brown, the anterior third of the head, mandibles and antennae brownish yellow. The thorax, which is slightly more piceous than the head, has two dull, orange bands extending longitudinally across the scutum. The three anterior gastric segments bear a small basal spot of yellow at either side.

Male: Length 6.0 mm.

Head including the eyes approximately one and one-third times as broad as long, broadest through the middle of the eyes, much narrowed behind with the posterior border evenly convex. The very large eyes occupy more than half of the side of the head, the anterior border extending almost to the insertion of the mandible. Clypeus with an elevated subtriangular median lobe, the lateral portions flat and sharply marked off from the median lobe. Mandibles linear, with a single long terminal tooth and one or two small denticles on the inner margin. First joint of the antenna approximately one and one-half times as long as broad, rather expanded basally and narrowed apically. Second joint short, much broader than long, broader than the following joint. Third joint more than twice as long as broad. Fourth joint one and one-half times as long as broad, the remaining joints gradually increasing in length and decreasing in thickness, the terminal joint about six times as long as broad. Ocelli prominent, the area between the lateral ocelli very feebly or not at all impressed.

Thorax bulky, elliptical, approximately three-fifths as broad as long, and one and two-fifths times as broad as the head. Thorax in profile with a narrow pronotum, the scutum very bulky but only slightly overhanging the pronotum, scutellum feebly convex and somewhat higher than the scutum. Basal face of the epinotum consisting of two distinct portions, a large, transversely convex, slightly sloping upper portion and a smaller, triangular, strongly sloping posterior portion; declivious face short and perpendicular.

Node of the petiole in profile low, the anterior face forming a continuous low slope with the rather thick peduncle, the posterior face short and declivious. Postpetiole in profile very low and thick, the dorsum evenly convex. Seen from above both nodes are strongly transverse, the postpetiole slightly wider and shaped like a much-compressed spindle. Wings hyaline with yellow veins.

Head rugulose, the epinotum, peduncle and base of the petiolar nodes finely striate, the rest smooth and shining with moderately large and rather sparse punctures. Hairs of medium length, yellow, erect. Abundant on the head, the sides of the scutum, the posterior half of the scutellum, epinotum, petiolar nodes and ventral surface of the gaster. Sparse on the pronotum, epimera, mesothoracic sternite and the dorsum of the gaster. Completely absent from a narrow, median, longitudinal area on the scutum. The color varies from a piceous brown to a very deep piceous black. The abdomen is always a trifle lighter than the head and thorax. The thorax sometimes bears two longitudinal brownish yellow stripes. Legs and antennae brownish yellow to dirty brown.

Localities: Brazil, Minas Geraes, Pirapora (Gerbe). (Thayer Exp.)
Rio de Janeiro, Petropolis. (T. J. Borgmeier.)
Santa Catharina, Blumenau (type loc.) (Moeller, Reichensperger, Witte.)
Sao Paulo, Sao Paulo (von Ihering.)
Tipuca River. (No collector.)

Rio Grande do Sul, Taquara (von Ihering.)
San Lourenco (von Ihering.)
Camaquam (von Ihering.)
Argentina, Missiones (von Stieger.)

This insect has been reported by Bruch from Mendoza, Jujuy and Tucuman, Argentina, but I believe that these records belong to the subspecies *electra*. If *moelleri* occurred so far to the west and southwest we should certainly expect to have records from the intervening territory, particularly as this would include the much-worked region between Buenos Aires and Sante Fe. There is nothing to indicate that this form extends to the south of Rio Grande do Sul.

The unusually complex synonymy of the variety *moelleri* has already been taken up in the introduction. For a full discussion of the matter the reader is referred to page 72.

Solenopsis (Solenopsis) saevissima var. **perfida** Santschi.

S. saevissima var. *perfida*, Santschi, Rev. Suisse Zool. Vol. 30, p. 266, (1923). ☿

I have seen no specimens referable to this variety. A translation of the original description is given below:

"Worker: dull yellowish or reddish with the appendages and the border of the gastric segments reddish yellow, the middle of the femora and the tibiae brownish. Two small teeth on the clypeus. Head of the minor workers not sensibly narrowed behind. Sixth and seventh funicular joints no longer than thick. Otherwise as in the type."

Type locality: Brazil, Minas Geraes, Piracicabo (E. Luja).

Solenopsis (Solenopsis) saevissima var. **quinquecuspis** Forel.

S. saevissima var. *quinquecuspis*, Forel, Bull. Soc. Vaud. Sci. Nat. Vol. 49, p. 224 (1913). ☿ (*pylades* var.)

S. geminata subsp. *saevissima* var. *quinquecuspis*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 379 (1915).

S. saevissima var. *quinquecuspis*, Santschi, Physis, Buenos Aires, Vol. 2, p. 381 (1916). ☿

As the name implies this variety was originally founded upon a supposed variation in the number of clypeal teeth. The variety *richteri*, to which *quinquecuspis* is closely related was assumed to have only two teeth on the anterior edge of the clypeus. Actually *richteri* is highly variable in this respect and consequently the number of clypeal teeth is of no significance for varietal status. In the case of *quinquecuspis*, however, there is another difference which permits separation. Although there is some ambiguity in the original description, I believe I am correct in assuming that this insect lacks the yellow band at the base of the first gastric segment which characterizes *richteri*. Should this prove not to be the case I can see no way in which *quinquescuspis* could be separated from *richteri*.

Localities: Argentina, Buenos Aires, Bahia Blanca (Type loc.)
(Zelenka and Olavaria.)
Puerto Militar (Zelenka and Olavaria.)
Buenos Aires (Weise).
Monte Hermoso. (Carette.)

Solenopsis (Solenopsis) saevissima var. **richteri** Forel.

S. saevissima var. *richteri*, Forel, Deutsche Ent. Zeitschr., p. 267 (1909). ☿ ♀ (*pylades* var.)

S. geminata subsp. *saevissima* var. *richteri*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 297 (1915).

S. saevissima var. *richteri*, Santschi, Physis Buenos Aires, Vol. 2, p. 381 (1916). ☿

S. pylades var. *tricuspis*, Forel, Mem. Soc. Ent. Belg., Vol. 20. p. 4 (1912). ☿

S. geminata subspecies *saevissima* var. *tricuspis*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 397 (1915).

S. saevissima var. *tricuspis*, Santschi, Physis Buenos Aires, Vol. 2, p. 381 (1916). ☿

The majors and medias of this variety are easy to distinguish because of their very characteristic coloration. The entire insect is a deep, piceous brown with the exception of the first gastric segment which bears at its base a broad band of reddish yellow. The smallest workers lack this band and are sometimes difficult to distinguish from those of *moelleri* although they are usually much darker. The thorax of the smaller medias and minors is stout as in the variety *moelleri*. The size of *richteri* ranges from 2.8 mm. to 6.0 mm. and in the largest workers the head is more cordate than in *moelleri* or the typical *saevissima*. The number of teeth on the clypeus is variable. There may be two, three or five but the carinal teeth are always much longer and thinner than in *moelleri* or the typical *saevissima*.

With the exception of its smaller size, longer carinal teeth, slightly thicker petiole and different color the female of *richteri* exactly resembles that of the typical *saevissima*. Its color is a dirty yellowish brown with the clypeus, the frontal area of the head and two longitudinal stripes on the scutum sordid yellow. In certain specimens other thoracic sclerites bear markings of yellow but these appear to be rather variable. The gaster is uniformly brown, except the base of the first segment which is reddish yellow. The presence of this yellow band on the first gastric segment is the most outstanding difference between the female of *richteri* and that of *moelleri,* although the longer carinal teeth of *richteri* furnish another separatory character. Length 7.7 mm.

I can detect no differences between the male of *richteri* and that of *moelleri* other than the somewhat greater size of the former.

Localities: Uruguay, Montevideo (Richter, Ducloux, von Steiger.)
Neuva Helvecia. (von Steiger.)
Herrero. (Ducloux.)
Argentina, Missiones. (Bruch.)
Salta, Jujuy. (Scheul.)
Mendoza. (Carette.)
Buenos Aires, Buenos Aires. (Type loc.) (Richter, von Steiger.)
La Plata. (Bruch.)
Tigre. (von Steiger.)
Delta Parana. (von Steiger.)
Sierra Ventana. (Bruch.)

As has already been noted in the introduction, it is necessary that *tricuspis* be regarded as a synonym of *richteri,* since the number of clypeal teeth in *richteri* is too variable to permit the recognition of a variety based upon this character alone.

That the variety *richteri* has been introduced into the United States is a fact which may in the future attract the attention of economic entomologists. Three or four years ago I was surprised to find specimens of this insect in material collected in Mobile, Alabama. Mr. H. P. Löding, who has made an intensive study of the insects of that region is of the opinion that its first appearance may be dated about eleven or twelve years ago. It apparently preceded the Argentine ant, for upon the arrival of that pest, it was driven out of the infested area with the native ants. My own observations in Mobile are too meager to be of much value but they indicate that *richteri* is spreading slowly to the north of the city. The area occupied at present is small, not extending more than six or seven miles from Mobile, but there is no reason why the insect should not spread widely since its range in South America lies almost entirely in the south temperate zone. In certain sections of Mobile *richteri* has become a nuisance by disfiguring lawns with its unsightly crater nests. These vary somewhat according to the surroundings but when in the open are usually in the form of flattened domes, of smooth texture on the outside but of very loose construction within. One particularly large nest which I found at the edge of a pine wood was neatly thatched with bits of pine needles. This ant frequently makes long tunnels close to the surface of the soil. Portions of the roof subsequently fall in, leaving irregular trench-like areas through which the insects pass.

Solenopsis (Solenopsis) saevissima subsp. **interrupta** Santschi.

S. saevissima var. *interrupta*, Santschi, Physis Buenos Aires, Vol. 2, p. 397 (1916).

S. saevissima var. *macdonaghi*, Santschi, Physis Buenos Aires, Vol. 2, p. 397, 380 (1916).

S. geminata subsp. *pylades* Bruch, Rev. Mus. La Plata, Vol. 23, p. 313 (1916).

The color of the subspecies *interrupta* is very similar to that of the typical *saevissima* and it seems fairly certain that many of the older records from Argentina attributed to the latter form really belong to *interrupta*. The presence of a caste of very large workers with distinctly cordate heads is the most distinguishing characteristic of *interrupta*. Some of these large individuals reach a length of 6.4 mm. and these show the cordate condition of the head to a very marked degree. However, it is quite noticeable in specimens 5 mm. in length and serves to separate these from *saevissima* workers of corresponding size.

In the largest workers the head is widest at the posterior third, the occiput is divided by a strong V-shaped median impression into two feebly convex lobes. Frontal groove very prominent, extending from the base of the antennal lobes to the occiput. In it lies a single ocellus. Clypeal teeth long and very stout, lateral denticles well developed, median denticle present and variable in size. The innermost mandibular tooth reduced to a mere angle between the masticatory and inner borders. The antennal scapes in repose reach the posterior fifth of the head. The eyes are bounded by a shallow groove or trench which extends forward as a fan-shaped area of striae to the insertion of the mandibles. Thorax rather bulky. Promesonotal suture strongly marked. The mesonotum in profile feebly convex, slightly higher than the pronotum, its posterior portion very little higher than the adjacent portion of the epinotum, the latter with a long and very feebly convex basal face. This gives the entire thorax a broadly arcuate outline which is broken only by the narrowly impressed mesoepinotal suture. Seen from above the sides of the thorax are only slightly constricted at the mesoepinotal suture. Petiolar nodes much as in the typical *saevissima*, but somewhat higher and thinner in profile. The postpetiole bears a prominent anterio-ventral tooth. Seen from above both nodes are strongly transverse, the petiole slightly more than three-quarters as wide as the postpetiole. Gaster truncate at the base. Color rich, reddish yellow, the abdominal segments bordered with brown, the bordering interrupted at the median line. The width of the bordering varies to some extent but in the majors and large medias never covers the entire segment.

The small medias and minors of *interrupta* are exceedingly similar to those of the typical *saevissima*. There is, of course, no difference in the shape of the head and the color is virtually identical.

Female: Length 7.0–8.0 mm.

I have not seen the female of *interrupta*. Santschi describes it as similar to the major worker except for the presence of three brown bands on the thorax. This character appears to be quite distinct since it is not shown by the typical *saevissima*, and in *richteri*, in which the thorax is also banded, the conditions are reversed there being two yellow bands on an otherwise brown thorax. For this reason it seems fairly certain that the insect described by Bruch in his publication of 1916 (*vide supra*) as *S. geminata* subsp. *pylades* is really *interrupta*. Although the size of the female which he describes is smaller than that mentioned by Santschi the relationship of the two is quite obvious. A translation of Bruch's description of the sexual forms is given below:

"Female: Length about 6.0 mm. Color castaneous yellow as in the worker. Dorsum of the thorax with three longitudinal brown bands, a median anterior band and two lateral bands situated more to the rear. The rest of the dorsum is darker, the same as the nodes. The abdomen is nearly black with the anterior part as in the worker, castaneous yellow, the segments bordered with the same color but deeper. The wings are subhyaline, their veins very pale yellow. Pilosity the same as in the worker. The head is square, not enlarged behind. The eyes are very large and very convex. The clypeal teeth are very short and very obtuse. The head is smooth and glistening with the punctuation feeble and sparse.

Male: completely black, very shining and brilliant, the funiculi and tarsi are a brownish yellow, the tibiae and the articulations of the joints sordid brown. The epinotum is somewhat opaque, finely reticulate and partially striate, the sides of the nodes are also reticulate. The head is very small, subglobular. The thorax is much broader than in the female and more convex. The first node is much wider at the apex than at the base, terminating in a horizontal edge and not rounded as in the worker and female. The second node is subglobular, the anterio-lateral part angular."

Localities: Argentina, Cordoba, Cordoba. (von Steiger.)
Santa Fe, Santa Fe. (von Steiger.)
Mendoza, Mendoza. (Jensen-Haarup.)
Chibuto. (Durione.)
Entre Rios, Estacion Sosa. (MacDonagh.)
Villaguay. (Bruch.)
Buenos Aires, Moreno. (Nordenskjold.)
Monte Hermoso. (Carette.)
Bajo Hondo (type loc.) (Carette.)
Rio Negro, Colonia Frias. (Lehmann-Nitsche.)

I have synonymized *macdonaghi* with *interrupta* for the following reasons:

The color of the two forms is identical except for a difference in the amount of infuscation of the abdomen. This difference is most clearly shown by the minor workers. The gaster of the minor of *interrupta* is said to be clear yellow with the bordering of the segments virtually obliterated. In the minor of *macdonaghi* the gaster is uniform piceous brown. While I have not seen a large series of workers of *interrupta*, I have repeatedly observed, in the typical *saevissima*, fluctuations in gastric infuscation among workers from the same nest. These are frequently of a sufficient magnitude to include both conditions described above. Such variations must be considered individual differences and cannot be used as a basis for varietal status. In the second place the majors and large medias of *macdonaghi* show an interruption of the bordering of the gastric segments. This is less conspicuous than in *interrupta*, where the fuscous band is narrower, but is quite distinct nevertheless. It is hardly necessary to add that the large workers of both have cordate heads. It is my opinion, therefore that *interrupta* and *macdonaghi* represent insignificant variations of a single form.

Solenopsis (Solenopsis) saevissima subsp. **electra** Forel

S. saevissima subsp. *electra*, Forel, Bull. Soc. Vaud. Sci. Nat., Vol. 50, p. 274, (1914). ☿ (*pylades* subsp.)

S. geminata subsp. *electra*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 397 (1915).

S. saevissima subsp. *electra*, Santschi, Physis Buenos Aires, Vol. 2, p. 381, (1916). ☿

Worker major: Length 4.7 mm. (Plate II, figs. 6, 7.)

Head strongly cordate. Mesonotum not convex, together with the basal face of the epinotum forming an almost rectilinear dorsum to the thorax. Epinotum with a long basal face and a very short declivious face, the latter entirely covered with transverse striae. Petiolar nodes relatively narrow, the postpetiole scarcely wider than the node of the petiole. In other regards the structure is the same as that of the typical *saevissima*.

The minors of *electra* are difficult to distinguish from those of the variety *moelleri*. The eyes are slightly larger, being composed of fifty to fifty-five facets. The epinotum is longer and lower with the short declivious face transversely striate throughout. The minors of the variety *richteri* also have the declivious face of the epinotum transversely striate but these can usually be distinguished from those of *electra* by their darker and more uniform color. Length of the minor worker 2.8 mm.

The color of the subspecies *electra* varies from a clear reddish brown with the abdomen piceous brown to dirty piceous brown with the abdomen deep piceous. In the larger specimens the base of the first gastric segment bears a band of dirty yellow.

The female of the subspecies *electra* has not been described. I have seen a single specimen taken by T. J. Borgmeier at Tintina, Argentina, which seems without doubt to belong to this form. Unfortunately the largest worker of the series taken with the female measures only 3.7 mm. and consequently I am deprived of the most outstanding characteristic for determination, the cordate head of the major worker. Nevertheless the workers so exactly correspond to those of similar size in a series sent me by Dr. Santschi that I have little hesitancy in regarding the two as belonging to the same subspecies. The female is described below.

Female: Length 6.4 mm. (Plate II, fig. 8.)

Head exclusive of the mandibles as broad as long, quadrate, the sides narrower in front of the eyes than behind them, occiput with a narrow, concave, median impression. Clypeus feebly projecting except in the middle where the two carinae project well forward. Carinae moderately divergent in their anterior two-thirds, parallel in their posterior third. The anterior edge of the clypeus between the carinae feebly concave. Carinal teeth unusually stout and rather blunt, lateral denticles absent, a small median denticle occurs between the carinal teeth. Mandibles with four teeth, the innermost much smaller than the others. Eyes large, oval, strongly convex, their anterior fifth flattened so that the eye is not uniformly convex. The antennal scape in repose slightly surpasses the lateral ocellus. First funicular joint a little longer than the following two together. Second joint approximately twice as broad as long, third joint one and one-half times as long as broad, the remaining small joints all somewhat longer than broad. Club slender, two-thirds as long as the remainder of the funiculus, the terminal joint one and three-fourths times as long as the penultimate.

Thorax oval, very slightly less than twice as long as broad, five-sixths as wide as the head. The two faces of the epinotum rather confluent, the basal face with a broad and shallow median longitudinal impression. Petiole and postpetiole as in the female of *sacvissima* except that the node of the petiole is thinner. Seen from above the petiole is five-sixths as wide as the postpetiole.

Cephalic punctures rather prominent and moderately numerous. The mandibles coarsely and irregularly striate. The area between the anterior border of the eye and the insertion of the mandible and the entire epinotum, with the exception of the anterior part of the basal face, finely and evenly striate. Peduncle and base of the petiolar nodes finely rugulose. Hairs long, erect, golden. Color rich golden yellow, the gaster reddish brown with a broad band of yellow at the base of the first segment, the remaining segments each with a pair of lateral yellow spots.

Localities: Argentina, Jujuy. (Type loc.) (Scheub.)
Tucuman. (Shipton.)
Salta, Rosario del Frontera. (Weiser.)
Cordoba, Cordoba. (Scott.)
Santiago del Estero. (Merkle?)

Solenopsis (Solenopsis) saevissima subsp. **electra** var. **wagneri,** Santschi

S. saevissima var. *wagneri,* Santschi, Physis Buenos Aires, Vol. 2, p. 328, (1916). ☿. Rev. Suisse Zool. Vol. 30, p. 266, (1923) ☿.

I have seen no workers which could be certainly referred to this form. It is apparently very similar to *electra* from which it differs principally in its larger size and the lack of the yellowish band at the base of the first gastric segment.

Localities: Bolivia, Carandaiti. (Lizer and Deletang.)
Argentina, Santiago del Estero, Icano. (Type loc.) (Wagner.)
Uruguay, Trinidad. (Joergensen.)

S. (SOLENOPSIS) TRIDENS FOREL.

Only two forms of this rare insect are known. They may be separated as follows:

1. Basal face of the epinotum entirely smooth; color brownish black, antennae and legs brown, mandibles brownish red............*tridens*
Basal face of epinotum at least in part covered with feeble transverse striae; color reddish yellow, the gaster and petiolar nodes black, appendages and occiput tinged with brown......var. *substituta*

S. (Solenopsis) tridens Forel.

S. tridens, Forel, Deutsche Ent. Zeitschr., p. 298 (1911.) ☿

I have seen no specimens referable to the typical *tridens.* A translation of Forel's description is given below.

"Worker: Length 2.8–3 .2 mm.

"Mandibles 4-toothed, smooth with coarse scattered punctures and several striae. The two carinae or keels of the clypeus are sharp, divergent, rather strong towards the front and ending in two teeth. Between these but further under lies a longer, more pointed, very prominent median tooth (much stronger than in *wassmani* Em.). Moreover at each side of the teeth there is a blunt scarcely tooth-shaped angle. The whole body slender. Head about one-fifth longer than broad, broader in front, significantly narrowed behind with the sides behind the eyes moderately convex and with convex although still furrowed posterior border. Frontal furrow absent. Frontal area significant. Eyes convex, relatively large, very slightly in front of the middle of the head, numbering about 60–70 facets. The antennal scape surpasses the posterior border of the head by about its own thickness. Funicular joints all longer than thick, seventh and eighth only slightly so. Promesonotal suture absent. Thorax strongly constricted. Basal face of the epinotum a good one and a half times longer than broad, almost rectangular, somewhat beveled (feebly convex transversely, more strongly convex longitudinally), terminated by two very blunt rounded protuberances. Declivious face much shorter, very steep without being perpendicular. First node much broader than long, seen from the side bluntly conical, feebly convex before and behind, thickened at the base, peduncle very thin (as long as the base of the nodes). Second node as broad as long but narrowed in front and behind, scarcely broader than the first, the summit sloping at the back but rounded throughout, as high as the first. Legs slender.

"Mesopleurae finely wrinkled, otherwise completely smooth and strongly shining. Head and abdomen with distinct though scattered small punctures. Pubescence not distinct. Hairs sparse, fine, golden and pointed, somewhat shorter, more numerous and more inclined on the tibiae and antennal scapes.

"Brownish black; epinotum, antennae and legs brown, mandibles brownish red."

Sexual forms unknown.

Type locality: Brazil (East Bahia) Villa Nova. (Garbe.)

Although Forel does not mention the presence of well developed humeral angles in the type we may assume that the insect shows this character since the angles are unusually prominent in the variety *substituta* Sants., a form which appears to differ mainly in its lighter color. These angles together with the blunt protuberances at the posterior edge of the basal face of the epinotum form the principal means of distinguishing this species from *saevissima*. The presence of the median clypeal tooth which Forel utilized as a means of recognition is of no value since the same structure occurs in some of the forms of *saevissima*.

S. (Solenopsis) tridens var. **substituta** Santschi.

(Plate IV, figs. 1, 2.)

S. tridens, var. *substituta*, Santschi, Bull. Soc. Ent. Belg., Vol. 65, p. 236, (1925). ☿, ♀.

This form differs from the typical *tridens* in its lighter coloration. The head and thorax are reddish yellow, the occiput, antennae, distal half of the femora and the remaining joints of the legs tinged with brown, the gaster and in some specimens the upper parts of the petiolar nodes piceous. The basal face of the epinotum is sculptured with indistinct transverse striae.

A single cotype which I was able to examine through the kindness of Dr. Santschi shows a very marked development of the humeral angles of the pronotum. Seen from above the anterior edge of the pronotum is very feebly convex and meets the sides at a sharp angle. The resulting outline of the pronotum is completely different from that of *saevissima* in which the anterior edge is strongly and evenly convex and the humeral angles so rounded that they can hardly be said to exist. The epinotum of *tridens* is unique. Seen from above the lateral bordering is sinuate and diverges at the junction of the basal and declivious faces to form two broad, low, obtusely angular carinae.

Santschi has given a brief account of the female of the variety *substituta* a translation of which is included here:

"Female: Length 6.5 mm.

"Except for the gaster which is black the color varies from a clear reddish yellow to a much darker reddish yellow with the upper side of the nodes, the thorax and the vertex brownish. Wings hyaline, veins yellowish. The head is scarcely longer than broad. The scapes slightly surpass the posterior border of the head."

Male unknown.

Type locality: Brazil (Sao Paulo) Pitangora. (Luderwaldt.)

S. XYLONI (MACCOOK).

Introduction.

After a careful study of a large amount of material from the southern and southwestern United States I am convinced that the forms *xyloni*, *maniosa*, *aurea* and *amblychila* are closely related variants of a single species. The specificity of *aurea* has already been established but since *xyloni* was described thirty years earlier this name must be given priority. The forms may be arranged as follows:

S. *xyloni* MacCook.
    var. *maniosa* Wheeler.
  subsp. *aurea* Wheeler.
  subsp. *amblychila* Wheeler.

The members of the group thus set up differ from *geminata* through the absence of the largest worker caste and through the more rounded head and evenly curved, three-toothed mandibles of the large workers. The absence of the fourth mandibular tooth also serves to separate these forms from *saevissima* but a much more striking difference is offered by the uniformly shorter antennal scapes of *xyloni*. These, even in the minor worker where they are longest, never reach the occipital border. In this character *xyloni* shows an approach to *gayi* of Chile but in the latter species the head of the large worker is more quadrate.

The first mention of *xyloni* seems to be attributable to Buckley. In 1866 he described a *Myrmica* (*Atta*) *sabeana* which may have been the insect we now designate as *xyloni*. Buckley's descriptions have been the cause of endless vituperation by myrmecologists and there is no need to increase the stock of expletives expended on this misguided worker. His description has long been abandoned as utterly worthless and MacCook's account of *xyloni*, published in 1879, is considered the original recognition of this form. To an impartial observer such procedure might seem a jump from the frying pan into the fire. MacCook's description is even less complete than that of Buckley but his figures prove that he was describing a *Solenopsis* and his specifications of size and color enable us to recognize the form with reasonable accuracy. Before describing *xyloni* MacCook sent specimens to Forel for examination. These were pronounced identical with *geminata* but MacCook refused to accept this interpretation since he had noted the absence of the largest soldier caste in his specimens and rightly considered them representatives of a separate species. It is interesting to note that thirty years later Forel valiantly defended the specificity of his *pylades* for this very reason. In the years following MacCook's description *xyloni* completely dropped from sight. Emery must have known this insect since there are two specimens in the collection of Dr. Wheeler which bear the note "*S. geminata*, color var., Emery det." Nevertheless there are no references to it in the literature until 1915 at which time it was made a subspecies of *geminata* by Wheeler. In the meantime (1906) he had described *aurea* as a variety of *geminata*. Three years later Forel raised *aurea* to specific rank. In his publication of 1915 Wheeler assigned a new subspecies, *maniosa*, to *geminata* and another, *amblychila*, to *aurea*. This arrangement has been followed up to the present. By uniting *xyloni*, *maniosa*, *aurea* and *amblychila* within a single species a much more consistent arrangement is secured since the group thus produced is of very uniform character and at the same time *geminata* is relieved of forms whose relation to it is obviously distant.

S. xyloni occurs throughout the southern United States from Florida to California. Specimens from the eastern half of this range belong without exception to the typical *xyloni*, a form which is found from Florida to Arizona. Its range to the north is much more extensive than was formerly supposed. It is abundant in northern Alabama and undoubtedly gets into that portion of Tennessee which lies to the west of the Appalachian highland. In central Texas the range of the typical *xyloni* is overlapped by that of the subspecies *aurea*, the two forms occurring together through western Texas, New Mexico and Arizona. The variety *maniosa* is the only form of this species found in California. *Maniosa* occurs as far east as New Mexico but is less abundant in New Mexico and Arizona than in California. The subspecies *amblychila* is found only in the mountains of southern Arizona.

Key to the subspecies and varieties of S. xyloni

1. Eyes of the major worker consisting of 70–80 facets (50 in the minor), separated from the insertion of the mandibles by a distance one and one-half times as great as their maximum diameter..........2
Eyes of the major worker consisting of not more than 50 facets (20 in the minor), separated from the insertion of the mandibles by a distance twice as great as their maximum diameter................3
2. Major worker brownish red to blackish red, the minor much darker *xyloni*
Color in all castes light reddish yellow, the posterior edges of the abdominal segments marked with black................var. *maniosa*
3. Clypeal teeth of the major worker present..............subsp. *aurea*
Clypeal teeth of the major worker absent............subsp. *amblychila*

S. (Solenopsis) xyloni (MacCook.)

S. xyloni, MacCook, in Comstock, Rep. Cotton Worm, p. 188, (1879). ☿, ♀.
S. geminata subsp. *xyloni*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 395 (1915). ☿.
?*Myrmica* (*Atta*) *sabeana*, Buckley, Proc. Ent. Soc. Phila., Vol. 6, p. 343 (1866). ☿.

Worker major: Length 5.8 mm. (Plate III, figs. 1, 4.)

Head, exclusive of the mandibles, slightly broader than long, broadest at the posterior third, the sides moderately convex, narrowed from the posterior third towards the mandibles, the occipital angles much rounded, occiput with a pronounced but rather narrow impression; frontal furrow present. Clypeus feebly projecting, forming a virtually straight line except where the carinal teeth project forward, the latter short and stout, lateral denticles reduced to feeble lobes or simply forming a sinuate portion of the edge of the clypeus, clypeal carinae clearly marked from the teeth to the frontal lobes but extending only a short distance between the latter before fusing with the clypeus. Mandibles evenly curved, armed with three large teeth. A small denticle on the inner margin represents the vestige of the fourth tooth. Eyes suboval, more or less pointed at the anterior end, composed of 70–80 facets, separated from the insertion of the mandibles by a distance one and one-half times as great as their maximum diameter, the outermost ring of facets usually unpigmented. The antennal scapes in repose slightly surpass the posterior fifth of the head, first funicular joint as long as the following two together, second and third funicular joints approximately twice as long as broad, the remaining small joints decreasing in length and increasing in width; club slender, about five times as long as broad, the terminal joint only slightly more than one and a half times as long as the penultimate.

Thorax seen from above with a well-developed promesonotal suture, the pronotum transversely rectangular, mesonotum longer than broad, suboval and truncate behind at the mesoepinotal suture, epinotum approximately as broad as long, subquadrate with the sides somewhat convex and not distinctly marked, the declivious face with a shallow impression which extends to the posterior third of the basal face. Seen in profile the pronotum appears somewhat angular with an abruptly sloping, straight anterior portion and a slightly convex less sloping posterior part, the promesonotal suture broad and shallow, the mesonotum evenly convex except at the extreme posterior portion where it descends abruptly to the deep mesoepinotal suture. Epinotum lower than the mesonotum, sharply angular, the basal face much longer than the declivious. Node of the petiole in profile thick and obtuse, the anterior face sharply sloping in a straight line from the summit of the node to the peduncle, the posterior face convex at the summit of the node but descending perpendicularly throughout most of its length, peduncle almost as long as the base of the node, thick and with a ventral lamella which ends anteriorly in an obtuse tooth. Postpetiole in profile dome-shaped, higher than thick and only slightly lower than the node of the petiole, the ventral face forms a sharp angle at its anterior edge and bears two conspicuous rugae near its posterior border. Seen from above the postpetiole is slightly narrower than the transversely oval postpetiole. Anterior edge of the abdomen narrowly truncate.

The color varies from a brownish red with the posterior portion of the abdomen castaneous brown to a deep blackish red with only the frontal area of the head and the mesonotum yellowish. In the latter case the posterior part of the abdomen is piceous. The mandibles and the area between their insertion and the anterior border of the eye coarsely striate. Basal face of the epinotum, meso- and metapleurae and frontal lobes with very fine striae. Lower portions of the petiolar nodes finely striato-rugulose. Cephalic punctures small but rather numerous, those of the abdomen minute but more abundant. Hairs long, stout, erect and golden.

Worker minor: Length 1.6 mm. (Plate III, figs. 7, 8.)

Head exclusive of the mandibles, as long as broad, sides virtually parallel except in the posterior third where they are somewhat narrowed. Occipital angles much less rounded than in the worker major, occiput flat. Clypeus more projecting than in the major, the carinal teeth relatively longer, the edge of the clypeus between them with a concave impression, lateral denticles poorly defined, carinae continued well back between the frontal lobes. Mandibles with four teeth, the innermost small but distinct and borne in a line with the other three. Eyes rounder than in the major worker, not pointed anteriorly, composed of 40–50 facets and separated from the insertion of the mandibles by a distance only slightly exceeding their maximum diameter. The antennal scapes in repose fail to reach the occipital border by a distance less than the length of the first funicular joint; second and third funicular joints notably longer than broad, the remaining small joints slightly or not at all longer than broad; club slender, the terminal joint twice as long as the penultimate.

Thorax without promesonotal suture, in some specimens there is a slight ridge in the position where the suture should occur. Promesonotum in profile feebly convex, mesoepinotal suture wide and deep, epinotum only slightly lower than the adjacent portion of the mesonotum, less angular than in the major worker, declivious face not impressed, approximately as long as the basal face. Node of the petiole in profile thicker and lower than in the worker major, the summit very obtuse, both anterior and posterior faces somewhat convex; peduncle without the ventral lamella, the tooth small and acute. Postpetiole in profile very similar in shape and size to the node of the petiole but slightly lower and inclined forward. Seen from above the node of the petiole is about three-fourths as wide as the postpetiole, the latter is much less transverse than in the major worker. Anterior edge of the abdomen truncate.

Color blackish red to piceous, the mandibles, clypeus and anterior portion of the head yellow. Mandibles coarsely striate, a few feeble striae between their insertion and the anterior margin of the eye; meso- and metapleurae finely striate. Punctures fine and rather sparse, the hairs which they bear long, erect, and golden.

The medias show characters transitional between those of the two castes described above.

Female: Length 6.6 mm. (Plate III, fig. 6.)

Head exclusive of the mandibles one-sixth longer than broad, subquadrate, the sides virtually straight, the posterior angles very slightly rounded, occiput flat or very feebly impressed. Clypeus and mandibles as in the major worker. The antennal scapes in repose reach the lateral ocelli; funicular joints and club as in the major worker. Eyes large, oval, their posterior border only slightly surpassing the middle of the side of the head. Thorax slightly narrower than the head, ellipsoidal, one and two-third times as long as broad. Epinotum without a sharp separation between the basal and declivious faces. Node of the petiole in profile somewhat thinner than that of the major worker; postpetiole more angular and a trifle lower. Seen from above the postpetiole is very strongly transverse, almost twice as broad as long. Wings hyaline, iridescent, the veins and stigma pale yellow.

Color piceous red, mandibles, anterior two-thirds of the head, antennae and legs yellow; abdomen brownish black, sometimes with two yellow spots at the base of each segment except the first. Sculpture and pilosity as in the major worker.

Male: Length 5 mm. (Plate III, fig. 5.)

Head trapezoidal, eyes large, oval about two-thirds as long as the side of the head, their anterior border almost reaching the insertion of the mandibles. Clypeus not projecting; mandibles bidentate, the inner tooth small. Second antennal joint much shorter and far broader than either the preceding or succeeding joints, approximately doughnut shaped, third joint as long as the following two together.

Thorax one and one-third times as broad as the head. Seen in profile the scutum is greatly swollen and overhangs the pronotum. Node of the petiole in profile very low with an acute summit. Postpetiole in profile much depressed; seen from above transverse and slightly wider than the node of the petiole. Wings as in the female. Color reddish black, appendages yellow, the pronotum with three broad yellow bands. Hairs numerous and fine, those on the petiolar nodes somewhat longer and coarser.

Localities: Alabama, Mobile. (H. P. Loding, W. S. Creighton.)
Montgomery, Huntsville. (W. S. Creighton.)
Louisiana, Shreveport. (No collector.)
Texas, Austin, Ft. Davis. (W. M. Wheeler.)
Arizona, Phoenix, Huachuca Mts., Oracle, Texas Pass. (W. M. Wheeler.)

The specimens from Alabama and Louisiana are somewhat lighter in color than those from Texas but not as light as the variety *maniosa*. The difference is too slight even for varietal distinction.

Xyloni nests under stones, at the base of tufts of grass or in sandy "draws" near the edge of streams. The crater is very irregular. and consists of a mass of loose, friable soil or sand without definitely formed passages.

S. (Solenopsis) xyloni var. **maniosa** (Wheeler.)

S. xyloni var. *maniosa*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 396 (1915). (*geminata* subsp.)

Except for their lighter color the workers of this form are identical with those of *xyloni*. As long as *maniosa* was placed with *geminata* it could be regarded as a subspecies but since there are no significant structural differences between it and *xyloni* I am forced to consider *maniosa* as a variety of that species.

The major workers of *maniosa* are of a clear reddish yellow with the abdomen more or less marked with piceous brown. In most specimens the brown coloration is confined to a band at the posterior edge of each gastric segment but more rarely the entire abdomen is dark. The minor workers usually show the complete infuscation of the abdomen but the head and thorax are reddish yellow as in the major worker. They are, therefore, easily distinguished from the uniformly dark minors of *xyloni*. The color of the female is a light reddish brown with the abdomen black. The males of *maniosa* cannot be distinguished from those of *xyloni*.

Localities: New Mexico; Almagordo. (G. von Krockow.)
Arizona; Tucson, Yuma, Tempe, Yucca, Gila Bend Mts., Benson. (W. M. Wheeler), Thatcher (Chamberlin.)
California; Pasadena, San Gabriel Mts., San Ysidro, Del Mar, Needles, (W. M. Wheeler), Friant, Fresno, Los Angeles, (Chamberlin), Berkeley Hills, Los Gatos Canyon, (J. P. Bradley), La Jolla, (J. F. McClendon), Visalia, (Culbertson), Whitter, (H. L. Quayle).

S. (Solenopsis) xyloni subsp. aurea (Wheeler.)

(Plate III, fig. 2.)

S. xyloni subsp. *aurea*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 21, p. 336 (1906). ☿, ♀, ♂. (*geminata* var.) Ibid., Vol. 24, p. 425, (1908). ♀, ♂.

S. aurea, Forel, Deutsche Ent. Zeitschr., p. 269 (1909).

The workers of all sizes of this subspecies may be distinguished from those of corresponding castes in *xyloni* by their smaller eyes and the more rectilinear dorsum of the thorax. The eyes of the major worker of *aurea* consist of about 50 facets which are separated from the insertion of the mandibles by a distance twice as great as their maximum diameter. Those of the minors have twenty facets and are separated from the insertion of the mandibles by a distance slightly less than twice their greatest diameter. The epinotum of the major worker is not depressed and in profile forms an almost straight line with the posterior portion of the epinotum. This is completely rectilinear in the minor worker. The clypeal teeth of the larger workers are slightly smaller than in *xyloni* but this character is difficult to employ unless one is familiar with both forms. There also appears to be a difference in size. I have never seen workers of *aurea* whose length exceeded 4.6 mm. but such a character cannot be depended upon, particularly when, as in this case, the females are of equal size.

The color of all sizes of workers in the subspecies *aurea* is a beautiful clear yellow with the mandibles and the posterior margins of the gastric segments tinged with brown.

The female of *aurea* very closely resembles that of *xyloni*. The head is slightly wider in proportion to its length and the postpetiole more transverse but both these differences are subject to confusing variations. By far the most satisfactory criterion lies in the paler coloration of the female of *aurea*. The insect is a rich golden yellow with the posterior edges of the gastric segments narrowly bordered with castaneous brown. The color of the male is blackish brown with the yellow bands on the pronotum absent or much obscured. I can detect no other differences between this insect and the male of *xyloni*.

Localities: Texas; Austin. (Type loc.), W. M. Wheeler. Fort Davis. (W. M. Wheeler.)
New Mexico; desert near Magdalena. (W. S. Creighton.)
Arizona; Yuma, Grand Canyon, Phoenix, Casa Grande, Post Canyon Pinaleno Mts., Coyote Mts. (W. M. Wheeler.)

Aurea nests under stones and dried dung and does not construct craters. It appears to be nocturnal or crepuscular in habits. The workers are rarely found outside the nest during the day and avoid direct sunlight as much as possible. Several workers which I found early one morning near the bottom of Bright Angel Trail in the Grand Canyon were dragging about a much crushed beetle. When I first observed them they were still protected from the direct rays of the sun. An hour later when I returned the place was in brilliant sunlight. The beetle had been abandoned and what few ants I could find had taken refuge under nearby stones. Although the workers in the nests which I found in the desert country near Magdalena, N. M. were close to the cake of dried dung which covered the upper passages of the nest, they almost immediately disappeared into the lower passages when exposed to the light.

S. (Solenopsis) xyloni subsp. **amblychila** (Wheeler.)

(Plate III, fig. 3.)

S. xyloni subsp. *amblychila*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 394 (1915). ☿. (*aurea* subsp.)

The major worker of *amblychila* may be distinguished from that of any other form in the species through the absence of clypeal teeth. The anterior edge of the clypeus forms a perfectly straight line, the carinae terminating behind the margin. The punctures, particularly those on the head are notably sparse and small. In other characters *amblychila* closely resembles *aurea*. The mesonotum of *amblychila* is, however, more convex and the small funicular joints a trifle more elongate.

The clypeal teeth are present in the medias and minors of *amblychila* and I can find no satisfactory character by which these may be separated from those of *aurea*.

The female of *amblychila* shows the unarmed clypeus and reduced punctures of the major worker. In other regards it is identical with that of *aurea*. The males of the two forms cannot be distinguished.

Localities: Arizona; Huachuca Mts. (Type loc.) (W.M.Wheeler.)
Mexico, Guadalajara. (McClendon.)

SUBGENUS EUOPHTHALMA, SUBGEN. NOV.

The characteristics of this subgenus have already been discussed in the introduction. It is composed of species of small or moderate size in which polymorphism is, at best, feebly developed. The eyes are large and are ordinarily composed of 20–60 facets. The second and third funicular joints are usually as broad as long or broader than long. In the case of *globularia*, which constitutes an exception to both statements, the second and third funicular joints are slightly longer than broad and the eyes of the smallest workers in some of the subspecies are made up of only twelve facets. Despite these irregularities *globularia* obviously belongs to the subgenus *Euophthalma* and its peculiar postpetiole is so distinct that confusion in regard to this species is extremely unlikely.

The synonymy of the group is as follows:

S. (*Euophthalma*) *andina* Santschi
S. (*Euophthalma*) *globularia* Fred. Smith
  = var. *curta* Forel
  subsp. *lucayensis* Wheeler
  subsp. *littoralis* subsp. nov.
  subsp. *pacifica* Wheeler
    var. *descheoensis* Mann
    var. *rubida* Wheeler
  subsp. *steinheili* Forel
    = var. *borinquenensis* Wheeler
    = var. *cubaensis* Wheeler
S. (*Euophthalma*) *huachucana* Wheeler
S. (*Euophthalma*) *macrops* Santschi
S. (*Euophthalma metanotalis* Emery
    var. *argus* Santschi
  subsp. *emiliae* Santschi
  subsp. *pelontona* Forel
  subsp. *shiptoni* Forel
    var. *steigeri* Santschi

S. (*Euophthalma*) *nigella* Emery
subsp. *gensterblumi* Forel
= subsp. *praevalens* Santschi
S. (*Euophthalma*) *occulata* Santschi
S. (*Euophthalma*) *picquarti* Forel
S. (*Euophthalma*) *schilleri* Santschi
S. (*Euophthalma* *silvestrii* Emery

My reasons for placing *gensterblumi* and *occulata* in the subgenus *Euophthalma* are discussed at the end of the description of those forms. A key to the various species is presented below:

1. Postpetiole greatly dilated, seen from above subcircular, more than one-half as wide as the greatest width of the abdomen....*globularia*

Postpetiole not dilated, seen from above transverse in the major workers, transversely oval to subglobose in the minors, never more than one-third as wide as the greatest width of the abdomen....2

2. Epinotum entirely rugose or reticulo-punctate....7

Epinotum smooth except for a few rugae on the metaplurae and the declivious face....3

3. Eyes very large and elongate with fifteen facets in their greatest length, their anterior border extending almost to the insertion of the mandibles....*macrops*

Eyes with eight or less facets in their greatest length, their anterior border separated from the insertion of the mandible by a distance equal to or exceeding their length....4

4. Major worker with the antennal scape in repose reaching the posterior third of the head, minor worker with the scape in repose reaching the posterior fourth of the head; occiput of major with a strongly impressed median furrow....*nigella*

Major worker with the antennal scape in repose slightly surpassing the posterior fourth of the head, minor with the scape in repose reaching the posterior sixth of the head; occiput flat or scarcely impressed....5

5. Eyes of forty-five facets....*andina*

Eyes of twenty to thirty facets....6

6. Greatest length of thorax (neck excluded) approximately two and one-half times its greatest width, sides feebly constricted at mesoepinotal suture, color piceous brown....*occulata*

Greatest length of thorax (neck excluded) less than twice its greatest width, sides strongly constricted at the mesoepinotal suture, color yellow....*huachucana*

7. Head distinctly broader than long, color pale yellow, the posterior part of the abdomen castaneous brown....*silvestrii*

Head as long as broad or longer than broad, color reddish brown to dark brown, the abdomen wholly or in large part piceous..*metanotalis*

S. picquarti Forel has been omitted since the characters given in the original description are not suitable for use in the key.

S. (Euophthalma) andina Santschi.

S. andina, Santschi, Rev. Suisse Zool., Vol. 30, p. 262, fig. 3 (1923). ☿

Worker: Length 2.8–3.0 mm. (Plate VII, figs. 1, 2.)

Head, exclusive of the mandibles, very slightly longer than broad (about one-seventh), the sides virtually straight, very feebly narrowed in front of the eyes, the posterior angles moderately rounded, the occiput approximately straight. Clypeus moderately projecting, the prominent and divergent clypeal carinae terminating in short, stout teeth. The lateral denticles are not distinct but there is an angular impression between the carinal teeth and the adjacent edge of the clypeus. Mandibles smooth, armed with four teeth, the outer three large, the innermost reduced to an angle which limits the internal border of the mandible. Eyes of about forty-five facets, separated from the insertion of the mandible by a distance slightly exceeding their greatest diameter. The antennal scapes in repose reach the posterior fourth of the head. First funicular joint slightly shorter than the following three together, joints 2–7 approximately as long as broad, club slender, the last joint two and one-quarter times as long as the penultimate.

Thorax with sharply defined humeral angles. Promesonotum in profile moderately convex, the mesoepinotal suture with a broad but rather shallow impression. Epinotum angular, the basal face very feebly convex and much longer than the declivious. Node of the petiole in profile triangular, the summit rounded but narrow, the anterior face at least one-third longer than the posterior, the peduncle with a very large ventral tooth. Postpetiole in profile rather angular and strongly inclined forward, about three-fourths as high as the node of the petiole. Seen from above the node of the petiole is approximately two-thirds as wide as the postpetiole. The latter is strongly transverse and trapezoidal in shape.

Color, thorax and abdomen castaneous brown, the head a somewhat more piceous brown, mandibles (except the teeth, which are brown), clypeus, anterior tarsal joints and articulations of the legs bright yellow. The meso- and metapleurae and the declivious face of the epinotum are finely striate. There are also a few irregular striae on the area between the eye and the insertion of the mandible. Punctures uniformly fine and sparse. Cephalic hairs very fine and subappressed, those on the thorax and abdomen stouter and erect. The appendages bear numerous rather stout, suberect hairs.

Male and female unknown.

Type locality: Cueva d'Iturbe (3700 m. alt.) Jujuy, Argentina. (Weiser.)

In the original description of *andina* Dr. Santschi states that the color is black. This is not the case in the cotype which he sent me for examination. The differences which I have noted above may, however, be due to the drying of the specimen.

S. (EUOPHTHALMA) GLOBULARIA (FRED. SMITH.)

Introduction.

In *globularia*, as in so many of the older species, the exact nature of the type has always been a matter for considerable speculation with the result that much confusion has arisen in the establishment of subspecies and varieties. In 1859 Frederick Smith described a *Myrmica globularia* from Brazil. The description is virtually worthless since it fails to mention any structural characters which would permit even generic recognition. It is not surprising, therefore, that when, in 1881, Forel described *Solenopsis steinheili* from St. Thomas, W. I., he did not recognize its relation to the already described *globularia*. Three years later, however, Forel placed *steinleili* in the synonymy of *globularia* when Mayr, to whom he had sent specimens, compared the two and pronounced them identical. Unfortunately Mayr appears to have overlooked differences which we now regard as varietal or subspecific characters. Although Smith failed to note major structural features he did give a fairly satisfactory account of the color of his Brazilian form. I quote here from his description:

"Pale rufo-testaceous, very smooth and shining with a rufo-fuscous band on the first segment of the abdomen."

The color of *steinheili*, on the other hand, is described by Forel as follows:

"Reddish brown-yellow to light brown with somewhat darker head and darker brown middle of the abdomen. Mandibles, anterior end of the head, antennae and legs dirty yellow."

From our present knowledge of the distribution of the species we know that the majority of the specimens which come from the islands of the West Indies have the darker coloration specified for *steinheili*. Conversely those from Brazil are notably pale and agree very well with Smith's description. However, such pale specimens are usually referred to Forel's variety *curta* which was described in 1912 from material taken in Colombia. The color of *curta* is said to be a reddish yellow with a very distinct brown band on the abdomen. It seems rather obvious that *curta* is identical with the original form described by Smith and that the confusion has arisen through Mayr's failure to note color differences between *steinheili* and the typical *globularia*. Such an interpretation necessitates the resuscitation of *steinheili* and this, in turn, eliminates the varieties *borinquenensis* and *cubaensis*, both described by Wheeler, which become synonyms of that form. It may be advantageous to repeat here the synonymy given at the beginning of the subgenus.

S. (*E.*) *globularia* (Fred. Smith)
    = var. *curta* Forel
  subsp. *lucayensis* Wheeler
  subsp. *littoralis* subsp. nov.
  subsp. *pacifica* Wheeler
    var. *descheoensis* Mann
    var. *rubida* Wheeler
  subsp. *steinheili* Forel
    = var. *borinquenensis* Wheeler
    = var. *cubaensis* Wheeler

A full discussion of the reasons for synonymizing the varieties *borinquenensis* and *cubaensis* with *steinheili* is presented at the end of the description of that form.

The distribution of *globularia* is very similar to that of *geminata*. The subspecies *steinheili* appears to be the most abundant member of the complex, occurring in a number of the islands of the West Indies. There are, however, two other known insular variants, *lucayensis* from the Bahamas and *descheoensis* from Descheo Island, P. R. In the portions of South America which border the Carribean we find the typical *globularia*, while along the Gulf seaboard of the United States the subspecies *littoralis* occurs. Two forms, the subspecies *pacifica* and its variety *rubida* are known from the Galapagos Islands. There appears to be but one record of this species from Central America and none from Mexico.[1] This hiatus in collections is much to be regretted since we have no connecting links between the northern and southern forms except the insular variants whose significance is doubtful because of changes brought about by isolation.

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[1] Since writing the above I have, through the kindness of Dr. W. M. Wheeler, examined specimens of *globularia* collected by Dr. Elizabeth Skwarra at Tamarinda (Vera Cruz) Mexico. There appears to be no difference between these specimens and those which are described in this paper as the subspecies *littoralis*, subsp. nov. The discovery of the Mexican specimens greatly extends the range of the subspecies *littoralis* and there is every reason to expect future records of this form from the coast of Texas.

I have had the good fortune to examine the types of the majority of the forms of *globularia* and have seen examples of all except the variety *rubida*. From a study of this material the following key to the subspecies and varieties has been prepared:

1. Head of the minor worker one-eighth longer than broad, that of the major worker as long as broad; eyes of 25–30 facets..........*globularia*
   Head of the minor worker one-sixth longer than broad, that of the major one-eighth longer than broad; eyes of 12–22 facets..........2
2. Epinotum subopaque, completely covered with fine, wavy striae subsp. *littoralis*
   Epinotum wholly or in part shining, at least the anterior portion of the basal face without striae...3
3. Node of the petiole one-half as wide as the postpetiole; color very pale yellow ....subsp. *lucayensis*
   Node of the petiole approximately two-thirds as wide as the postpetiole; color reddish yellow to piceous brown....................4
4. Eyes of 12–18 facets; length 1.8–2.0 mm.............subsp. *steinheili*
   Eyes of 18–22 facets; length 2.0–2.2 mm..........................5
5. Color deep piceous brown..........................var. *descheoensis*
   Color clear golden yellow to red, the abdomen brown or black........6
6. Color golden yellow, the first gastric segment banded with brown subsp. *pacifica*
   Color red, the abdomen in large part black (ex Wheeler)......var. *rubida*

S. (Euophthalma) globularia (Fred. Smith).

S. globularia (Fred. Smith), Cat. Hym. Brit. Mus., Vol. 6, p. 131 (1858). (*Myrmica.*)
S. globularia subsp. *lucayensis* var. *curta*, Forel, Mem. Soc. Ent. Belg., Vol. 20, p. 4 (1912).

Worker minor: Length 1.7 mm.

Head, exclusive of the mandibles, one-eighth longer than broad, the sides moderately convex, widest just behind the eyes, the occiput very feebly impressed. Clypeus feebly projecting, the two carinae very prominent and terminated by two strong, stout teeth, lateral denticles poorly developed, virtually absent in some specimens; a small median denticle is present on the anterior edge of the clypeus. Mandibles with four large teeth. The antennal scape in repose fails to reach the occipital border by a distance slightly exceeding the length of the first funicular joint; joints 2–7 all slightly longer than broad; club slender, the terminal joint about two and one-half times as long as the penultimate. Eyes of about twenty-five facets, separated from the insertion of the mandible by a distance slightly less than their greatest diameter.

Promesonotum in profile flattened, forming a straight line with the epinotum, the mesoepinotal impression confined entirely to the suture which is narrow and slot-like. Epinotum in profile angular, the basal face much longer than the declivious. Node of the petiole in profile thick with a very bluntly rounded summit, the anterior face considerably longer than the posterior; the peduncle about two-thirds as long as the base of the node, rather slender and without a ventral tooth. Postpetiole in profile about two-thirds as high as the petiole, its anterior face strongly inclined forward and about equal in length to the unusually long ventral face, the latter bearing a prominent, tooth-like projection near the middle of its anterior edge; dorsum of the postpetiole evenly convex. Seen from above the node of the petiole is a trifle more than half as wide as the postpetiole, the latter is greatly dilated, subcircular with a truncate posterior border. The anterior edge of the first gastric segment is truncate, the truncation being about equal in width to the postpetiole.

Color reddish yellow; the mandibular teeth, the anterior edge of the clypeus and a broad band on the first gastric segment reddish brown, the occiput is also sometimes tinged with brown. The mesopleurae and the entire epinotum are covered with fine striae which give these portions a submatte appearance in contrast to the rest of the insect which is very smooth and shining with small and sparse punctures. Hairs sparse, rather short, stout and erect.

Worker major: Length 2.0 mm. (Plate VI, figs. 1, 2.)

Head as broad as long, the sides less convex than in the minor worker. Eyes of about thirty facets. Median clypeal tooth almost as long as the carinal teeth. The area between the eye and the insertion of the mandible bears a few irregular striae. In other regards as in the minor worker.

The female of the typical *globularia* appears to be known only from Smith's totally inadequate treatment. It may be of interest to include it here, since it speaks volumes for the difficulties encountered in attempting to utilize his descriptions:

"Female: Length: 1¾ line (Approximately 4.2 mm.)

Rather darker red than the worker; the thorax oblong-ovate, the wings hyaline and beautifully iridescent; the node of the petiole of the same relative proportions as in the worker; abdomen oblong-ovate, rufo-fuscous with the base and apical margins of the segments rufo-testaceous."

Male: Length: 3.0 mm.

Head, including the eyes, slightly broader than long, the sides moderately convex from the eyes to the occiput, the latter flat. Ocelli large and prominent. Clypeus not projecting when seen from above, angular with a well-defined central portion when seen from the side. Mandibles narrow, bidentate. Eyes large, strongly convex, nearly circular when seen from the side, their anterior edge virtually reaches the insertion of the mandible. Antennal scape short, cylindrical, twice as long as the globose second joint, the following joints all two or more times as long as broad. Thorax wider than the head, approximately twice as long as broad. Node of the petiole in profile angular, depressed, not sharply separated from the peduncle. Postpetiole in profile also depressed, the dorsum feebly convex, slightly lower than the node of the petiole, the lower face with a prominent ventral tooth. Seen from above the node of the petiole and the postpetiole are both strongly transverse.

Color brownish black, the legs, antennae and mandibles dirty yellow. Smooth and shining with numerous, long, erect, yellow hairs. Wings hyaline, iridescent and clothed with numerous, short hairs.

The descriptions of the workers and male given above are based upon specimens collected by Dr. W. M. Mann at Natal, Brazil. Owing to the great confusion in the earlier taxonomy of this species it is difficult to compile a satisfactory list of localities. I have given only the records which appeared to be unquestionably those of the typical *globularia*.

Localities: "Brazil" (Type locality.) (F. Smith.)
Brazil, Ceara. (Diaz da Rocha.)
Natal. (W. M. Mann.)
Colombia, Barranquilla. (A. Forel.)

The specimens taken by Biolley at Puntarenas, Costa Rica probably belong here also.

S. (Euophthalma) globularia subsp. **lucayensis** Wheeler.

S. globularia subsp. *lucayensis*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 24, p. 131, pl. 11, fig. 8 (1908). ♀.

S. globularia, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 21, p. 89 (1905).

The workers of this subspecies are unique in the feeble sculpture of the epinotum and their exceedingly pale coloration. The basal face of the epinotum is virtually free from striae, while those on the sides are very weak and confined to the pleurae. The color is a pale, almost milky yellow with a faint brownish band on the abdomen. *Lucayensis* may further be distinguished from *steinheili* and *pacifica* to which it is most closely related, through the narrow petiolar node which is only one-half as wide as the postpetiole.

The sexual forms are unknown.

Type locality: Andros Island, Bahamas. (W. M. Wheeler.)

S. (Euophthalma) globularia subsp. **littoralis** subsp. nov.

The worker of this form may be distinguished from that of the typical *globularia* by its greater size, its more elongate head, its smaller eyes and the much broader petiolar node. The shape of the head in *littoralis* is approximately that of the subspecies *pacifica* and *steinheili* (about one-sixth longer than broad in the minors and slightly longer than broad in the majors) but it differs from them in the more pronounced sculpture of the epinotum. This, in *littoralis* is completely covered with rugae and subopaque as in the typical *globularia*. The node of the petiole in *littoralis* is unusually broad, in both the large and small workers it is fully two-thirds as wide as the postpetiole. The eyes of *littoralis* consist of 18–22 facets depending upon the size of the worker. The color is yellowish brown, the head and abdomen blackish brown, the appendages dingy yellow. Length: 2.0–2.2 mm. (Plate VI, fig. 3.)

Female: Length 3.8 mm.

Head, exclusive of the mandibles, as broad as long, broadest just behind the eyes, the sides straight and slightly narrowed from the eyes to the insertion of the mandibles, evenly convex from the eyes to the occiput, the latter flat. Clypeus moderately projecting with very coarse, short carinal teeth. The antennal scape in repose reaches the lateral ocellus. Funicular joints 2–7 all slightly longer than broad; club slender, the terminal joint two and one-half times as long as the penultimate. Eyes large, strongly convex, their posterior border situated at a point half way between the insertion of the mandible and the occiput.

Thorax seen from above narrowly elliptical, twice as long as broad. Epinotum angular, the basal face considerably longer than the declivious. Node of the petiole in profile relatively thinner than in the worker, its anterior face less sharply separated from the peduncle. Postpetiole in profile about four-fifths as high as the node of the petiole, the ventral tooth very large. Seen from above the petiole is slightly more than two-thirds as wide as the postpetiole, the latter is almost twice as broad as long and suboval in shape. Anterior edge of the first gastric segment truncate.

Color reddish brown, the head and thorax fuscous. The entire epinotum, the area between the eye and the insertion of the mandible and the base of the mandible striato-rugose. Hairs long, erect, golden.

The male of the subspecies *littoralis* differs very slightly from that of the typical *globularia*. In *littoralis* the eyes are slightly less convex and a trifle more elongate but in other regards the two appear to be identical.

Described from a number of workers and a single female taken by T. S. VanAller in Baldwin County (near Mobile) Alabama. The nest was in a rotten log on the shore of the bay. I was at first hesitant to accept this insect as a native form, since most of our southern ports have become asylums for a fauna of exotic species. Further investigation, however, has produced evidence that *littoralis* is endemic to the Gulf coast. Through the coöperation of Dr. M. R. Smith two localities in Mississippi were added and recently, when I spent two months in Mobile, further data were secured. The distribution of *littoralis* in the vicinity of Mobile is the reverse of that of the introduced species. It is rare in the neighborhood of the docks but occurs in increasing numbers as one goes towards the Gulf. On Dauphin Island, thirty miles south of Mobile, I found it in great abundance. Practically every rotten log on the beach harbored one or more nests. It appears to be a strictly littoral form and nests only in logs which are badly rotted and partially buried in the sand. Winged males and females were taken from a nest on June 30, but their scarcity and the absence of callow sexual forms leads me to believe that the nuptual flight had occurred prior to that time.

Localities: Alabama, Baldwin County (type locality). (T. S. Van Aller.)
Barrett's Beach. (W. S. Creighton.)
Dauphin Island. (W. S. Creighton.)
Mississippi, Ocean Springs. (J. P. Kislanko.)
Perkinston. (J. P. Kislanko.)
Pascagoula. (W. S. Creighton).

S. (Euophthalma) globularia subsp. **pacifica** Wheeler.

S. globularia subsp. *pacifica*, Wheeler, Proc. Cal. Acad. Sci., Vol. 2, p. 273 (1919). ☿, ♂. Wheeler, Zoologica, N. Y. Zool. Soc., Vol. 5, No. 10, p. 108, fig. 22 (1924). ♀.

The workers of this subspecies very closely resemble those of *steinheili* but they are of slightly greater size (2.0–2.2 mm.) and the eyes, which consist of 18–22 facets, are larger. The color is a deep, golden yellow with the first gastric segment bearing a broad brown band which extends almost to the base of the gaster. (Plate VI, fig. 5.)

The female of *pacifica* differs from that of *littoralis* in the following particulars:

The color is a clear golden yellow with the borders of the gastric segments tinged with brown, the eyes are larger and more elongate, a median denticle is present on the anterior edge of the clypeus, the sculpture of the epinotum is feebler, the postpetiole is smaller.

I have not seen the male of *pacifica*. Wheeler describes it as follows: "Eyes more convex and perhaps a little longer, head narrower behind, mandibles smaller, epinotum more sloping, more rounded and less angular than in the typical *globularia*. The color is also different, the body being dark brown, the antennae and legs pale yellow, whereas in the typical form the body is black and the appendages brown."

Localities: Galapagos Islands, Albemarle Is. (Type loc.) (Albatross Exped.) Indefatigable Is., Tower Is., Daphne Is. (F. X. Williams.)

The nests of *pacifica* are usually constructed under small logs on sandy beaches. The specimens from Daphne Island, however, were found under stones at the bottom of the crater. It is to be regretted that our ignorance of the sexual forms of *steinheili* prevents a comparison of these with those of *pacifica*, since this might offer suggestive evidence concerning the supposed West Indian origin of certain species in the Galagagos fauna.

S. (Euophthalma) globularia subsp. **pacifica** var. **rubida** Wheeler.

S. globularia subsp. *pacifica* var. *rubida*, Wheeler, Proc. Cal. Acad. Sci., Vol. 2, p. 273 (1919). ☿.

The single type from which this variety was described is now in the Museum of the California Academy of Sciences. I have not seen it. The original description is as follows:

"Worker: Length 1.8 mm.

"Differing from the other forms of the species in color, the body being red with the mandibles, antennae and legs yellow and the first gastric segment, except at its anterior and posterior borders, black. The postpetiole is globular but a little broader than long."

Type locality: Hood Island, Galapagos. (F. X. Williams.)

S. (Euophthalma) globularia subsp. **pacifica** var. **descheoensis** Mann.

S. globularia var. *descheoensis*, Mann. Bull. Amer. Mus. Nat. Hist. Vol. 42, p. 428 (1920). ☿.

This form must be regarded as a variety of the subspecies *pacifica* rather than of the typical *globularia* to which it was first related. Structurally it is very similar to *pacifica*, the principal difference lying in its darker coloration. The entire insect is a deep piceous brown except the antennae and legs which are yellowish brown.

The sexual forms are unknown.

Type locality: Descheo Island, Porto Rico. (F. E. Lutz.)

S. (Euophthalma) globularia subsp. **steinheili** Forel.

S. steinheili, Forel, Mitt. Munchen Ent. Ver. Vol. 5, p. 11 (1881). ☿

S. globularia, Forel, Bull. Soc. Vaud. Sci. Nat. (2), Vol. 20, p. 376 (1884).

S. globularia var. *borinquenensis*, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 24, p. 131, pl. 11, fig. 7 (1908). ☿.

S. globularia var. *cubaensis*, Wheeler, Bull. Mus. Comp. Zool. Harvard, Vol. 54, p. 485, (1913). ☿.

Aside from their darker coloration both the major and the minor workers of *steinheili* may be distinguished from the corresponding castes in the typical *globularia* by their smaller eyes, the more elongate head and the feebler sculpture of the epinotum. The head of the minor worker of *steinheili* is approximately one-sixth longer than broad, that of the major about one-eighth longer than broad. The eyes are composed of 12–16 facets and are separated from the insertion of the mandibles by a distance slightly exceeding their greatest diameter. The striations of the epinotum are feeble and are entirely absent on the upper portions of the sides and the anterior part of the basal face. The postpetiole of the major worker is transversely oval (subcircular in the typical *globularia*). The color varies from a clear, yellowish brown with the mandibles and appendages honey yellow to a dirty piceous brown, with the appendages sooty yellow. Length 1.8–2.0 mm.

The female is unknown. The male is identical with that of the typical *globularia*.

Localities: St. Thomas, W. I. (Type loc.) (A. Forel.)
St. Vincents, W. I. (H. H. Smith.)
Grenada, W. I. (H. H. Smith.)
Haiti, Port au Prince. (Keitel.)
Manneville. (W. M. Mann.)
Virgin Islands, Culebra Is. (W. M. Wheeler.)
Porto Rico, San Juan. (W. M. Wheeler.)
Cuba, Cogimar. (W. M. Wheeler.)
San Francisco de Morales. (W. M. Wheeler.)
Soledad (Cienfuegos). (W. S. Creighton.)

Following Mayr's comparison which led to the synonomizing of *steinheili* and *globularia*, one could employ either Smith's or Forel's description of the insect. Forel, naturally, compared such forms as subsequently came into his hands with *steinheili*, while Wheeler utilized the criteria given in Smith's original description. It thus happens that the diagnostics of color given for the variety *borienquenensis* by Wheeler are almost identically those pointed out by Forel as characteristic of *steinheili*. The only difference lies in the fact that Forel does not specifically mention the occurrence of a piceous band on the thorax of *steinheili*. This, however, becomes of no consequence when one studies the types of *borinquenensis*. These display a variation in color which extends from individuals with a clear, reddish brown head and thorax to specimens which have a piceous head and a piceous band on the thorax. Thus, while Wheeler was justified in considering *borinquenensis* distinct from the typical *globularia*, there is no reason to separate it from *steinheili*.

I have also ventured to synonymize the variety *cubaensis* with *steinheili*, although there are a number of objections which might be raised to such procedure. *Cubaensis* was originally given varietal status because of its unusually dark coloration and its smaller and more rounded postpetiole. As to the color one might repeat what has just been said for *borinquenensis*, there is a wide variation from brownish to piceous individuals with, I am bound to admit, a preponderance of the latter. The shape and particularly the size of the postpetiole are matters of more concern. Since a rounded postpetiole is characteristic of the minor worker throughout the species, little value can be attached to this criterion. Its size, on the other hand, is of considerable importance. After carefully measuring a number of specimens of both forms I find that although the largest workers of *steinheili* have postpetioles larger than any found in *cubaensis* and the smallest workers of that form have postpetioles smaller than any found in *steinheili*, there is considerable overlapping in the middle of the scale. Thus while it might be possible to separate the extremes on size variations, a procedure entailing the use of differences of one-twentieth of a milimeter, this could not be used to distinguish the individuals at the middle of the range. Since the determination of such small differences is impractical for ordinary work it seems decidedly more advantageous to synonymize the two forms.

A possible explanation for the variation of color displayed by *steinheili* is offered by the character of the nesting sites. The specimens from Culebra Island and Porto Rico were taken from nests in white sand while the Cuban material was found under stones in damp loam. It is a recognized fact that dry, sandy nesting sites often tend to produce light colored forms in distinction to the darker condition usually associated with a damp and shady type of site. The statement just made might, of course, be used in proof that the two forms are distinct, but until it is proven that the nesting sites are habitually different such an argument lacks weight.

S. (Euophthalma) huachucana Wheeler.

S. huachucana, Wheeler, Bull. Amer. Mus. Nat. Hist., Vol. 34, p. 393 (1915.) ☿, ♀.

Worker minor: Length 1.8 mm. (Plate VII, fig. 5.)

Head, exclusive of the mandibles, one-sixth longer than broad, the sides straight throughout their anterior half, slightly tapering towards the occiput in the posterior half, the occiput with a broad and extremely feeble impression. Mandibles rather narrow, armed with four teeth, the outermost very thin, twice as long as the adjacent tooth, the innermost tooth small and acute but very distinct. Clypeus feebly projecting, the carinae prominent, well separated and only feebly divergent. Carinal teeth stout, tapering and moderately acute, the edge of the clypeus between them not impressed. Lateral denticles very poorly developed, separated from the carinal teeth by a small but distinct impression. The antennal scape in repose fails to reach the occipital border by a distance equal to the length of the first funicular joint. Second and third funicular joints slightly longer than broad, fourth and fifth joints as broad as long, sixth and seventh joints broader than long. Club slender, slightly longer than the remainder of the funiculus, the terminal joint two and one-quarter times as long as the penultimate. Eyes composed of 20–30 facets, separated from the insertion of the mandible by a distance equal to their greatest diameter.

Thorax in profile with the mesonotum flat, the strong mesoepinotal suture narrowly but deeply impressed. Seen from above the humeral angles are moderately rounded and the sides are strongly constricted at the mesoepinotal suture. Basal face of the epinotum in profile slightly convex, much longer than the declivious face, the angle between the two considerably rounded. Basal face of the epinotum unbordered, the edges strongly rounded, those of the declivious face more angular but scarcely bordered. Node of the petiole in profile obtusely triangular, its anterior and posterior faces of equal length, its summit very obtuse, its base about twice as long as the peduncle which bears a very small ventral tooth. Postpetiole in profile scarcely thicker than the node of the petiole and about four-fifths as high, its summit much rounded, its anterior face perpendicular. Seen from above the node of the petiole is triangular and about three-fourths as wide as the globose postpetiole. The anterior edge of the first gastric segment is truncate and very slightly concave.

Color honey yellow, the posterior third of the head and the hinder edges of the gastric segments tinged with brown. Very smooth and shining with sparse, small punctures which bear long, erect, rather stout hairs.

Worker major: Length 2.3 mm. (Plate VII, figs. 6, 7.)

Head one-eighth longer than broad, the occipital border somewhat more impressed than in the minor. The antennal scape in repose only slightly surpasses the posterior fourth of the head. Eyes composed of 27–30 facets, separated from the insertion of the mandible by a distance considerably exceeding their greatest diameter. Promesonotum in profile evenly convex. The declivious face of the epinotum feebly bordered. Postpetiole seen from above transversely oval. In other regards as in the worker minor.

Female: Length 6.1 mm.

Head, exclusive of the mandibles, as broad as long, the sides slightly narrowed in front, the occiput feebly impressed in the middle, the occipital angles not much rounded. Clypeus very feebly projecting, its anterior edge almost straight, carinae more divergent than in the worker, their terminal teeth short and very stout, lateral denticles obsolete. Mandibles tridentate. The antennal scape in repose reaches the lateral ocellus. All funicular joints distinctly longer than broad, the terminal joint of the club a little less than twice as long as the penultimate. Eyes large, convex, oval, their posterior border slightly surpassing the middle of the side of the head. Thorax about four-fifths as wide as the head, elongate-elliptical, epinotum in profile without a sharp angle between the basal and declivious faces, the basal face with a shallow, median longitudinal impression, the declivious face feebly bordered and transversely wrinkled. Node of the petiole in profile with a long, sloping, concave anterior face and a short, perpendicular posterior face, the summit much compressed. Postpetiole in profile notably thicker than the node of the petiole and about three-fourths as high.

Smooth and moderately shining. Punctures proportionately larger and more numerous than in the worker, the hairs which they bear much stouter. The genae behind the insertion of the mandibles and also the metapleurae striato-rugose; the peduncle and base of the petiole and the sides of the postpetiole finely rugulose. Head and thorax a rich reddish yellow, the basal portion of the first gastric segment a somewhat paler yellow, the gastric segments edged with reddish brown.

Male unknown.

Type locality: Miller Canyon, Huachuca Mts., Arizona. (W. M. Wheeler.)

Huachucana is closely related to the South American *andina* but differs in its more globose postpetiole, smaller eyes, less angular epinotum and yellow color. It is known only from the type material which was taken by Dr. W. M. Wheeler in 1910. Two nests were found under stones in Miller Canyon at an altitude of about fifty-five hundred feet.

S. (Euophthalma) macrops Santschi.

S. macrops, Santschi, Ann. Soc. Cien. Argent. Vol. 84, p. 280, (1914). ♀.

Worker: Length 1.4 mm. (Plate VI, fig. 4.)

Head, exclusive of the mandibles, one-sixth longer than broad, somewhat narrower in front than behind, the sides slightly convex, the occiput broadly and very feebly impressed. Clypeus moderately projecting, its anterior edge deeply impressed between the carinae which are well marked and feebly divergent. Carinal teeth short, stout and obtuse, lateral denticles poorly defined and much rounded. The antennal scape in repose reaches the posterior fourth of the head. First funicular joint as long as the following three together, the small joints of the funiculus all strongly transverse, club short and thick, the terminal joint almost three times as long as the penultimate. Eyes very large, rather convex, about two and one-half times as long as wide, with twelve or thirteen facets in their greatest length (about fifty in all), their anterior border, which extends forward and downward, closely approximated to the most ventral point of the insertion of the mandible.

Thorax without humeral angles, the anterior edge of the pronotum evenly convex, the constriction of the sides at the mesoepinotal suture only moderately strong. Promesonotum in profile very feebly convex, dorsum of the thorax with a very feeble impression at the mesoepinotal suture, the latter very shallow. Basal face of the epinotum in profile feebly convex and passing to the somewhat shorter declivious face without a well marked angle. Basal face not bordered, the edges much rounded, those of the declivious face more angular. Node of the petiole in profile triangular, its base more than twice as long as the peduncle, the summit narrow but rather blunt, the peduncle with a small but distinct ventral tooth. Postpetiole in profile slightly higher than thick, its anterior face perpendicular, its posterior face forming a continuous convex slope with the rounded summit. Seen from above the node of the petiole is rather constricted, being only two-thirds as wide as the subglobose postpetiole. Anterior edge of the first gastric segment truncate and slightly concave.

Color black, the mandibles, antennae, legs and joints of the petiole more or less tinged with brown. Smooth and shining with very sparse and minute punctures. Hairs very fine, whitish and closely appressed except on the funicular joints and tarsi where they are suberect. Sexual forms unknown.

Type locality: Argentina, Tandil (Bruch).

Through the kindness of Dr. Santschi I have been able to examine one of the two types upon which this remarkable species was founded. The peculiar shape and position of the eyes sharply separate *macrops* from the other species in the subgenus *Euophthalma*. In regard to the significance of the unusually elongate eyes Dr. Santschi gives the following note:

"It (*macrops*) is distinguished particularly by the shape of the eyes which are like those in *Oxyopomyrmex*. It shows, perhaps, a case of convergence for adaptation. The color also indicates an epigaeic life. I have been able to observe that the workers of *Oxyopomyrmex* stop at the opening of their nest before going out in such a manner that only the head projects, which permits the two eyes, by reason of their placement, to embrace a vast visual sector."

One additional note on *macrops* may be added. In the original description of this insect Santschi notes that the mandibles bear three teeth. The mandibles of the specimen which I examined were closed so that the teeth of the inner mandible were shielded. In the mandible which could be examined there were only three teeth, but at the point where the long outermost tooth should have occurred there was a truncated stump, strongly suggestive of the remains of a broken tooth. Since the mandibular teeth are frequently broken or worn down, it seems to me that Dr. Santschi's statement concerning the tri-dentate mandibles of *macrops* should be regarded with caution until more material can be examined.

S. (EUOPHTHALMA) METANOTALIS EMERY.

Introduction.

The strongly sculptured epinotum of *metanotalis* is a character sufficiently distinct to have saved it from the confusion which has fallen upon many of the less striking species in the genus. All the forms placed under *metanotalis* appear to have been correctly assigned, but when we compare these with others, not included under *metanotalis*, but showing the same characteristic of epinotal sculpture, difficultues at once arise. *Metanotalis* is unquestionably a highly variable insect. Such a condition is ordinarily accompanied by an abundance of material which permits extensive comparison. In the case of *metanotalis*, however, not only is the insect relatively rare but also the number of specimens collected is seldom large. Since its recognition by Emery in 1896, *metanotalis* has been taken less than a dozen times and on three occasions the specimens showed sufficient differences to justify varietal status. I have been fortunate in having for examination cotypes of the varieties *emiliae* and *argus* which Dr. Santschi generously sent me and there are, in the collection of Dr. W. M. Wheeler, two specimens of the typical *metanotalis* identified by Emery. A study of this material shows that it can be arranged in a series in which the size of the insect steadily decreases, the head becomes narrower, the occiput, at first concave, is later flat and then feebly convex, the eyes diminish in size, the flat mesonotum grows convex, the epinotum loses its angularity and the node of the petiole increases in height and decreases in thickness. It thus happens that the smallest member of the series, Santschi's *emiliae*, shows characteristics similar to those of *shiptoni*, a species first described by Forel in 1914. Unfortunately I have seen no specimens of the typical *shiptoni* but I have examined a cotype of the variety *steigeri* which was established by Santschi in 1916. Moreover Forel's description of *shiptoni* deals with many of the characters enumerated above and from it one may see that *shiptoni* is transitional between *emiliae* and the variety *steigeri*. At the time when Forel described *shiptoni steigeri* was unknown and it is doubtful if he had seen *emiliae*, although in 1912 he described a similar variety which he called *pelotana*. Nevertheless *shiptoni*, if compared directly with the typical *metanotalis*, might justifiably be considered a separate species. Indeed, there is no reason, even when we consider the transitional forms, why such a view might not be defended, but in this event consistency would demand that the intermediate forms also have specific status. That this is far from desirable appears self-evident and I, therefore, propose to consider *shiptoni* a subspecies of *metanotalis*. The various forms may be arranged as follows:

S. (*E.*) *metanotalis* Emery
    var. *argus* Santschi
  subsp. *emiliae* Santschi
  subsp. *pelotana* Forel
  subsp. *shiptoni* Forel
    var. *steigeri* Santschi

The subspecies *pelotana* I have not seen but it is evidently related to *emiliae* since Santschi mentions the existence of transitional forms between the two.

The distribution of *metanotalis* is largely restricted to the La Plata basin. If one takes as a center a point slightly to the north of Santa Fe, Argentina, and describes around it a circle whose radius is four hundred and fifty miles, all the known forms will be included in the area thus delimited. The subspecies *shiptoni* and its variety *steigeri* occur in the northwestern part of this area, while the subspecies *pelotana* marks its eastern boundary. The remaining forms come from the region around the mouth of the La Plata river.

It is difficult to separate the forms of *metanotalis* since they present such a completely intergrading series of characters. The following key may facilitate the recognition of the various subspecies and varieties:

1. Dorsum of the thorax in profile rectilinear, mesonotum flat and forming a straight line with the basal face of the epinotum........2
Dorsum of the thorax in profile not rectilinear, mesonotum feebly convex, mesoepinotal suture feebly impressed.....................3
2. Occiput flat...*metanotalis*
Occiput slightly impressed..............................var. *argus*
3. Eyes with approximately forty facets..................subsp. *emiliae*
Eyes with not more than twenty-five facets.......................4
4. Carinal teeth long, head broader behind than in front, length 1.6–1.8 mm..subsp. *shiptoni*
Carinal teeth short, head as broad in front as behind, length 1.4 mm.
var. *steigeri*

Forel's *pelotana* has been omitted since the brief description does not permit its certain inclusion. It would probably run down to *emiliae* from which it seems to differ in its longer head and lighter color.

S. (Euophthalma) metanotalis Emery.

S. metanotalis, Emery, Bull. Soc. Ent. Ital., Vol. 28, p. 86 (1896). ☿.

Worker: Length 2.5 mm. (Plate V, figs. 2, 3.)

Head as broad as long, the sides feebly convex, the occiput flat. Clypeus strongly projecting, the carinae prominent and terminating in rather long, stout, blunt teeth, the anterior edge of the clypeus between these strongly impressed, lateral denticles entirely absent. Outer border of the mandibles evenly curved, their masticatory margin armed with three large teeth, the outermost almost twice as long as the adjacent tooth, a very small denticle present at the junction of the masticatory and inner margins. The antennal scape in repose almost reaches the posterior sixth of the head, first funicular joint as long as the following three together, joints 2–5 as broad as long; joints six and seven slightly broader than long; club stout, the terminal joint slightly more than twice as long as the penultimate. Eyes very feebly convex, composed of about fifty-five facets, separated from the insertion of the mandible by a distance very little less than their greatest diameter.

Thorax with well marked humeral angles. Seen in profile the anterior portion of the pronotum forms a sharply descending, evenly convex slope. Its posterior portion forms, with the mesonotum and the basal face of the epinotum, a flat plane in which the unimpressed mesoepinotal suture appears as a narrow slot. Basal face of the epinotum separated from the much shorter declivious face by a wide but sharply defined angle. Posterior half of the basal face feebly bordered, declivious face strongly bordered throughout. Node of the petiole triangular, about as high as the base is thick, the summit bluntly rounded, the peduncle almost as long as the base of the node and bearing a prominent ventral tooth, the portion of the petiole posterior to the peduncle and ventral to the node rather bulky and forming with the peduncle a sinuate ventral outline. Postpetiole in profile approximately two-thirds as high as the node of the petiole, somewhat thicker than high, the summit evenly rounded, the anterior face very short, the posterior face much longer and gradually sloping. Seen from above the node of the petiole is narrow, scarcely three-fourths as wide as the transversely oval postpetiole. Abdomen oval, the edge of the first gastric segment not truncate.

Smooth and moderately shining, the punctures sparse and very fine. Hairs of moderate length, sparse and erect. The entire epinotum and the mesopleurae striato-rugulose. The base of the mandible and the area between its insertion and the anterior margin of the eye irregularly striate. The lower portions of the sides of the petiole and postpetiole densely punctate. Sides of the pronotum with irregular and extremely feeble striae.

Head ferrugineous, thorax and clypeus, mandibles, appendages, petiolar joints and base of the gaster yellowish red, posterior portion of the gaster piceous.

Sexual forms unknown.

Localities: Argentina, La Plata (Type loc.) (Spegazzini.)
Buenos Aires. (Silvestri.)
Rio Negro, Colonia Frias. (Lehmann-Nitsche.)
Uruguay, Montevideo. (Ris.)

Redescribed from two specimens taken by Silvestri and compared with the types by Emery. *Metanotalis* is not likely to be confused with any other species in the subgenus except *silvestrii*, which also has a sculptured epinotum. The head of *silvestrii*, however, is distinctly broader than long, a character not shown by any of the forms of *metanotalis*.

S. (Euophthalma) metanotalis var. **argus** Santschi

S. metanotalis var. *argus*, Santschi, Rev. Suisse Zool. Vol. 30, p. 260 (1923). ☿, ♂.

The distinctly impressed occiput and slightly larger eyes of sixty or more facets distinguish this variety from the typical *metanotalis*. The clypeal teeth of *argus* are somewhat longer and the basal and declivious faces of the epinotum meet at a sharper angle. The abdomen is narrowly truncate at its anterior edge and uniformly piceous. Length 2.2–2.5 mm. (Plate V, fig. 1.)

The female of *argus* is unknown but Santschi has given a good description of the male, a translation of which is presented below:

"Male: Length 4.5 mm.

"Black, appendages mottled with brown, genital armature a greyish white. Wings hyaline with light brown veins. Gaster glistening and smooth, the rest dull, densely reticulo-striate or reticulo-punctate. Pilosity fine and oblique on the gaster, longer, thicker and more erect on the head, short and not very abundant on the appendages which are very densely pubescent. The head is more than a third broader than long, rounded behind; the eyes occupy all of the projecting portion of the sides. The clypeus has a semilunar impression near the anterior border in front of the median lobe. The frontal furrow reaches the median ocellus. Mandibles rugose, very narrow with parallel borders, armed with two teeth. The scape is one quarter longer than thick, the following joint rounded, as long and as thick as the scape. Joints 2–5 of the funiculus subequal in length, one-fourth to one-third longer than thick, the remaining joints progressively elongated. Thorax robust, basal face of the epinotum convex, a little longer than the declivious, which is concave in profile, forming with it a rounded angle. The two petiolar nodes have almost the same profile as in the worker."

Localities: Argentina, Entre Rios. (Type loc.) (Bruch.)

S. (Euophthalma) metanotalis subsp. **emiliae** Santschi.

S. metanotalis var. *emiliae*, Santschi, Rev. Suisse Zool. Vol. 20, p. 526 (1912). ☿.

The subspecies *emiliae* differs from the typical form in the following characteristics:

The eyes have only forty-two facets, the head is slightly narrower with the occipital angles more rounded, the mesonotum in profile is feebly convex and does not form a straight line with the basal face of the epinotum, the mesoepinotal suture is feebly impressed, wider and not slot-like, the epinotum is more rounded, without a sharp angle between the two faces, the basal face is very slightly convex; the node of the petiole is higher and thinner, its height greater than the width of its base; the color is a more brilliant red; the anterior edge of the first gastric segment is narrowly truncate.

Length 2.0 mm. (Plate V, figs. 5, 6.)

Sexual forms unknown.

Localities: Uruguay, Colonia Helvetia. (Type loc.) (von Steiger.)
Argentina, Rio de la Plata, Isla Martin Gracia. (Bruch.)

S. (Euophthalma) metanotalis subsp. **pelotana** Forel.

S. metanotalis var. *pelotana*, Forel, Mem. Soc. Ent. Belg., Vol. 20, p. 5 (1912). ☿, ♀. Santschi, Rev. Suisse Zool. Vol. 30, p. 260 (1923). ☿.

Forel's description of *pelotana* deals only with the shape of the head and the color. The head is said to be markedly longer than broad with a slightly convex posterior border. The color is described as reddish yellow, the abdomen brown with the base reddish.

To judge from the shape of the head *pelotana* is transitional between *emiliae* and *shiptoni*. Forel has given us the description of the female of *pelotana* which is the only form in the species in which this caste is known. A translation of his description is presented below:

"Female: Length 6.0 mm.

"Mandibles tridentate. Clypeal carinae very strong. Thorax as broad as the head. Epinotum with two feeble tubercles (subbidentate) with the declivious face bordered and transversely wrinkled, less distinct on the basal face, the rest dull and reticulo-punctate as in the worker. Posterior half of the abdomen densely reticulate and subopaque or dull. Reddish yellow, two longitudinal brownish bands on the mesonotum and a transverse, more or less cloudy brownish band on each abdominal segment. Wings lacking. In other regards as in the worker."

Localities: Brazil, Pelotas. (Type loc.) (Gensterblum.)

S. (Euophthalma) metanotalis subsp. **shiptoni** Forel.

S. shiptoni, Forel, Bull. Soc. Vaud. Sc. Nat., Vol. 50, p. 267 (1914). ☿.

The subspecies *shiptoni* may be distinguished from any of the foregoing forms by its smaller eyes. In the original description these are said to be composed of "at least fifteen large facets," but I am inclined to believe that the number given is too conservative. *Shiptoni* is smaller than *emiliae* which it seems to resemble closely. The head is one-seventh longer than broad and the occiput appears to be more convex than in *emiliae*. The node of the petiole is high and thin as in *emiliae*, the postpetiole somewhat thinner than in that subspecies. The head is brown, thorax petiolar joints and mandibles yellowish red, the appendages more yellowish, abdomen brownish black.

Length 1.6–1.8 mm.

Sexual forms unknown.

Type locality: Argentina, Tucuman. (Shipton.)

S. (Euophthalma) metanotalis subsp. **shiptoni** var. **steigeri** Santschi.

S. shiptoni var. *steigeri*, Santschi, Physis Buenos Aires, Vol. 2, p. 378, (1916). ☿.

In the original description of *steigeri* Santschi notes that it differs from *shiptoni* in the lack of infuscation of the occiput, the larger eyes and the somewhat broader anterior portion of the head which is as wide as the posterior portion (wider behind than in front in *shiptoni*). The number of facets composing the eye is said to be 18–22, but in the cotype sent me by Dr. Santschi the number is only sixteen. As I have already noted, Forel's statement in regard to the size of the eyes in *shiptoni* is rather loose and I doubt that Dr. Santschi is justified in drawing conclusions as to the relative number of facets in the two forms. In both *shiptoni* and *steigeri*, however, the facets are very coarse and much rounded so that they stand out sharply from each other. This gives the eye an appearance which Forel has very aptly compared to a black-berry.

Since I have not seen specimens of the subspecies *shiptoni* it is not possible to give a more complete comparison of that form and the variety *steigeri*. It seems advantageous, however, to contrast *steigeri* with the subspecies *emiliae*, but it must be borne in mind that the differences given below may or may not be shown by the typical *shiptoni*. In the variety *steigeri* the node of the petiole is higher and thinner than in *emiliae*, its height much greater than the width of its base. The postpetiole in profile, while thinner than in *emiliae* is still scarcely higher than thick. Seen from above the postpetiole is more strongly compressed from front to back than in *emiliae*, but no wider in comparison to the width of the node of the petiole than in that form. The anterior edge of the first gastric segment is more broadly truncate than in *emiliae* and with a pronounced concave impression. Length: 1.5 mm. (Plate V, figs. 4, 7.)

Type locality: Argentina, Chaco. (von Steiger.)

S. (EUOPHTHALMA) NIGELLA EMERY.

Introduction.

In his original treatment of *nigella* Emery combined the descriptions of the major and minor workers. This is to be regretted since it is difficult to know whether some of the characteristics apply to the major, the minor or to both. Moreover the species is strongly dimorphic with pronounced differences between the major and minors. The latter caste has never been fully described and this omission undoubtedly accounts for some of the subsequent confusion in the taxonomy of the species. In 1901 Forel described a subspecies of *S. picta* which he called *gensterblumi*. It is obvious that he did not, at the time, recognize the true nature of *picta* since *gensterblumi* has no relation to its whatsoever. I have examined the single cotype of *gensterblumi* in the collection of Dr. W. M. Wheeler and find that, except for the somewhat lighter color, it exactly corresponds to the cotypes of the small workers of Santschi's subspecies *praevalens*, a form of *nigella* described in 1923. Since the difference in color is, in my opinion, too slight to justify varietal status, I am forced to place *praevalens* in the synonymy of *gensterblumi* which becomes a subspecies of *nigella*. It is most unfortunate that Forel's name must take precedence over that of Santschi, since the former author was completely mistaken as to the affinities of *gensterblumi*, and the long association of the name with that of *picta* makes for possible confusion. Forel's failure to recognize the relationship of *gensterblumi* casts some doubt on the status of the insect which he described in 1912 as the female of *nigella*. I have included his description, since it is the only one available, but it should be used with considerable caution. *S. picquarti*, described by Forel in the Biologia Centrali-Americana, also seems to be very closely related to *nigella*. To judge from the description it can hardly be more than a subspecies of that form but, not having seen specimens, I prefer to retain *picquarti* as a separate species.

The typical *nigella* and the subspecies *gensterblumi* may be separated as follows:

Head of the major as broad as long, antennal scapes in the minor surpassing the posterior quarter of the head .... *nigella*

Head of the major one-sixth longer than broad, antennal scapes in the minor barely reaching the posterior quarter of the head. subsp. *gensterblumi*

S. (Euophthalma) nigella Emery.

S. nigella Emery, Bull. Soc. Ent. Ital., Vol. 19, p. 355 (1887). ☿ Forel, Mem. Soc. Ent. Belg., Vol. 20, p. 5 (1912). ♀.

Worker major: Length 2.4 mm. (Plate VI, figs. 6, 7.)

Head very large, twice as broad as the thorax, subquadrate, the sides feebly convex, narrowed throughout their anterior half, occiput with a strongly impressed median groove, frontal furrow well developed. Clypeus moderately projecting, the carinae prominent, slightly divergent in their anterior half and each terminating in a moderately long, stout tooth, lateral denticles absent, the anterior edge of the clypeus between the carinal teeth strongly impressed. Mandibles short, their external border strongly curved, their masticatory border armed with three large teeth (the fourth tooth sometimes represented by a small denticle). Antennal scapes very short, their tips not reaching the posterior third of the head. First funicular joint only slightly shorter than the following three together, second funicular joint as broad as long, the remaining small joints strongly transverse; club stout, the terminal joint less than twice as long as the penultimate. Eyes large and flat, composed of about thirty facets, separated from the insertion of the mandible by a distance one-third greater than the maximum diameter of the eye.

Thorax with much-rounded humeral angles. The pronotum in profile with an abruptly descending anterior face, mesonotum flat, the mesoepinotal suture very broad, deeply incised but not impressed. Epinotum in profile angular, the basal face perfectly straight and very slightly longer than the declivious. Node of the petiole in profile triangular, the anterior face much longer than the posterior, the summit rather thin and somewhat flattened, the peduncle short and thick with a very large ventral tooth. Postpetiole in profile barely three-fourths as high as the node of the petiole, the anterior face perpendicular and sharply cut off from the dorsum which is evenly convex. Seen from above the node of the petiole is only two-thirds as wide as the postpetiole, the latter is one-fourth broader than long, subpentagonal, with the anterior angle rounded. Anterior edge of the abdomen narrowly truncate. Femora somewhat swollen.

Color black, the mandibles, joints of the funiculus, legs and tarsi reddish. Punctures distinct but small and sparse. Hairs excessively short, fine and appressed. A few long hairs on the anterior edge of the clypeus, the mandibles and the gula. The base of the mandibles and the metapleurae with a few irregular striations. The area between the eye and the insertion of the mandible striato-punctate, the declivious face of the epinotum transversely rugulose, the peduncle and the base of the petiolar nodes punctato-rugulose.

Redescribed from two major workers taken by Silvestri at Cernadas, (Cordoba) Argentina and compared by Emery with the types. I have not seen the minor worker of *nigella* nor is there an adequate description of it. The following characteristics are compiled from Emery's original description and Santschi's comparison with the minor of the subspecies *gensterblumi:*

Worker minor: Length 1.5 mm.

Head more elongate and narrower than in the major, the frontal furrow virtually absent, the area between the eye and the insertion of the mandible smooth. The antennal scape considerably surpasses the posterior quarter of the head. The declivious face of the epinotum is transversely wrinkled. The postpetiole is subglobose.

A translation of Forel's description of the female follows:

"Female: Length 4.5 mm.

"Head square, a little enlarged behind, the sides rather convex, occiput grooved, the middle of the head much broader than the thorax. Mandibles strongly bent in their anterior half. Clypeal teeth strong. Epinotum with two large and strong subdentate tubercles. Node of the petiole squamiform. Head with scattered punctures. Wings hyaline with pale veins. Black, legs and scapes brown; mandibles, anterior border of the head, funiculus, tarsi and articulations of the legs reddish."

Male unknown.

Localities: Brazil, Rio Grande do Sul. (Type loc.) (von Ihering.)
Argentina (Cordoba) Cernadas. (Silvestri.)
(Buenos Aires) Tandil. (Bruch.)
(Entre Rios) Villaguay. (Bruch.)
(Cordoba) Tanti Viejo. (Durione.)
Huasan. (Bruch.)
Sierra de la Ventana. (Bruch.)

Nigella is easily distinguished from any other species in the subgenus *Euophthalma* by the unusually large head of the major worker. The proportion of the head to the body and the strong bending of the mandibles is suggestive of the major of *geminata*. Beyond the fact that this species nests in sand, nothing appears to be known of its habits, but it is safe to assume from the structure of the head of the major that it is graminivorous.

S. (Euophthalma) nigella subsp. **gensterblumi** (Forel).

S. nigella subsp. *gensterblumi* (Forel), Mitt. Schweiz Ent. Ges., Vol. 10, p. 298 (1901) ☿. (*picta* subsp.)

S. nigella subsp. *praevalens*, Santschi, Rev. Suisse Zool., Vol. 30, p. 257 (1923) ☿.

I have not seen the major worker of this subspecies but Santschi's description permits the recognition of a number of differences between it and the typical *nigella*. The more important of these are:

The head of *gensterblumi* is longer (one-sixth longer than broad) with a more feebly impressed occiput, the eyes are slightly larger, the mandibles have four teeth, the antennal scape reaches the posterior third of the head, the basal face of the epinotum is grooved (smooth in *nigella*) and meets the declivious face at a somewhat sharper angle. Length 2.2 mm.

Worker minor: Length 1.6 mm. (Plate VI, figs. 8, 9.)

Head, exclusive of the mandibles, as broad as long, the sides strongly convex, the occiput broadly and feebly impressed. Clypeus strongly projecting, the carinae prominent and each terminated by a moderately long, stout tooth, the anterior edge of the clypeus between the carinal teeth moderately impressed, lateral denticles represented only by much rounded and very feeble angles. External border of the mandible evenly curved, the masticatory margin with three large teeth and a fourth much smaller inner tooth at the junction of the masticatory and inner margins. The antennal scapes in repose barely reach the posterior quarter of the head, first funicular joint as long as the following three together, joints 2–7 all strongly transverse; club stout with the terminal joint swollen, three times as long as the penultimate. Eyes slightly convex, consisting of 18–22 facets, separated from the insertion of the mandible by a distance less than their greatest diameter.

Promesonotum in profile flattened, the mesoepinotal suture feeble and virtually unimpressed. Basal and declivious faces of the epinotum not sharply separated, together forming a feebly convex arc. Node of the petiole in profile triangular, the anterior face feebly concave and much longer than the straight posterior face, the summit narrow but rounded, the peduncle one-half as long as the base of the node and with a prominent ventral tooth. Postpetiole in profile about three-fourths as high as the node of the petiole. Seen from above the petiole is almost as wide as the postpetiole, the latter is approximately as broad as long with the anterior border strongly convex and the lateral and posterior borders straight.

Color piceous brown, the mandibles, clypeus, small funicular joints, base and apical third of the scape brownish red, legs brown, except the tarsi which are yellowish. Cephalic punctures small but distinct and fairly numerous, those on the thorax and abdomen minute and sparse. Metapleurae and the declivious face of the epinotum feebly striate. Otherwise smooth and shining. Hairs short, fine and appressed, a few long stout bristles are present on the clypeus, mandibles, gula and the edges of the posterior gastric segments.

Sexual forms unknown.

Localities: Brazil, Sao Paulo. (Type loc.) (von Ihering.)
Sao Paulo, Ypiranga. (Luderwaldt.)
Argentina, La Plata. (Bruch.)
(Buenos Aires) Sierra de la Ventana. (Bruch.)

For a discussion of the synonymy of *gentserblumi* the reader is referred to the introduction to *nigella*.

S. (Euophthalma) oculata Santschi.

S. oculata, Santschi, Com. Mus. Nac. Hist. Buenos Aires, Vol. 2, p. 161 (1925).
☿. (*angulata* subsp.)

Worker: Length 2.0–2.2 mm. (Plate VII, figs. 3, 4.)

Head as broad as long, broadest just in front of the eyes, posterior to the eyes the sides converge slightly toward the occiput, occipital angles not much rounded, occiput with a feeble and broadly concave impression. Clypeus strongly projecting, the carinae very divergent, rather thin and angular above, each terminated by a small, slender and somewhat acute tooth, lateral denticles well marked, much broader at the base than the carinal teeth but with the apex rather sharp. Mandibles quadri-dentate, the innermost tooth much smaller than the rest but distinct. The antennal scape in repose fails to reach the occipital border by a distance equal to its greatest thickness.

First funicular joint almost as long as the following three together, second joint slightly longer than broad, joints three, four and five as broad as long, joints six and seven slightly broader than long. Club very slender, one-sixth longer than the remainder of the funiculus, the terminal joint very slightly more than twice as long as the penultimate. Eyes composed of approximately thirty facets, separated from the insertion of the mandible by a distance equal to their greatest diameter.

Thorax seen from above rather narrow, the anterior edge of the pronotum considerably rounded, the sides only slightly constricted at the mesoepinotal suture. Promesonotum in profile evenly but moderately convex, mesoepinotal suture not impressed, shallow and slot-like. Basal face of the epinotum in profile flat, much longer than the declivious face from which it is separated by a well marked angle. Basal face of the epinotum not bordered, the edges much rounded, those of the declivious face angular but scarcely bordered. Node of the petiole in profile triangular, its height slightly greater than the length of its base, the summit obtusely rounded, posterior face perpendicular, anterior face somewhat longer than the posterior and sloping steeply to the peduncle, the latter approximately as long as the base of the node and bearing a prominent ventral tooth; a very short posterior peduncle occurs behind the node, the ventral surface of the petiole feebly sinuate. Postpetiole in profile about three-fourths as high as the node of the petiole, thicker than high with a short, perpendicular anterior face and an evenly convex summit which passes without sharp transition to the sloping posterior face. Seen from above the node of the petiole is rather narrow, subtriangular in outline and two-thirds as wide as the transversely oval postpetiole. Anterior edge of the first gastric segment truncate.

Head and abdomen piceous brown, thorax and petiolar joints a somewhat clearer and more yellowish brown, mandibles, base and apex of the scape, funicular joints and club and the articulations of the legs dirty yellow. Genae, between the insertion of the mandible and the anterior edge of the eye, with a few short and feeble striae, meso- and metapleurae and the sides and bottom of the mesoepinotal suture strongly striate, the rest of the insect smooth and shining with sparse and very small punctures. Hairs sparse, very slender, erect or suberect on the body, shorter stouter and subappressed on the appendages.

Type locality: Argentina, Catamarca. (Weiser.)

Oculata was originally considered by Dr. Santschi a subspecies of *angulata*. Since the female is unknown its relation to *angulata* was largely conjectural, and except for its somewhat similar petiole I can see no reason why *oculata* should have been assigned to that species. Its large eyes of thirty facets obviously place it in the subgenus *Euophthalma*. *Oculata* is closely related to *andina* and *huachucana*. It may be distinguished from *andina* by its longer antennal scapes and smaller eyes of only thirty facets. It differs from *huachucana* in its darker color and much more slender thorax in which the greatest width of the promesonotum is scarcely one and one-quarter times as wide as the epinotum. The constriction of the sides at the mesoepinotal suture is feeble. In *huachucana* the greatest width of the promesonotum is more than one and one-half times as wide as the epinotum and the sides are strongly constricted at the mesoepinotal suture. The node of the petiole, seen from above, is much narrower in *oculata* than in *huachucana*.

Although I have given *oculata* specific status I suspect that it may subsequently be necessary to make it a subspecies of *andina*. Indeed the differences between the two are of such a nature that I would not be greatly surprised if it should prove to be the minor worker of *andina*.

S. (Euophthalma) picquarti Forel.

S. picquarti, Forel, Biol. Centrali-Amer. Hymenop., Vol. 3, p. 80, (1900). ☿.

This species is known only from type material which I have not seen. A translation of Forel's description is given below:

"Worker: Length 1.6–1.7 mm.

"Head rather large, distinctly impressed behind, broader behind than in front, the sides feebly convex, rather flattened. Mandibles smooth, glistening. Eyes large, having seven or eight facets in a line in their greatest diameter, situated slightly in front of the anterior third of the head. Anterior border of the clypeus impressed in the middle, strongly projecting and armed with two short teeth, broad, directed outward and lightly curved outward. The clypeus is depressed, feebly convex, the two carinae are almost obsolete and the teeth scarcely form more than the lateral angles of the median lobe. At either side of the median lobe the clypeus is strongly impressed and has rather a prominent angle at the lateral border of this impression. Frontal furrow not very distinct but visible. The scapes slightly surpass half the distance between the eye and the occipital border (almost reaching the posterior third of the head). The pronotum rounded near the anterior angles, dorsum of the pronotum rather depressed, rectilinear in profile, mesoepinotal suture deeply incised. Epinotum very short, in profile subdepressed above. Nodes of the petiole small and narrow, the second as broad as long. Femora rather strongly swollen. Joints 2–7 of the funiculus much broader than long. Last joint three times as long as the penultimate. Very smooth and very glistening with sparse piligerous punctures. Genae with dense, longitudinal, wrinkled striae. Some reticulations on the anterior part of the pronotum, on the metanotum and on the peduncle. Hairs regular, sparse everywhere, yellowish. Color dull, brownish yellow, tarsi, antennae and mandibles more clear, antennal club and a band on the abdomen brownish."

Sexual forms unknown.

Type locality: Costa Rica. (Tonduz.)

This insect is very closely related to *nigella* if not actually a subspecies of it. I have seen no material which can be assigned to *picquarti* and, other than the original description, there are no references to it in the literature. It is probably rare or very local in distribution.

S. (Euophthalma) schilleri Santschi.

S. schilleri, Santschi, Rev. Suisse Zool., Vol. 30, p. 263, fig. 3, c (1923). ☿.

This species is known only from a single type in the collection of Dr. Santschi. A translation of his original description is given below:

"Worker: Length 2.8 mm.

"Black, mandibles, border of the clypeus, joints of the funiculus, neck, joints of the legs, the petiole and the last tarsi ferrugineous red. Smooth and glistening except the front of the head and the sides of the epinotum, which are striate and the dorsum of the epinotum and the foot of the peduncle, which are finely reticulate. Pilosity regular, very sparse, fine, whitish, short on the head, very rare on the thorax, longer and more oblique on the abdomen. The appendages have only a slightly erect pubescence.

Head almost square, scarcely longer than broad, the sides a little convex, the posterior border straight with the angles rounded. The eyes oval, rather convex, occupying the space between the first anterior quarter and the middle of the head. No frontal furrow. Frontal area very narrow. The clypeal carinae progressively diverge up to the two lone teeth of the anterior border which are as long as one-fourth of the distance between them. Mandibles striate, the terminal border a little oblique, armed with three teeth besides a denticle on the internal border. The scape fails to reach the posterior border of the head by twice its own thickness. The first joint of the funiculus is as long as the following three together, joints 2–8 scarcely longer than thick, the terminal joint of the club slightly more than twice as long as the penultimate. Pronotum bordered in front and sub-shouldered. The profile of the thorax is almost straight from the middle of the pronotum to the epinotal angle, with a slight meso-epinotal incision. The epinotal faces sub-bordered, the basal face rectangular, about twice as long as broad, forming a rounded angle with the declivious face which is only half as long. The node of the petiole shows a triangular profile with a blunt summit, its anterior face more prolonged than the posterior and not higher than the length of its base. Peduncle with a subvertical tooth below. Postpetiole as high as the preceding node, much thicker in profile and a third broader. First segment of the gaster one-third longer than broad and truncate at the base."

Sexual forms unknown.

Type locality: Argentina (Neuquen) Challacito. (Schiller.)

Santschi relates this species to *andina* but, from his description, I consider it much more closely related to *metanotalis*. The straight dorsum of the thorax and the reticulate sculpture of the epinotum both indicate affinities with *metanotalis*. *Schilleri* would apparently run down in the key to *metanotalis* from which it seems to differ in its more rounded pronotum and darker color.

S. (Euophthalma) silvestrii Emery.

S. silvestrii, Emery, Bull. Soc. Ent. Ital. Vol. 37, p. 120 (1905). ☿ ♀.

Worker: Length 1.6–1.9 mm. (Plate V, figs. 8, 9.)

Head almost one-sixth broader than long, the sides feebly convex, somewhat narrowed toward the occiput, more feebly narrowed in front of the eyes. Occiput flat. Clypeus strongly projecting, its carinae each terminating in a long, rather thin tooth, the edge of the clypeus between these with a broad, cresentric impression, lateral denticles absent. Mandibles smooth, armed with four teeth, the innermost tooth very small and borne on the inner margin. The antennal scape in repose fails to reach the occipital border by a distance slightly exceeding the length of the first funicular joint. This joint almost as long as the following three together, joints 2–7 only slightly broader than long, club rather thin, the terminal joint twice as long as the penultimate. Eyes consisting of about forty-five rather feebly pigmented and poorly defined facets, separated from the insertion of the mandible by a distance equal to their greatest diameter.

Promesonotum in profile feebly arcuate, the mesoepinotal suture narrowly but clearly impressed. Epinotum in profile evenly curved, the two faces not separated by an angle. Node of the petiole in profile triangular, its summit rounded, the anterior and posterior faces of almost equal length, the peduncle short and thick with a well developed ventral tooth. Postpetiole in profile notably lower, thicker, and more rounded above than the node of the petiole. Seen from above the node of the petiole is constricted, barely twice as wide as the peduncle and only two-thirds as wide as the transversely oval postpetiole. The anterior edge of the first gastric segment is narrowly truncate.

Color: thorax and node of the petiole pale yellow, the head, postpetiole and appendages slightly more reddish yellow, the abdomen castaneous brown, except for a yellow patch at the base of the first segment. Mesopleurae, metapleurae and the entire epinotum opaque, covered with fine coriaceous reticulations; the sides of the pronotum, the area between the eye and the insertion of the mandible and the base of the node of the petiole with a few irregular and rather feeble striae. Punctures fine and sparse. Hairs on the body unusually sparse, erect and very stout, those on the appendages more numerous, finer and shorter.

I have not seen the female of *silvestrii*. A translation of Emery's description of this caste follows:

"Female: Length 6.5 mm. with the gaster much distended.

"Pale ferrugineous, the posterior part of the gaster brown. Shining, the metapleurae, epinotum and sides of the postpetiole opaque, striatulate-punctate with the punctures stronger than in the worker. Hairs of the body acuminate. Head subquadrate with rounded sides; sculpture as in the worker but with the pilegerous punctures very much larger. Clypeus as in the worker but its teeth much larger and not spine-shaped. Node of the petiole more squamiform than wedge shaped, postpetiole much broader than the petiole, more than twice as broad as long, somewhat hollowed out in front and for this reason semilunar with the angles rounded. The wings lacking."

Male unknown.

Type locality: Uruguay, La Sierra. (F. Silvestri.)

The description of the worker given above is based upon two cotypes in the collection of Dr. W. M. Wheeler. *Silvestrii* is known only from type material. It is closely related to *metanotalis* but differs in its broader head, less pronounced sculpture and paler color. No data on the habits of this species are available but its pale coloration and the feeble pigmentation of the eyes indicate a hypogaeic existence.

Subgenus Diagyne Santschi.

Diagyne, Santschi, Rev. Suisse Zool., Vol. 30, p. 267 (1923).

Female: scarcely larger than the largest workers (female 2.6–2.8 mm.) with ten jointed antennae. Clypeus without teeth or carinae. Mandibles rather narrow, armed with a large, stout, sharp outermost tooth which is somewhat bent inward. Middle tooth very small. Innermost tooth unusually large, composed of two teeth more or less fused together. Inner border of the mandible very short.

Worker: Clypeus with neither carinae nor teeth. Eyes consisting of four or five poorly defined facets. Second funicular joint as broad as long, joints 3–7 broader than long.

Type of the subgenus: *S. succinea* Emery.

The subgenus *Diagyne* is founded upon a single species, *S.* (*Diagyne*) *succinea* Emery. This insect as it is now known is represented only by the typical form and the subspecies *nicai* described by Forel in 1913. In the present work I have treated *S. inermiceps* of Wheeler and Mann as a synonym of *succinea*. This matter is fully discussed at the end of the description of *succinea*.

S. (Diagyne) succinea Emery.

S. succinea, Emery, Bull. Soc. Ent. Ital., Vol. 22, p. 52 (1890). ☿ ♀ ♂.
S. inermiceps, Wheeler and Mann, Bull. Amer. Mus. Nat. Hist. Vol. 33, p. 20, fig. 7, (1914). ☿.

Worker: Length 1.9–2.5 mm. (Plate VIII, figs. 2, 3.)

Head, exclusive of the mandibles, slightly less than one-sixth longer than broad, the sides convex, the head broadest a little behind the middle, the occiput with a broad and shallow, concave impression. Clypeus very little projecting, its anterior edge sinuate, the median lobe broad and slightly concave, without carinae or teeth. Mandibles smooth, narrow and armed with four teeth, the innermost tooth much smaller than the rest. Eyes of three or four poorly defined facets, situated approximately at a point one-third of the way from the insertion of the mandible to the occipital border (in using this measurement it is well to note that the gena at the insertion of the mandible is deeply notched and the distance as given refers to a measurement from the bottom of this notch to the eye). The antennal scape in repose slightly surpasses the posterior fifth of the head. First funicular joint as long as the following three together, second joint approximately as broad as long, joints 3–7 broader than long. Club stout, the terminal joint three times as long as the penultimate.

Thorax with the humeral angles moderately rounded, the sides somewhat constricted at the mesoepinotal suture. Seen in profile the promesonotum is evenly and moderately convex except for the short and steep anterior face which descends to the neck. Mesoepinotal suture distinctly impressed. Epinotum in profile much higher than long, the two faces not clearly separated, together forming a steeply descending slope which is slightly convex in its upper half. The declivious face bears a broad and feeble median impression. Node of the petiole in profile low and very obtuse, scarcely higher than thick, the summit rather flattened, feebly convex and sloping slightly to the perpendicular posterior face. Anterior face longer than the posterior and steeply sloping forward, peduncle short and very thick with a long ventral lamella which ends anteriorly in an angular projection. Postpetiole in profile thicker and lower than the node of the petiole, the summit somewhat more evenly convex, virtually without a posterior face and with the anterior face extremely short. Seen from above both nodes are transversely oval in outline, the postpetiole being slightly wider and thicker than the node of the petiole. Edge of the first gastric segment truncate and with a strong concave impression at the base of the postpetiole.

Very smooth and shining with sparse, minute punctures which bear fine erect or suberect hairs. Punctures on the thorax, petiolar nodes and abdomen much sparser than on the head, the hairs which they bear coarser and longer. Color, head and thorax rich golden yellow, the mandibles, anterior edge of the clypeus and the articulation of the petiole and postpetiole yellowish brown, the appendages, petiolar nodes and the abdomen pale yellow.

Female: Length 2.6 mm. (Plate VIII, fig. 1.)

Head very slightly longer than broad, the sides behind the eyes narrowed toward the occiput, occipital angles well marked not much rounded, the occiput flat. Clypeus feebly projecting, the anterior edge without teeth, the median lobe with a broad, shallow sulcus extending backwards to the elongated frontal lobes, the carinae represented only by the rounded lateral areas which bound the sulcus. Mandibles rather long and narrow, tridentate, the terminal tooth bent inward, the middle tooth small and rounded, the innermost tooth very large, consisting of two more or less fused teeth. Eyes of medium size, strongly convex, with their posterior border approximately at the middle of the side of the head. Antennae ten-jointed. Antennal scape in repose slightly surpassing the occipital border, first funicular joint almost as long as the following three together, joints 2–7 all slightly longer than broad, club rather short and strongly swollen, the last joint twice as long as the penultimate.

Thorax only slightly narrower than the head. The basal and declivious faces of the epinotum not sharply separated, virtually forming a single very declivious slope. Node of the petiole in profile obtusely rounded with a very long anterior face not sharply separated from the peduncle, the posterior face short and slightly sloping forward. Postpetiole in profile large and thick, only slightly lower than the node of the petiole. Seen from above the petiole is rather narrow, the postpetiole is strongly transverse and one and one-half times as broad as the node of the petiole. Anterior edge of the first gastric segment truncate. Just posterior to the base of the postpetiole the segment bears a conspicuous, concave and narrowly triangular impression.

Punctures of moderate size and fairly numerous. Hairs long, stout and erect. Color tawny yellow, the triangular area between the ocelli deep brown. Wings hyaline, the veins and stigma clear yellow.

The male of *succinea* is known only from Emery's very brief description a translation of which is given below:

"Male: Length: 3.0 mm.

Extremely smooth with sparse, very feeble punctures, abdomen almost impunctate. Black, legs piceous, articulations and tarsi reddish, antennae an obscure yellow-brown. Head strongly narrowed behind, eyes very large, antennal scape very short. Mesonotum with a profound anterior, longitudinal, median sulcus, punctate toward the middle.

Localities: Costa Rica, Jimenez. (Type loc.) (No coll.)
Costa Rica, San Mateo. (P. Biolley.)
St. Vincents, W. I. (H. H. Smith.)
Cocos Keys, W. I. (A. Alfaro.)
Haiti, Petionville. (W. M. Mann.)
Haiti, Grande Riviere. (W. M. Mann.)

We may now discuss the reasons for synonymizing *inermiceps* with *succinea.* Although there are a number of species in the genus in which the carinal teeth are greatly reduced in size and one (the major worker of *S. xyloni* subsp. *amblychila*) in which they are lacking entirely, the complete absence of both teeth and carinae is met with in but two cases, that of *succinea* and of *inermiceps.* That the two are related by this unusual character appears too patent to require comment. Further relationship may be shown by a reference to Emery's description of the worker of *succinea.* In addition to describing the clypeal peculiarities Emery notes that the head is moderately elongate with the occipital border scarcely impressed. The first funicular joint is as long as the three following joints together. The color of the abdomen is lighter than that of the head and thorax. This last is a character of peculiar significance since the reverse is usually the case. The cotypes of *inermiceps* agree perfectly in all the points mentioned above, despite the fact that in the original description of *inermiceps* the first funicular joint is said to be nearly as long as the following four joints together. I also find that I cannot agree to the shape of the head of *inermiceps* as given in the original description, since it appears to me that the length has been greatly overestimated. This may be due, however, to the fact that I have excluded the mandibles in making measurements of the head. The size range of *inermiceps* is also slightly greater than specified in the description, extending from 1.9–2.3 mm. Since I can detect no differences which would permit the separation of the two forms I have been forced to synonymize them. In view of the fact that Emery's description is too brief to deal with differences which might be subspecific this procedure may be questioned. However, even though slight variations may exist, it is impossible at present to evaluate them. Finally, the presence of the typical *succinea* in other localities in the Antilles (*vide supra*) indicates that the Haitian specimens may also be typical.

Practically nothing is known of the habits of this interesting species. It nests in rotting logs in upland forests and the workers have been taken on decaying fruit. As Dr. Santschi has observed the small size and aberrant mandibles of the female point strongly to habits of a temporary social parasite.

S. (Diagyne) succinea subsp. nicai Forel.

S. succinea subsp. *nicai*, Forel, Bull. Soc. Vaud. Sci. Nat. Vol. 49, p. 222, (1913). ☿. Santschi, Rev. Suisse Zool. Vol. 30, p. 267 (1923). ♀ ♂.

I have not seen any material referable to this subspecies. Forel describes the worker as follows:

"Worker: Length 2.0–2.7 mm.

"Differs from the typical *succinea* through its much flatter promesonotum, almost entirely flat except at its anterior and posterior extremities, and particularly through its longer petiolar joints, especially the peduncle of the first (much shorter in the type). The petiole has a distinct posterior face, although convex. The postpetiole is also much longer, yet a little broader than long and as long as it is high. The head and body in general are almost without punctures. The thoracic impression is a little wider. For the rest similar to the type."

The insect which Santschi has described as the female of *nicai* appears, from his description, to be very similar to that of the typical *succinea*. The principal differences seem to lie in its slightly greater size (2.8 mm.), the presence of tubercles in advance of the frontal lobes and the slightly larger eyes which occur nearer the middle of the sides of the head. Santschi also notes that the mandibles are quadridentate but as one of the cotypes of *succinea* which I examined showed the apex of the large innermost tooth divided, I am doubtful as to the significance of this difference. The color of the female of *nicai* appears to be much as in the typical *succinea* except that the infuscation of the vertex seems somewhat more extensive.

A translation of Santschi's description of the male of *nicai* is given below, since it will serve to amplify Emery's very brief notes on the male of *succinea* and may also be useful in contrasting the two:

"Male: Length 2.5 mm.

"Blackish brown, the appendages dull yellow, a little brownish on the femora. Smooth and glistening. The head trapezoidal, a little longer than broad, the same with the eyes which occupy the anterior two-fifths of the sides. Clypeus feebly convex, without carinae. The scape as long as the second joint of the funiculus, the following joints, shorter at first, are progressively elongated until they become much longer at the tip, the first joint of the funiculus globular and longer than thick. Thorax as broad as the head, the two epinotal faces more distinct than in the female, the declivious one-third shorter than the other. Peduncle as in the female but the nodes a little lower. Gaster not longer than the thorax."

Localities: Brazil, Sao Paulo. (Type loc.) (von Ihering.)
Argentina, near Rosario de Santa Fe. (Bruch?)

SUBGENUS OEDALEOCERUS, SUBGEN. NOV.

Worker: Small, monomorphic with rudimentary eyes of 5–10 facets, small joints of the antennae as broad as long or broader than long, dorsum of the thorax not impressed at the mesoepinotal suture (at least in the typical *angulata*), epinotum angular.

Female: Scape of the antenna greatly thickened and abruptly bent at the basal third, eyes large and strongly convex, postpetiole approximately two and one-half times as wide as the node of the petiole.

Type of the subgenus: *S. angulata* Emery.

Oedaleocerus contains only one species, *S.* (*O.*) *angulata*.

S. (OEDALEOCERUS) ANGULATA EMERY.

Introduction.

The case of *angulata* presents a number of difficulties not encountered elsewhere in the genus. It is unique in possessing a female whose characters are so aberrant that subgeneric rank is a necessity, yet its worker shows none of these peculiarities. On the contrary, practically every feature of that caste appears to place it in the subgenus *Diplorhoptrum*. It would even be possible to identify it with the *molesta* group within that subgenus, since the only outstanding difference shown by the worker of *angulata* is a somewhat shorter and more cordate head than is usual with those species. This distinction is at most a specific difference and totally unsuitable for subgeneric evaluation in comparison with a subgenus which contains such extremes as *S. leptanilloides* and *S. wasmanni*. This lack of similarity between the worker and female of *angulata* is sufficiently unusual to suggest that the female may actually be a workerless parasite. This interpretation would explain the differences between the two castes and at the same time permit the inclusion of the worker in the subgenus *Diplorhoptrum*, where it obviously belongs. Unfortunately our present knowledge of the sexual forms of *angulata* is so meager that any such consideration must be largely speculative. Aside from the type series, workers and females of *angulata* have been taken together on only one occasion. The females in this instance differed very slightly, if at all, from those of the type series. This fact in itself may be taken as very strong evidence that the theory of parasitism is untenable. On the other hand, despite the fact that females showing the same aberrant characters have been taken at widely different times and places with workers of *angulata*, the idea that they are parasitic may still be defended. It is a well known fact that the sexual phases of the workerless parasites are produced in very large numbers. This results in conspicuous swarms at the time of mating and such a factor might have operated in the case of the female of *angulata* to call attention to a species which, because of its small size, would otherwise pass unnoticed. Furthermore, when Forel, in 1913, received workers and females of *angulata*, he was able to detect a number of structural differences in the worker upon which he gave it subspecific rank (the subspecies *nigelloides*). In the female no comparable differences were found. Indeed, the only difference between the female of the typical *angulata* and that of the subspecies *nigelloides* appears to be the completely black thorax and darker stigma of the latter. It is usually true that structural differences in the head of the worker, as was the case with the subspecies *nigelloides* are also shown, at least to some extent, in the female. Their absence in the female of that form constitutes an additional puzzle to the enigma of *angulata*. The close similarity of the females of the two forms, instead of overthrowing the idea of parasitism, may actually strengthen it.

There is an additional difficulty in connection with *angulata* which arises largely from the similarity of the worker to some of the species in the subgenus *Diplorhoptrum*. Because of this similarity *angulata* has become a dumping ground for all manner of forms whose relationship to it is very questionable. I refer in particular to the subspecies *huasanensis*, *mendozensis* and *carettei*, all described by Forel. Only the workers of these forms are known. If I am correct in assuming that Dr. Bruch has presented accurate figures in his interesting publication of 1916, none of these can be regarded as rightfully belonging to *angulata*. The first two show a strong impression of the dorsum of the thorax at the mesoepinotal suture, a character which clearly separates them from *angulata*, in which the dorsum of the thorax is unimpressed, and relates them to the *basalis* group in the subgenus *Diplorhoptrum*. This is also true of the configuration of the petiolar joints, in which the proportions are more like those of the *basalis* group than like *angulata*. In the case of the subspecies *carettei* the relationship appears to me to lie in an entirely different direction. Although the eyes are composed of only ten or twelve facets, the shape of the thorax and petiolar joints as shown in Bruch's figure, strongly suggests a relationship to *nigella* in the subgenus *Euophthalma*. I have already shown that Santschi's subspecies *oculata* must be regarded as a separate species in that subgenus. The suggestions made above demand the examination of type material before satisfactory changes can be made. Since I have seen only cotypes of the typical *angulata*, the forms just discussed must be left provisionally in the subgenus *Oedaleocdrus*. They may be listed as follows:

S. (*O.*) *angulata* Emery
    subsp. *carettei* Forel
    subsp. *dolichops* Emery
    subsp. *huasanensis* Forel
    subsp. *mendozensis* Forel
    subsp. *nigelloides* Forel

S. (Oedaleocerus) angulata Emery.

S. angulata, Emery in Ihering, Berlin Ent. Zeitschr., Vol. 39, p. 393, nota (1894) ☿. Emery, Bull. Soc. Ent. Ital., Vol. 28, p. 88 (1896). ♀.

Worker: Length 1.7–2.0 mm. (Plate VIII, figs. 4, 7.)

Head slightly longer than broad, the sides moderately convex, somewhat more narrowed in the anterior half than in the posterior half, the occiput broadly and feebly concave with the angles well rounded. Clypeus strongly projecting, the carinae prominent and divergent, each terminated by a large, stout and rather blunt tooth, lateral denticles reduced to sinuousities in the edge of the clypeus. Mandibles smooth, armed with four sharp teeth, the outermost tooth at least twice as long as the adjacent one, the innermost tooth reduced to a small denticle but still sharp. The antennal scape in repose reaches the posterior fifth of the head, first funicular joint as long as the following three together, second joint approximately as broad as long, joints 3–7 all broader than long; club moderately stout, the last joint considerably wider than the penultimate and almost four times as long as the latter. Eyes of five or six poorly defined facets which are somewhat separated and borne on an oval, pigmented area, separated from the insertion of the mandible by a distance only a little less than twice their greatest diameter.

Humeral angles of the prothorax rather well marked, the promesonotum one and two-fifths times as wide as the epinotum, the mesoepinotal suture broad and prominent, the sides only moderately constricted at the suture except at the dorsum where the constriction is pronounced. Seen in profile the pronotum descends to the neck in a convex slope, mesonotum flat, mesoepinotal suture wide and deep but not impressed. Basal face of the epinotum virtually flat and feebly sloping to the rear, much longer than the steep declivious face which it meets at a well marked angle. Node of the petiole in profile bluntly triangular, its height somewhat greater than the length of its base, the summit feebly convex and sloping to the rear, not sharply separated from the almost perpendicular posterior face, anterior face longer than the posterior, steeply sloping forward and sharply separated from the peduncle which bears a prominent ventral tooth. Postpetiole approximately four-fifths as high as the node of the petiole, thicker and with the summit more obtusely rounded, posterior face feebly convex and sloping to the rear, not sharply separated from the rounded summit, anterior face somewhat shorter and perpendicular. Seen from above the node of the petiole is subtriangular in outline, slightly broader than long and only a little narrower than the transversely oval postpetiole. Anterior edge of the first gastric segment narrowly truncate. Very smooth and shining with sparse small punctures which bear rather thick, curved suberect hairs. A few feeble striae on the meso- and metapleurae. Color yellowish brown, the abdomen and occiput somewhat darker.

Female: Length 4.3 mm. (Plate VIII, figs. 5, 6.)

Head (eyes excluded) one-seventh broader than long, the sides much narrower in front of the eyes than behind them, slightly tapering from the eyes to the insertion of the mandibles, the sides behind the eyes strongly convex forming well marked occipital angles, occiput flat. Clypeus strongly projecting, the two carinae moderately divergent, each ending in a short, coarse, blunt tooth, the anterior edge of the clypeus between these teeth with a feeble concave impression, the median lobe of the clypeus concave throughout, lateral denticles very obtuse and not particularly prominent. Mandibles quadridentate, the teeth with the same relative proportions as in the worker. Eyes strongly convex, suboval in outline, separated from the insertion of the mandible by a distance equal to one-half their greatest diameter. Ocelli large, the two lateral ones separated from each other by a distance one and one-half times as great as that which separates each from the median ocellus. The antennal scapes strongly swollen and sharply bent at their basal third where they are thickest, slightly tapering toward the apex and scarcely tapering toward the base. First funicular joint not quite as long as the following three together, second joint approximately as broad as long, joints three and four slightly broader than long, the remaining small joints much broader than long; club slender, the terminal joint two and one-half times as long as the penultimate.

Thorax elliptical, rather bulky, as wide as the head and a little less than twice as long as wide. Basal face of the epinotum steeply sloping and much longer than the declivious which is feebly convex. Both faces feebly bordered. Node of the petiole in profile triangular, its height greater than the length of its base, the summit rather narrow, the anterior face longer than the posterior, both steeply declivious. The base of the node bears a sinuate lateral expansion which extends forward to form the dorsum of the peduncle, the latter short and stout with a rather poorly developed ventral lamella which ends anteriorly in a tooth-like angle. Postpetiole in profile much depressed, the summit broadly and feebly convex, no definite posterior face, the anterior face straight and somewhat tilted forward (i. e. the dorsum in advance of the ventral portion). Seen from above both nodes are strongly transverse, the postpetiole elliptical, almost twice as broad as long and one and three-fourths times as wide as the node of the petiole. Truncation at the anterior edge of the first gastric segment very little, if any, wider than the postpetiole, with a deeply concave impression at the base of the latter.

Smooth and moderately shining with coarse and fairly numerous punctures on the head, thorax and first abdominal segment, punctures on the remaining abdominal segments much smaller. Mandibles and the genae between the insertion of the mandible and the eye with a few coarse, irregular striae. Metapleurae and most of the sides of the epinotum finely striate, basal face of the epinotum transversely striate, ventral portions of the petiole and postpetiole granulate. Blackish brown, the mandibles, funicular joints, except the club, articulations of the legs, tibiae and tarsi reddish; occiput, scape, club and part of the thorax, petiolar joints and abdomen brown.

Redescribed from two cotypes in the collection of Dr. W. M. Wheeler. The color characteristics given in the description of the female are those of Emery. The specimen from which the present description was drawn is not fully colored.

Localities: Brazil, Rio Grande do Sul. (Type loc.) (von Ihering.)
Uruguay, Montevideo. (von Steiger.)
Argentina, Buenos Aires. (von Steiger.)

S. (Oedaleocerus) angulata subsp. **carettei** Forel.

S. angulata subsp. *carettei*, Forel, Bull. Soc. Vaud. Sci. Nat., Vol. 49, p. 223 (1913). ☿. Bruch, Rev. Mus. La Plata, Vol. 23, p. 319, Text fig. 12, b, e, pl. 9, fig. 11 (1916). ☿.

Forel's description follows:

"Worker: Length 1.7–1.9 mm.

"A little longer than Emery's type from which it may be distinguished by the large eyes composed of ten to twelve facets, further by the second node which is a great deal narrower. The head is also a little shorter."

Localities: Argentina, Mendoza. (Type loc.) (Carette.) (Bruch.)
Delta Parana. (Bruch.)
Arroyo Chana. (Bruch.)
Alto Pencoso. (Bruch.)

Dr. Bruch notes that the worker of this form reaches a length of 2.5 mm.

S. (Oedaleocerus) angulata subsp. **dolichops** Emery.

S. angulata subsp. *dolichops*, Emery, Bull. Soc. Ent. Ital., Vol. 37, p. 123 (1905). ☿.

Emery's description follows:

"Differs from the type of the species through the very elongate eyes, at least half again as long as broad in the worker."

Localities: Argentina, Puerto Camerones. (Type loc.) (Coll.?)
La Plata. (Spegazzini.)
Puerto Madryn. (Biraben.)
Buenos Aires. (von Steiger.)
Santa Fe. (von Steiger.)

S. (Oedaleocerus) angulata subsp. **huasanensis** Forel.

S. angulata subsp. *huasanensis*, Forel, Mem. Soc. Ent. Belg., Vol. 20, p. 8, (1912). ☿. Forel, Bull. Soc. Vaud. Sci. Nat., Vol. 50, p. 287 (1914). ☿. Bruch, Rev. Mus. La Plata, Vol. 23, p. 318, Text fig. 12, a, d, pl. 19, fig. 10 (1916). ☿.

Forel 1912:

"Worker: Length 1.7–1.8 mm.

"Clypeal teeth shorter than in the type of *angulata*. The eyes are much larger than in the type, elongated as in the variety *dolichops* Em., but of about eight or ten facets, often flat and partly atrophied. The scape is long, failing to reach the occipital border by about two times its own thickness. The postpetiole is broader, twice as broad as the node of the petiole. Otherwise identical with the type of the species. Of a brownish red with the head reddish brown and the abdomen brown. Appendages and mandibles yellowish.

Localities: Argentina, Huasan. (Type loc.) (Bruch.)
San Luiz. (Bruch.)

S. (Oedaleocerus) angulata subsp. **mendozensis** Forel.

S. angulata subsp. *mendozensis*, Forel, Bull. Soc. Vaud. Sci. Nat., Vol. 49, p. 223 (1913). ☿ (*carettei* var.) Forel, Ibid., Vol. 50, p. 277 (1914). ☿. Bruch, Rev. Mus. La Plata, Vol. 23, p. 320, text fig. 12, c, f, pl. 9, fig. 12 (1916). ☿.

Forel 1913:

"Worker: Length 1.9 mm.

"Distinguished from the variety *carettei* by its more slender shape, narrower, by its much clearer color, a dull reddish yellow with a brown occiput, and particularly by the very narrow head, one and one-third times as long as broad, with the sides scarcely convex. The eyes have only eight or nine facets but they are, nevertheless, much larger than the type of angulata."

Forel 1914:

"I am obliged to make this form a special subspecies and not a simple variety of *carettei*, for the node of the petiole is much thicker and differently shaped than in the subspecies *carettei*. It is almost cubical, as thick above as below and almost flat above, whereas that in *carettei* is conical with a feebly obtuse summit."

Localities: Argentina, Rio Negro, Colonias Frias. (Type loc.)
(Lehmann-Nitsche.)
Alto Pencoso. (Bruch.)

S. (Oedaleocerus) angulata subsp. **nigelloides** Forel.

S. angulata, subsp. *nigelloides*, Forel, Bull. Soc. Vaud. Sci. Nat., Vol. 49, p. 223 (1913). ☿, ♀, ♂.

Forel's description follows:

"Worker: Length 1.8–1.9 mm.

"Form of the thorax as in the type but the color almost black as in *nigella* Em. The teeth and the clypeal carinae are very strong and thicker than in the typical *angulata*, the carinae are strongly elevated. The head is slightly broader with the sides a little more convex than in the type of the species. The head is a little longer than broad but a little shorter than the typical *angulata*. The scapes are distinctly longer, they fail to reach the posterior border by only twice their thickness, whereas in the typical *angulata* they scarcely reach the posterior third of the head (see description of the worker of *angulata* W. S. C.). The mesoepinotal suture is, furthermore, slightly stronger than the typical *angulata*. In other regards the thorax is identical, also the petiolar nodes, the sculpture and the pilosity. The color is almost black with the mandibles, the base of the funiculus, the sides of the mesonotum, the epinotum and the petiolar nodes reddish. The antennal club and the tarsi are brown.

"Female: Length 4.3 mm.

"As in the type of the species with the same punctuation, spaced more closely than in the worker, especially on the head. Entirely black with the tarsi and the antennae brown and the mandibles and the base of the funiculus a yellowish red. The stigma is darker than in the type. The nodes are very large as in the type.

"Male: Length 3.2–3.5 mm.

"Thorax a little broader than the head, color as in the female but with the mandibles brown. Head rather dull and strongly reticulate. Second node twice as broad as long. Otherwise as in the type of the species."

Localities: Argentina, Buenos Aires. (Type loc.) (Rovereto.)
Monte Hermoso. (Carette.)

Forel has certainly underestimated the length of the antennal scape in the typical *angulata*. His last statement in the description of the male is also rather confusing, since the male of the typical *angulata* has never been described.

PLATE I

FIGURE 1. Head of the major of *S. geminata* Fabr.

FIGURE 2. Head of the major of *S. geminata* subsp. *eduardi* Forel.

FIGURE 3. Head of the major of *S. geminata* subsp. *medusa* Mann.

FIGURE 4. Head of the male of *S. geminata* Fabr.

FIGURE 5. Head of the female of *S. geminata* Fabr.

FIGURE 6. Head of the minor of *S. geminata* Fabr.

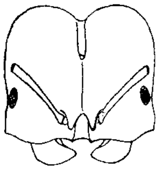
FIGURE 7, 8. Thorax of *S. geminata* subsp. *rufa* Jerdon showing variation in the form of the mesosternal spine.

FIGURE 9. Epinotum of minor of *S. geminata* subsp. *eduardi* Forel.

FIGURE 10. Epinotum of minor of *S. geminata* Fabr.

FIGURE 11. Thorax of the major of *S. geminata* Fabr.

FIGURE 12. Thorax of the minor of *S. geminata* Fabr.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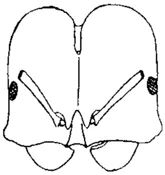
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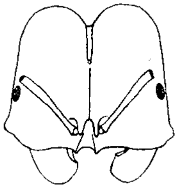
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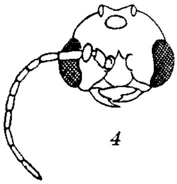
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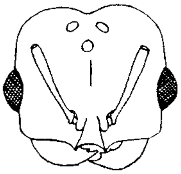
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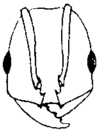
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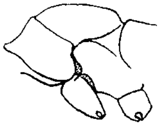
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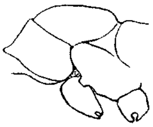
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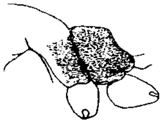
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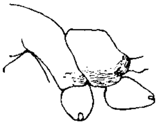
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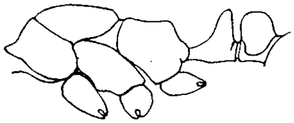
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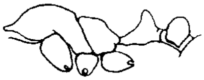
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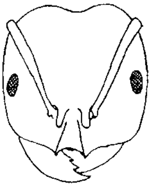
PLATE II

FIGURE 1. Head of the major of *S. saevissima* F. Smith.
FIGURE 2. Thorax of the major of *S. saevissima* F. Smith.
FIGURE 3. Head of the female of *S. saevissima* F. Smith.
FIGURE 4. Thorax of the major of *S. saevissima* subsp. *interrupta* Santschi.
FIGURE 5. Head of the major of *S. saevissima* subsp. *interrupta* Santschi.
FIGURE 6. Head of the major of *S. saevissima* subsp. *electra* Forel.
FIGURE 7. Thorax of the major of *S. saevissima* subsp. *electra* Forel.
FIGURE 8. Head of the female of *S. saevissima* subsp. *electra* Forel.
FIGURE 9. Thorax of the minor of *S. saevissima* F. Smith.
FIGURE 10. Head of the male of *S. saeevissima* var. *moelleri* Forel.
FIGURE 11. Head of the minor of *S. saevissima* F. Smith.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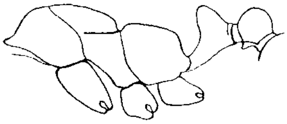
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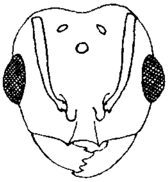
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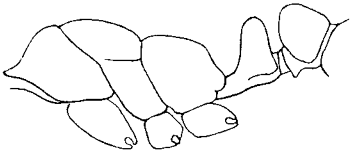
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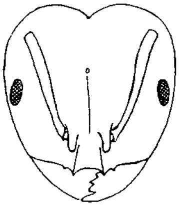
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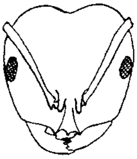
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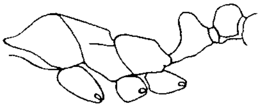
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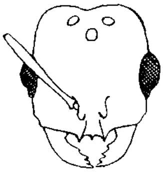
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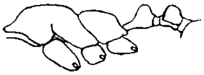
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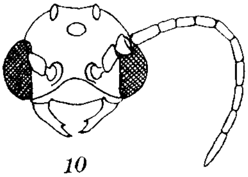
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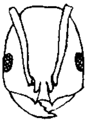
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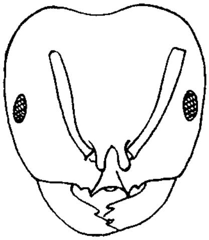
PLATE III

FIGURE 1. Head of the major of *S. xyloni* MacCook.
FIGURE 2. Head of the major of *S. xyloni* subsp. *aurea* Wheeler.
FIGURE 3. Head of the major of *S. xyloni* subsp. *amblychila* Wheeler.
FIGURE 4. Thorax of the major of *S. xyloni* MacCook.
FIGURE 5. Head of the male of *S. xyloni* MacCook.
FIGURE 6. Head of the female of *S. xyloni* MacCook.
FIGURE 7. Thorax of the minor of *S. xyloni* MacCook.
FIGURE 8. Head of the minor of *S. xyloni* MacCook.



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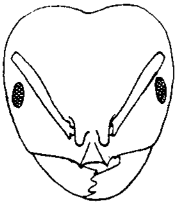

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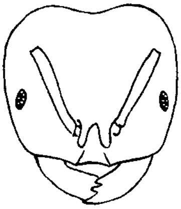

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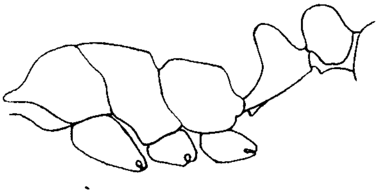

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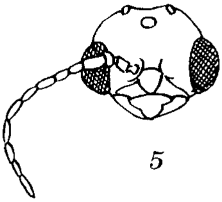

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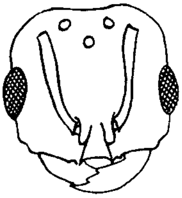

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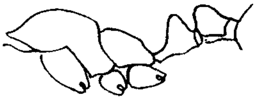

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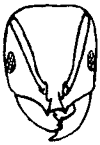

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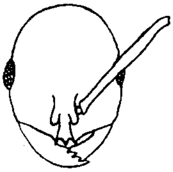
PLATE IV

FIGURE 1. Head of the media of *S. tridens* var. *substituta* Santschi.
FIGURE 2. Thorax of the media of *S. tridens* var. *substituta* Santschi.
FIGURE 3. Head of the female of *S. gayi*, var. *fazi* Santschi.
FIGURE 4. Head of the major of *S. gayi* subsp. *bruesi* subsp. nov.
FIGURE 5. Thorax of the major of *S. gayi* subsp. *bruesi* subsp. nov.
FIGURE 6. Head of the major of *S. gayi* Spinola.
FIGURE 7. Thorax of the major of *S. bondari* Santschi.
FIGURE 8. Petiole and postpetiole of the worker of *bondari* seen from above.
FIGURE 9. Head of the major of *S. bondari* Santschi.



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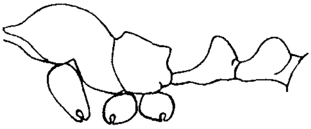

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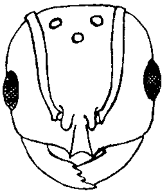

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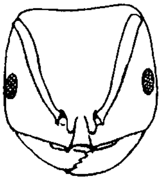

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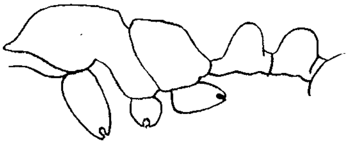

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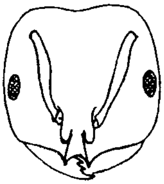

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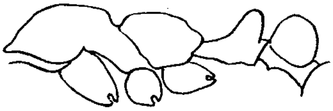

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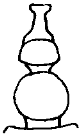

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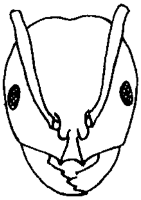

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PLATE V

FIGURE 1. Head of the worker of *S.* (*E.*) *metanotalis* var. *argus* Santschi.

FIGURE 2. Thorax of the worker of *S.* (*E.*) *metanotalis* Emery.

FIGURE 3. Head of the worker of *S.* (*E*). *metanotalis* Emery.

FIGURE 4. Thorax of the worker of *S.* (*E.*) *metanotalis* subsp. *shiptoni* var. *steigeri* Santschi.

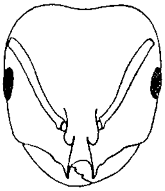
FIGURE 5. Head of the worker of *S.* (*E.*) *metanotalis* subsp. *emiliae* Santschi.

FIGURE 6. Thorax of the worker of *S.* (*E.*) *metanotalis* subsp. *emiliae* Santschi.

FIGURE 7. Head of the worker of *S.* (*E.*) *metanotalis* subsp. *shiptoni* var. *steigeri* Santschi.

FIGURE 8. Thorax of the worker of *S.* (*E.*) *silvestrii* Emery.

FIGURE 9. Head of the worker of *S.* (*E.*) *silvestrii* Emery.



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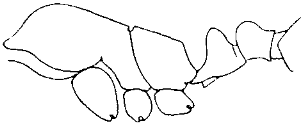

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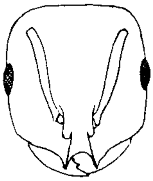

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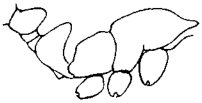

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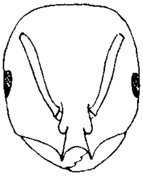

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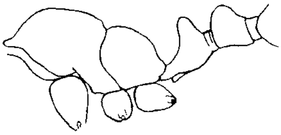

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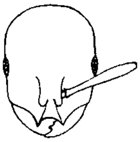

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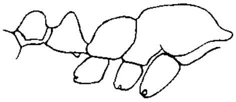

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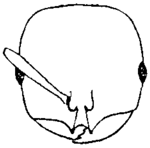

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PLATE VI

FIGURE 1. Head of the worker of *S.* (*E.*) *globularia* F. Smith.

FIGURE 2. Thorax of the worker of *S.* (*E.*) *globularia* F. Smith.

FIGURE 3. Head of the worker of *S.* (*E.*) *globularia* subsp. *littoralis* subsp. nov.

FIGURE 4. Worker of *S.* (*E.*) *macrops* Santschi.

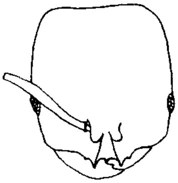
FIGURE 5. Petiole and postpetiole of *S.* (*E.*) *globularia* subsp. *pacifica* Wheeler, seen from above.

FIGURE 6. Thorax of the major of *S.* (*E.*) *nigella* Emery.

FIGURE 7. Head of the major of *S.* (*E.*) *nigella* Emery.

FIGURE 8. Thorax of the minor of *S.* (*E.*) *nigella*, subsp. *gensterblumi* Forel.

FIGURE 9. Head of the minor of *S.* (*E.*) *nigella*, subsp. *gensterblumi* Forel.



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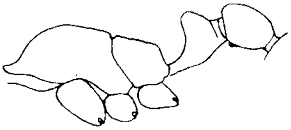

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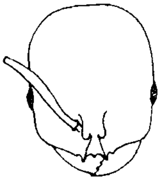

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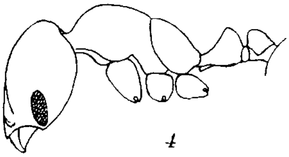

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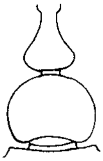

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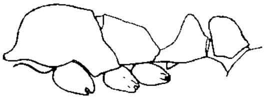

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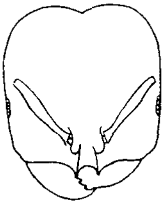

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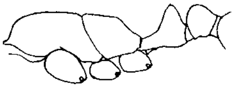

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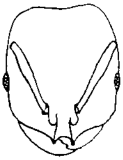

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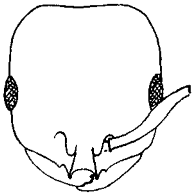
PLATE VII

FIGURE 1. Head of the worker of *S.* (*E.*) *andina* Santschi.
FIGURE 2. Thorax of the worker of *S.* (*E.*) *andina* Santschi.
FIGURE 3. Head of the worker of *S.* (*E.*) *oculata* Santschi.
FIGURE 4. Thorax of the worker of *S.* (*E.*) *oculata* Santschi.
FIGURE 5. Head of the minor of *S.* (*E.*) *huachucana* Wheeler.
FIGURE 6. Thorax of the major of *S.* (*E.*) *huachucana* Wheeler.
FIGURE 7. Head of the major of *S.* (*E.*) *huachucana* Wheeler.



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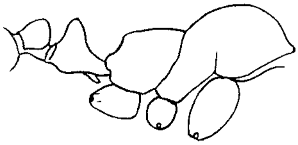

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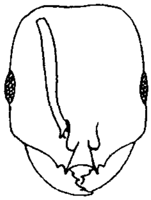

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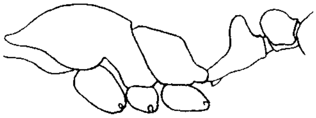

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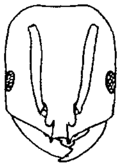

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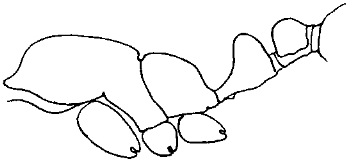

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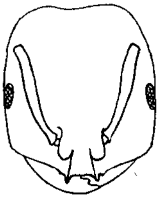

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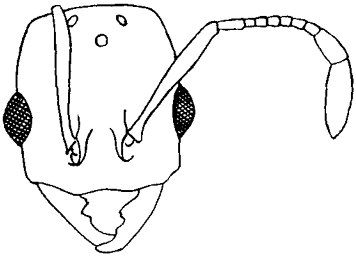
PLATE VIII

Figure 1. Head of the female of *S.* (*D.*) *succinea* Emery.
Figure 2. Head of the worker of *S.* (*D.*) *succinea* Emery.
Figure 3. Thorax of the worker of *S.* (*D.*) *succinea* Emery.
Figure 4. Head of the worker of *S.* (*O.*) *angulata* Emery.
Figure 5. Antennal scape of the female of *S.* (*O.*) *angulata* Emery.
Figure 6. Head of the female of *S.* (*D.*) *angulata* Emery.
Figure 7. Thorax of the worker of *S.* (*D.*) *angulata* Emery.



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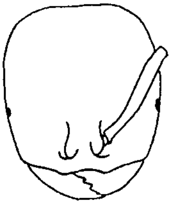

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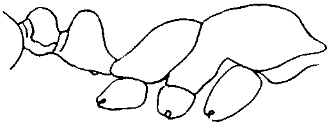

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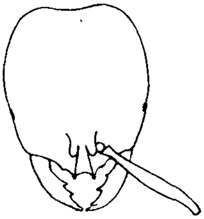

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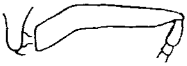

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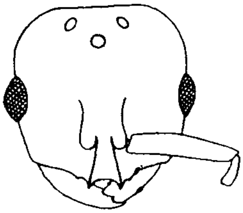

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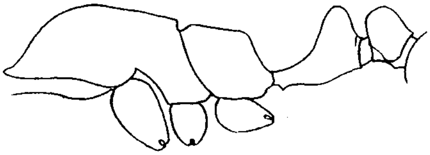

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EQUATIONS FOR VAPOR PRESSURES AND LATENT HEATS, INCLUDING APPROXIMATE EQUATIONS FOR SOLID COMPOUNDS CONTAINING A GASEOUS COMPONENT.

By Louis J. Gillespie.

EQUATIONS FOR VAPOR PRESSURES AND LATENT HEATS, INCLUDING APPROXIMATE EQUATIONS FOR SOLID COMPOUNDS CONTAINING A GASEOUS COMPONENT.*

By Louis J. Gillespie.

After early papers by Kirchoff[1] and Rankine[2] the subject of vapor pressure equations was thoroughly treated by Gibbs,[3] subject to the provisions that the vapor is an ideal gas of invariable specific heat, and the density and the specific heat at constant pressure of the liquid or solids are constant. Of the many vapor pressure equations that have been published we will mention here only that of Nernst,[4] which was applied to solids, and which assumes the vapor to be an ideal gas, and the specific heats to be functions of the temperature (usually in series expansion). Van der Waals and also van Laar have discussed the vapor pressure equation[5] in the light of the van der Waals equation of state.

An equation derived by Kirchoff and by Gibbs and discussed by Gibbs for either solids or liquids is

$$\log p = A - B \log T - C/T \tag{1}$$

where A, B, and C are constants.

Keyes and Hara[6] found that log p is linear in $1/T$ for the silver-silver oxide equilibrium up to 388 atmospheres and pointed out that the equation deduced by Pollitzer from the previous measurements of Lewis, with the aid of the Nernst heat theorem in connection with series expansions of specific heats, failed to represent the data. The failure of the equation to serve as an interpolation formula for the range covered by the measurements of Lewis is evident: it may be that it is not due entirely to the series expansion of specific heats at very low temperatures, now known to be erroneous, but possibly in part due to failure to apply corrections to the ideal gas law. It has often been found that experimental data on decomposition pressures may be satisfactorily correlated when B in equation (1) is put equal to zero, which corresponds in the derivation of (1) to the assumption that the increase of heat capacity in the evaporation is zero, and it may be that there is some advantage in either keeping both the term $B \log T$ and the corrections to the ideal gas law or rejecting both together.

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* Contribution from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, No. 248.

We shall here present the general vapor pressure equation, containing no assumptions beyond the two laws of thermodynamics, and then develop its application by means of simplifying assumptions, with especial reference to the decomposition pressures of solids.

2. DERIVATION OF THE GENERAL VAPOR PRESSURE EQUATION.

Let us consider a system in two compartments, separated by a membrane which we may at will imagine as solid and impermeable, or freely permeable to gas. Let a solid and its compound with a gas, for example, $BaCl_2$ and $BaCl_2 \cdot 8NH_3$, be present in one compartment, filling it to the exclusion of any gas, and let the pure gas which may be evolved by the compound, for example, ammonia, be present alone in the other compartment. When the membrane is impermeable we may change the condition of the gas by changing independently the temperature T and the pressure p. The change of chemical potential of the gas μ will be given by

$$d\mu = -s\,dT + v\,dp \tag{2}$$

where s is the entropy and v the volume of a mole of the gas, by Gibbs' equation 97. It will be convenient to write a similar equation for the chemical potential of the gaseous component in the compartment containing nothing but the solids:

$$d\mu = -\bar{s}\,dT + \bar{v}\,dp \tag{3}$$

where $\bar{s}$ and $\bar{v}$ will refer to the gaseous component in the solids and may be called the partial molal entropy and the partial molal volume of the gaseous component in the solids. The next four paragraphs deal with the exactness of this.

The solids, considered collectively in this way as if they were a single phase, may be called the condensate. As already pointed out,[7] "the condensate has then a potential, a partial molal volume, etc., with respect to a given component, and the Gibbs theory applies to these quantities just as to the usual quantities distributed among the individual phases, the arguments not being changed." This was justified by the consideration that there is no logical necessity for always dividing up the extensive thermodynamic properties like the energy, the thermodynamic potential, the volume, etc., among the individual phases, but they may for special purposes be divided up, one portion to the gas, and the rest to the condensed phases considered collectively. The reasoning, which was merely outlined, involves some difficulties, which will be avoided if we take advantage of Gibbs' equation 121.

The chemical equation for the formation of the compound may be written:

$$1/8\ BaCl_2 + NH_3 = 1/8\ BaCl_2 \cdot 8NH_3 \quad \text{(type reaction)} \tag{4}$$

$$\nu S + G = \nu SG \quad \text{(general reaction)} \tag{5}$$

where S represents the chemical symbol for the solid, G that for the gas, and SG that for the compound of solid with gas. Writing μ_S, μ, and μ_{SG} for the potentials of solid, gas, and compound, respectively, Gibbs' equation 121 becomes

$$\nu\mu_S + \mu = \nu\mu_{SG} \tag{6}$$

Now each potential in equation (6) may be regarded as a function of the two variables T and p. We have by differentiation

$$(d\mu/dp)_T = \nu(d\mu_{SG}/dp)_T - (d\mu_S/dp)_T \tag{7}$$

$$(d\mu/dp)_T = \nu(v_{SG} - v_S) \tag{8}$$

where v_S and v_{SG} are the volume of a mole of solid and compound respectively. The last term in equation (8) is the increase in volume of the solids attending the reaction represented by equation (5). It may be regarded as the partial molal volume of the gaseous component in the condensate, since if V be the total volume of the condensate, n_1 the number of moles of gaseous component and n_2 the number of moles of solid S it contains, the partial volume of the gaseous component would be defined as $(\partial V/\partial n_1)$, where the variation is at constant T, p, and n_2, and by reason of equation (5) we should have

$$(\partial V/\partial n_1) = \nu(v_{SG} - v_S) \tag{9}$$

This is therefore, by equation (8), the value of the quantity written in equation (3) as $\bar{v}$ for brevity, as the potential μ is the same regardless of the choice of the other components, as shown by Gibbs.

Similarly it may easily be seen that

$$(d\mu/dT)_p = -\nu(s_{SG} - s_S) = -\bar{s} \tag{10}$$

It will be observed that the concept of the potential of ammonia in a *compound* such as $BaCl_2 \cdot 8NH_3$ is inconvenient since the partial volume of ammonia in it is hardly a fit subject of discussion, as the composition of a chemical compound is not susceptible of the continuous variation indicated by $\partial E/\partial n_1$ or $\partial V/\partial n_1$. The partial volume of ammonia in the condensate is however physically interpretable through equation (9). The potential of ammonia in the condensate is interpretable through equation (6). The difficulty due to the law of definite proportions is similarly removed in the case of other partial thermodynamic quantities, such as heat capacity. Such considerations apply to the solids of any univariant system, and are useful when the system considered has a variance greater than one, provided the extra variance is due to the presence of other gases which are inert, not reacting with the solids.

We may now proceed to derive a vapor pressure equation from equations (2) and (3) by integration and elimination of the potentials. Since $d\mu$ is a perfect differential, the increase of potential $\Delta\mu$ of the gas in passing from a standard state defined by the values T_0 and p_0 to the variable state given by T and p may be found by integrating equation (2) along any path we please, and it is convenient to integrate first at the constant pressure p_0 to the values T, p_0, and then at constant temperature T to the values T, p. The state given by T_0 and p_0 is arbitrary, and will usually be taken at the lowest experimental temperature and vapor pressure under consideration. The variation of entropy at constant pressure is given by

$$(ds/dT)_p = C_p/T \qquad (11)$$

Hence, if s_0 is the molal entropy of the gas at T_0, p_0, the entropy at T, p_0 is

$$s = \int_{T_0}^{T} C_p d \ln T + s_0 \qquad (12)$$

where C_p is the constant pressure specific heat at p_0 and T. The increase of potential $\Delta\mu$ for the gas is therefore

$$\Delta\mu = -\int_{T_0}^{T}\left[\int_{T_0}^{T} C_p d \ln T + s_0\right] dT + \int_{p_0}^{p} (v - RT/p)\, dp + RT \ln (p/p_0) \qquad (13)$$

Here we have added and subtracted the integral of $RT\, dp/p$ from the right-hand side in order to display the term $RT \ln p$, which occurs in any vapor pressure equation as soon as a convenient equation of state is chosen.

For the solids,

$$\bar{s} = \nu(s_{SG} - s_S) = \int_{T_0}^{T} \nu(c_{SG} - c_S)\, d\ln T + \bar{s}_0 \qquad (14)$$

by equation (10) and the general equation (11) applied to each solid phase, where $\bar{s}_0$ is the difference $\nu(s_{SG} - s_S)$ at T_0, p_0. Let $\nu(c_{SG} - c_S)$ be represented by $\bar{c}$, which may be called the molal specific heat at constant pressure of the gaseous component in the condensate. Then the increase $\Delta\mu$ for the solids is

$$\Delta\mu = -\int_{T_0}^{T}\left[\int_{T_0}^{T} \bar{c}\, d\ln T + \bar{s}_0\right] dT + \int_{p_0}^{p} \bar{v}\, dp \qquad (15)$$

Now along the vapor pressure curve μ is the same for both gas and solid and hence also $\Delta\mu$; furthermore, T and p are the same for both, including the particular values T_0 and p_0. Elimination of $\Delta\mu$ from equations (13) and (15) gives the vapor pressure equation:

$$RT\ln p = RT\ln p_0 + \int_{T_0}^{T}\int_{T_0}^{T} (C_p - \bar{c})\, d\ln T\, dT$$
$$+ (s_0 - \bar{s}_0)(T - T_0) - \int_{p_0}^{p} (v - RT/p - \bar{v})\, dp \qquad (16)$$

which may be transformed into

$$\ln p = \ln p_0 + \frac{1}{RT}\int_{T_0}^{T}\int_{T_0}^{T} (C_p - \bar{c})\, d\ln T\, dT$$
$$+ \frac{\lambda_0}{RT_0} - \frac{\lambda_0}{RT} - \frac{1}{RT}\int_{p_0}^{p} (v - RT/p - \bar{v})\, dp \qquad (17)$$

by virtue of the thermodynamic relation:

$$\lambda_0 = T_0\,(s_0 - \bar{s}_0) \qquad (18)$$

where λ_0 is the latent heat of vaporization of 1 mole of the gaseous component at T_0, p_0. In these equations $(C_p - \bar{c})$ is for the constant pressure p_0, and $(v - RT/p - \bar{v})$ is for the temperature T, which is held constant for the last integration.

Equation (17) is the general vapor pressure equation. It is exact, depending only on the two laws of thermodynamics. It holds not only for solid compounds which evolve a gas, but also for liquids, if $\bar{c}$, $\bar{s}$, and $\bar{v}$ are understood to be the specific heat, the entropy and the volume of 1 mole of liquid.

3. Discussion of the Integrals.

The value of the single integral of equations (16) and (17) depends evidently on both T and p. But this integral proves to be of but small influence on the calculated value of ln p, compared with the influence of the term λ_0/RT, so that an approximate value of p serves to evaluate the integral closely enough to give a very good value of ln p. It is for this reason that the addition of a quantity equal to zero in obtaining equation (13) was significant.

The single integral of equation (17) may be discussed as regards the two parts $(v - RT/p)\,dp$ and $\bar{v}\,dp$. As regards the second, $\bar{v}$ is the volume of the gaseous component in the liquid or in the solids at the temperature T and the integration extends from p_0 to p, so that the condensed phase would from a physical standpoint be supersaturated with vapor all the way up to the upper limit of integration. Theoretically this is of no consequence, as a perfect differential $(d\mu)$ may be integrated along any path, irrespective of equilibrium conditions. Practically, the compressibility might be determined if necessary at higher pressures and extrapolated into the desired range with doubtless sufficient accuracy.

The integral of $(v - RT/p)\,dp$ may be calculated from the Beattie-Bridgeman equation of state in its original pressure form,[5] but this requires integration by parts, yielding functions of the volume which would necessitate solving the equation for the volume by the method of successive approximations.

Beattie[9] has given another form of the equation, the volume form, which is

$$v = RT/p + \beta/RT + \gamma p/R^2T^2 + \delta p^2/R^3T^3 \tag{19}$$

where

$$\left.\begin{aligned} \beta &= RTB_0 - A_0 - Rc/T^2 \\ \gamma &= -RTB_0b + A_0a - RB_0c/T^2 \\ \delta &= RB_0bc/T^2 \end{aligned}\right\} \tag{20}$$

which permits an easy integration of $(v - RT/p)\,dp$ in a series of powers of p:

$$\int_{p_0}^{p} (v - RT/p)\,dp = \frac{\beta}{RT}(p - p_0) + \frac{\gamma}{2R^2T^2}(p^2 - p_0^2) + \frac{\delta}{3R^3T^3}(p^3 - p_0^3) \tag{21}$$

Constants for 14 gases (including air) have been given for the pressure equation.[10] Constants for the volume equation have not yet been determined from the pressure data. Beattie has shown however[9] that constants as determined for the pressure equation may be used in the volume equation with good results for gases above their critical temperatures. The results were not so good for gases near the critical temperature and also near the saturation line, although they were considerably better than those obtained with the ideal gas law.

The double integral of equation (17) is a function of one variable T. If C_p and $\bar{c}$ are representable by a series expansion, which will usually be the case except at very low temperatures, an expression will hold such as

$$C_p - \bar{c} = \mathrm{A} + \mathrm{B}T + \mathrm{C}T^2 + \mathrm{D}T^3 \tag{22}$$

Substitution into the double integral J_2 of equation (17) and integration between the limits T_0 and T gives

$$\begin{aligned} J_2 = {} & \mathrm{A}T \ln T - \mathrm{A}T_0 \ln T_0 \\ & - \left[\mathrm{A} + \mathrm{A} \ln T_0 + \mathrm{B}T_0 + \frac{\mathrm{C}T_0^2}{2} + \frac{\mathrm{D}T_0^3}{3} \right] (T - T_0) \\ & + \frac{\mathrm{B}}{1 \cdot 2}(T^2 - T_0^2) + \frac{\mathrm{C}}{2 \cdot 3}(T^3 - T_0^3) + \frac{\mathrm{D}}{3 \cdot 4}(T^4 - T_0^4) \end{aligned} \tag{23}$$

4. APPLICATION OF THE VAPOR PRESSURE EQUATION TO EXPERIMENTAL DATA

When the specific heats, volumes, and the latent heat are sufficiently known, equation (17) furnishes of course a test for the thermodynamic consistency of the data.

When only the specific heats and the volumes are sufficiently known, λ_0, the latent heat at T_0, p_0, can be found by plotting for instance $(\ln p - J_2/RT + J_1/RT)$, where J_1 indicates the single integral and J_2 the double integral of equation (17), against $(1/T_0 - 1/T)$, λ_0 being found from the slope of the best straight line. The constant p_0 will have been determined from a preliminary smoothing of the data.

The case of liquids has not been studied to determine to what degree simplification by approximation may safely be carried. In this case much is demanded of a vapor pressure equation, on account of the high precision attainable in the experimental measurement of the vapor pressure of liquids.

In the case of solids which evolve a gas, the relations are more favorable. The precision attainable in measurements of decomposition pressures is not so high as in the case of liquids, probably on account of strains in the solids, which are not quite of negligible thermodynamic effect. The decomposition pressure of solids at any temperature is always less than the normal vapor pressure of the liquid, and often much less. Thus there is less demand on the equation of state for the gas, and equation (21) will often suffice when the constants used are those determined for the pressure form of the equation of state. Other approximations will usually be permissible, and are made below.

The quantity $\bar{v}$ is known for many solids for a particular temperature and pressure. It does not vary much in a series of similar compounds, and when it is unknown for some compound, the value for a similar compound may be used. It may be considered a constant with respect to temperature and pressure changes.

As respects the specific heats, the quantity $(C_p - \bar{c})$ will at present have to be regarded as a constant, at least in most cases, and as an unknown adjustable constant in many cases, because of the lack of data for calculating $\bar{c}$.

It appears inadvisable to retain terms expressing variations of $\bar{v}$ or $\bar{c}$ unless these are known from independent data. If they are kept and treated as adjustable constants, a better representation of data may be expected, but the resulting equation is likely to represent errors which ought to be disregarded, and in such a case not only will these constants receive fictitious values, but the values found for the latent heat will suffer a loss of accuracy.

Treating $(C_p - \bar{c})$ and $\bar{v}$ as constants, and using the volume equation for the gas, the following approximate equation is easily obtained especially for decomposition pressures of solid compounds.

$$\begin{aligned}\log p = \log p_0 &- \frac{M(C_p - \bar{c})}{R}\left(\frac{1}{M}\log\frac{T_0}{T} + 1 - \frac{T_0}{T}\right) \\ &+ \frac{M\lambda_0}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right) + \frac{M}{RT}\left(\bar{v} - \frac{\beta}{RT}\right)(p - p_0) \\ &- M\gamma(p^2 - p_0^2)/2R^3T^3 - M\delta(p^3 - p_0^3)/3R^4T^4 \qquad (24)\end{aligned}$$

where β, γ, and δ are calculated from equations (20) for the given value of the variable temperature T, and M is the modulus of logarithms, 0.43429 . . . , common logarithms being denoted by log.

If therefore we put

$$y = \frac{1}{1/T_0 - 1/T}\left[\log (p/p_0) - M\left(\bar{v} - \frac{\beta}{RT}\right)(p - p_0)/RT\right.$$

$$\left. + M\gamma\,(p^2 - p_0^2)/2R^3T^3 + M\delta(p^3 - p_0^3)/3R^4T^4 \right. \quad (25)$$

and

$$x = \frac{1}{1/T_0 - 1/T}\left(\frac{1}{M}\log\frac{T_0}{T} + 1 - \frac{T_0}{T}\right) \quad (26)$$

a plot of y against x should give a straight line whose slope dy/dx is $-M(C_p - \bar{c})/R$ and whose intercept on the axis: $x = 0$, is $M\lambda_0/R$. In this way the data can be treated when $(C_p - \bar{c})$ is unknown, and the constants be determined. The constant p_0 will have been found by a preliminary smoothing of the data, plotting $\log p$ against $1/T$. Points corresponding to values of T, p very close to T_0, p_0 may be found far off the line, and no attention need be paid to them in selecting the best line.

The constants $(C_p - \bar{c})$ and λ_0 having been determined, equation (24) may be used to calculate the decomposition pressure corresponding to the temperature T. The last three terms may be calculated by the use of an approximate value of p, as determined for instance by use of the preliminary plot or equation used for obtaining the value of p_0, and then corrected if necessary by the use of the improved value of p thus found. The convergence is rapid.

Equation (24) was applied in this way to the data of Gillespie and Lurie[11] on the decomposition pressures of the system

$$BaCl_2 + BaCl_2 \cdot 8NH_3,$$

which extend from 0° C. to about 50° and to about 9 atmospheres. No particular difficulty was encountered. In fact, the last two terms of equation (24) never exceeded 0.03 per cent of the pressure, a negligible matter in the case of an equilibrium involving solids. The quantity $\bar{c}$ was found to be about 16 calories per mole of ammonia in the solids, which may be compared with 11 cal/mole, the value found from available data[12] for $AlCl_3 + AlCl_3 \cdot 6\ NH_3$ at about 0° C. A value of $\bar{v}$ was known.

It seems probable that the terms of equation (24) involving p^2 and p^3 may often if not usually be omitted, giving, with elimination of β by equations (20) the relatively simple equation:

$$\log p = \log p_0 - \frac{M(C_p - \bar{c})}{R}\left(\frac{1}{M}\log\frac{T_0}{T} + 1 - \frac{T_0}{T}\right)$$
$$+ \frac{M\lambda_0}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right) + M\left(\frac{\bar{v} - B_0}{RT} + \frac{A_0}{R^2T^2} + \frac{c}{RT^4}\right)(p - p_0) \quad (27)$$

This equation is recommended for solids as the approximation usually next in order to equation (1) with $B = O$. Equation (1) does not appear to have been especially successful and it is but little simpler to apply it to data, when B and C are not known in advance, than equation (27) itself.

5. The Latent Heats of Evaporation.

The latent heat at T_0 in joules or calories will be obtained by multiplying λ_0, obtained as above in say liter-atmospheres (normal), by a factor Q, which may be 101.328 for (absolute) joules, or 24.2122 for 15°-gram-calories.[13]

The latent heat λ at another temperature T is equal to $T\ (s - \bar{s})$ and may be found by applying the exact thermodynamic equations (28) and (29):

$$s = s_0 + \int_{T_0}^{T} C_p d \ln T - \int_{p_0}^{p} (dv/dT)_p dp \quad (28)$$

$$\bar{s} = \bar{s}_0 + \int_{T_0}^{T} \bar{c}\ d \ln T - \int_{p_0}^{p} (d\bar{v}/dT)_p dp \quad (29)$$

which give the general relation (30):

$$\lambda = \frac{T}{T_0}\lambda_0 + T\int_{T_0}^{T} (C_p - \bar{c})\ d \ln T - T\int_{p_0}^{p} [d(v - \bar{v})/dT]_p dp \quad (30)$$

These integrations are to be performed as in the case of the potentials, that with respect to T being carried out at the pressure p_0, and that with respect to p being carried out at the temperature T.

Assuming that $(C_p - \bar{c})$ and $\bar{v}$ are constant and using the equation of state (19) we obtain the equation

$$\lambda = T\lambda_0/T_0 + \frac{T}{M}(C_p - \bar{c})\log\frac{T}{T_0} - \frac{RT}{M}\log\frac{p}{p_0}$$
$$- Q\left(\frac{A_0}{RT} + \frac{3c}{T^3}\right)(p - p_0) - Q\left(\frac{B_0 b}{2RT} - \frac{A_0 a}{R^2T^2} + \frac{2B_0 c}{RT^4}\right)$$
$$\times (p^2 - p_0^2) + \frac{5B_0 bc}{3R^2T^5}(p^3 - p_0^3) \quad (31)$$

to accompany equation (24). In the application to the data of Gillespie and Lurie the last two terms of equation (31) were negligible, and the approximate formula:

$$\lambda = T\lambda_0/T_0 + \frac{T}{M}(C_p - \bar{c}) \log \frac{T}{T_0} - \frac{RT}{M} \log \frac{p}{p_0} - Q\left(\frac{A_0}{RT} + \frac{3c}{T^3}\right)(p - p_0) \qquad (32)$$

is recommended for use in connection with equation (27). In equations (30) and (31) C_p, $\bar{c}$, and R will be in the same heat units as λ and λ_0, and Q is the conversion factor mentioned above.

6. CONTROLS ON CALCULATIONS.

The numerical work, the applicability of the simplifying assumptions used and the calculation of the latent heats may be controlled as follows.

Let the vapor pressures be calculated for the experimental temperatures by the equation derived, and designated by p_c. Let the corresponding pressures be calculated by the use of the linear relation (33):

$$\log p = A - C/T \qquad (33)$$

and designated by p_l. Let the values of the deviation: dev = $\log p_c - \log p_l$ be plotted against $1/T$. They must lie on a smooth curve, which may now be drawn. Let the slope of the smooth deviation curve at the temperature T be $d\,\text{dev}/d(1/T)$. Then

$$d \log p_c/d(1/T) = d\,\text{dev}/d(1/T) + d \log p_l/d(1/T)$$
$$= d\,\text{dev}/d(1/T) - C \qquad (34)$$

An exact form of the Clapeyron equation is

$$d \log p/d(1/T) = -M\lambda T/[p(v - \bar{v})] \qquad (35)$$

The last member of equation (34) should be equal to the right-hand side of (35). The value of this may be found from the value of λ as calculated in pv units and from a value of $(v - \bar{v})$ computed from the equation of state used. This equality may be tested by drawing a line through the point T, p of the deviation plot with the slope calculated from the heat, namely with slope equal to

$$C - M\lambda T/[p(v - \bar{v})];$$

the line so drawn must be tangent to the deviation curve.

A further control is obtained by plotting the deviation: $\log p' - \log p_l$, where p' is the observed pressure, on the same deviation plot. If these points fall on a smooth curve of their own, different from the other smooth curve, the approximations used for smoothing the pressures have been too severe, otherwise not. But if better values of $d\,dev/d(1/T)$ can be thus obtained from the observed points, the value of the latent heat in heat units may be calculated from these values together with the best possible values of $(v - \bar{v})$ from the relation:

$$\lambda = \frac{Qp(v - \bar{v})}{MT}[C - d\,\text{dev}/d(1/T)] \tag{36}$$

where C is the same as in equation (33).

7. Summary.

By methods similar to that of Gibbs, general thermodynamic equations have been derived for the vapor pressure and the latent heats. A difficulty is discussed and removed, which appears if we wish to use a partial quantity such as a partial volume for the case of chemical compounds, due to the combination of atoms in definite proportions.

Approximations which may be made when dealing with solid compounds are discussed, and approximate equations are given for the decomposition pressures of solid compounds and their latent heats of decomposition. Methods of applying the equations to experimental data are given, together with a procedure for controlling the accuracy of the calculations.

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AN EXPERIMENTAL STUDY OF THE ABSOLUTE TEMPERATURE SCALE. I. THE CONSTRUCTION OF SEVERAL TYPES OF PLATINUM RESISTANCE THERMOMETERS.

By James A. Beattie, David D. Jacobus, and John M. Gaines, Jr

AN EXPERIMENTAL STUDY OF THE ABSOLUTE TEMPERATURE SCALE. I. THE CONSTRUCTION OF SEVERAL TYPES OF PLATINUM RESISTANCE THERMOMETERS.

(Contribution from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, No. 241.)

By James A. Beattie, David D. Jacobus, and John M. Gaines, Jr.

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1. Introduction.

The ease and precision with which temperatures can be determined and reproduced by use of platinum resistance thermometers have brought this method of measuring temperatures into quite general use. The Siemens[1] resistance thermometer in which the wire was wound on a clay cylinder and enclosed in a wrought iron tube was not satisfactory for use at high temperatures due to contamination of the platinum. In the modern resistance thermometer the wire is wound on a mica frame and enclosed in a glass, quartz, or in special instances, metal sheath. This form of the instrument has given such excellent results in the hands of various investigators that by international agreement[2] the platinum resistance thermometer has been adopted as the instrument for the interpolation of temperatures on the thermodynamic scale between − 190° C. and + 660° C.

Three types of supporting frames are in general use: the Callendar[3] mica-cross, the Bureau of Standards calorimetric,[4] and the Bureau of Standards strain-free.[5] The calorimetric, or flat, thermometer was designed for calorimetric measurements and has a very small time lag in responding to thermal changes; the strainfree type is so constructed that the platinum wire cannot be subjected to mechanical strain as the wire expands and contracts on the mica form due to the heating and cooling of the thermometer. A detailed description of a method for constructing the calorimetric and strainfree types has been given by Sligh.[6]

During the course of an investigation of the absolute temperature scale methods of construction that are especially useful when several thermometers of one kind are to be made have been devised for each of the three types of windings. Some minor changes in design have been introduced. These are described in the present paper.

All of the resistance thermometers for which the methods of construction are given had a resistance at 0° C. of about 25.5 ohms and were wound with 0.1 mm. platinum wire, it requiring about 175 cm. of wire for each thermometer.

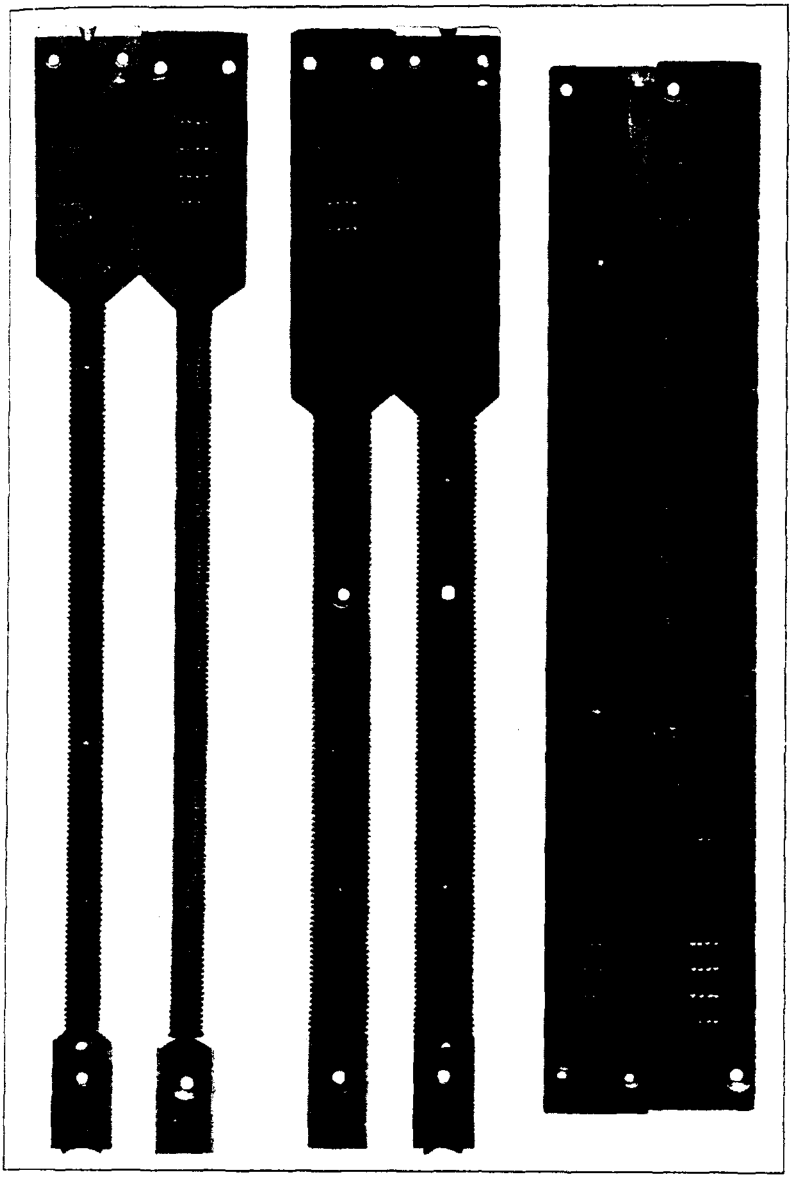
2. Methods of Compensating for Lead Resistance.

Of the three methods of compensation for variation in lead resistance: the Callendar four-lead, the Siemens three-lead, and the potential four-lead type, the latter is most suitable for work of the highest precision since the lead resistance is completely eliminated at all temperatures. All of the thermometers constructed for the present work were of the four-lead potential type and were used in conjunction with a Mueller[7] bridge and a commutator that is described in a later section of this paper.

3. Construction of the Templets.

When several thermometers of one kind are to be made, the use of a templet for the drilling and notching of the mica results in a saving of time and also in cleaner edges for, and more precise spacing of, the holes and notches than are obtained by operating directly on the mica. Photographs of the templets used in the construction of each of the three types of windings are shown in Figure 1 and some of the details of the dimensions are given in Figure 2.

General features. Each templet was made from two pieces of cold rolled steel, each 7½ to 8 inches long by ¾ inch wide by ⅛ inch thick, which were held in alignment by dowels and screws at the top and bottom. Smaller dowels served to hold the sheets of mica in place for drilling and notching. The main dowel pins should fit tightly into their respective sockets to prevent breakage of the fragile drills as they pass through the holes in the top plate of the



Mica-Cross Calorimetric Strain-Free

FIGURE 1. Photograph of the Templets.

templet into those in the bottom plate. Each plate was beveled slightly on its inner edge at one end in order that the top and bottom may be separated more easily.

Drill sizes. Uniform drill sizes were used for the corresponding holes of each of the templets. The holes for the larger dowel pins were drilled with a No. 45 twist drill and those for the smaller pins with a No. 59 drill.

For drilling the holes in the mica to take the 0.1 mm. platinum wire, a 0.008 inch "Improved Mascot" pivot drill was used. The top plate of the templet was drilled to within 0.01 inch of the bottom with a No. 66 drill which is a passing fit for the shank of the pivot drill. The two pieces of the templet were then placed together and an 0.008 inch hole drilled through the remaining metal of the top plate and for a distance of about 0.03 inch into the bottom plate.

The holes for the gold lead-wire were made with a No. 77 twist drill and extended through the top plate and for a distance of about 0.03 inch into the bottom plate. The holes through the top plate were enlarged by a No. 60 drill to within about 0.02 inch of the bottom. For the platinum supporting ring a No. 79 drill was used.

The mica-cross type. The two halves of the templet were fastened together, placed between the centers of a lathe and turned to a diameter of 7/16 inch for a distance of 3/4 inch and to 17/64 inch for a distance of 5 1/4 inches. A series of circular grooves were then cut by means of a tool ground to a 30° angle the point of which was flattened to a breadth of 0.008 inch. The grooves were spaced 0.040 inch apart and were 0.027 inch deep. The tool was held so that the bottom edge was perpendicular to the axis of the templet. Thus the platinum wire which was about 0.004 inch in diameter could not be held tightly in the bottom of the grooves as would be the case if they were V-shaped; and when the winding was held upright the wire rested on a horizontal surface as shown in Figure 2.

Since the platinum wire was to be wound "non-inductively" on the mica frame, a series of circular grooves gives an even spacing of the angle of the wire front and back, and has the advantage over a double thread that the bottom of the templet can be advanced into the collet of the lathe as the grooving progresses.

Immediately at the top of the series of grooves, two holes were drilled for threading the final turn of platinum wire through the strip and above these a series of holes for the gold lead-wire. The spacing and use of these holes are shown in Figure 2.

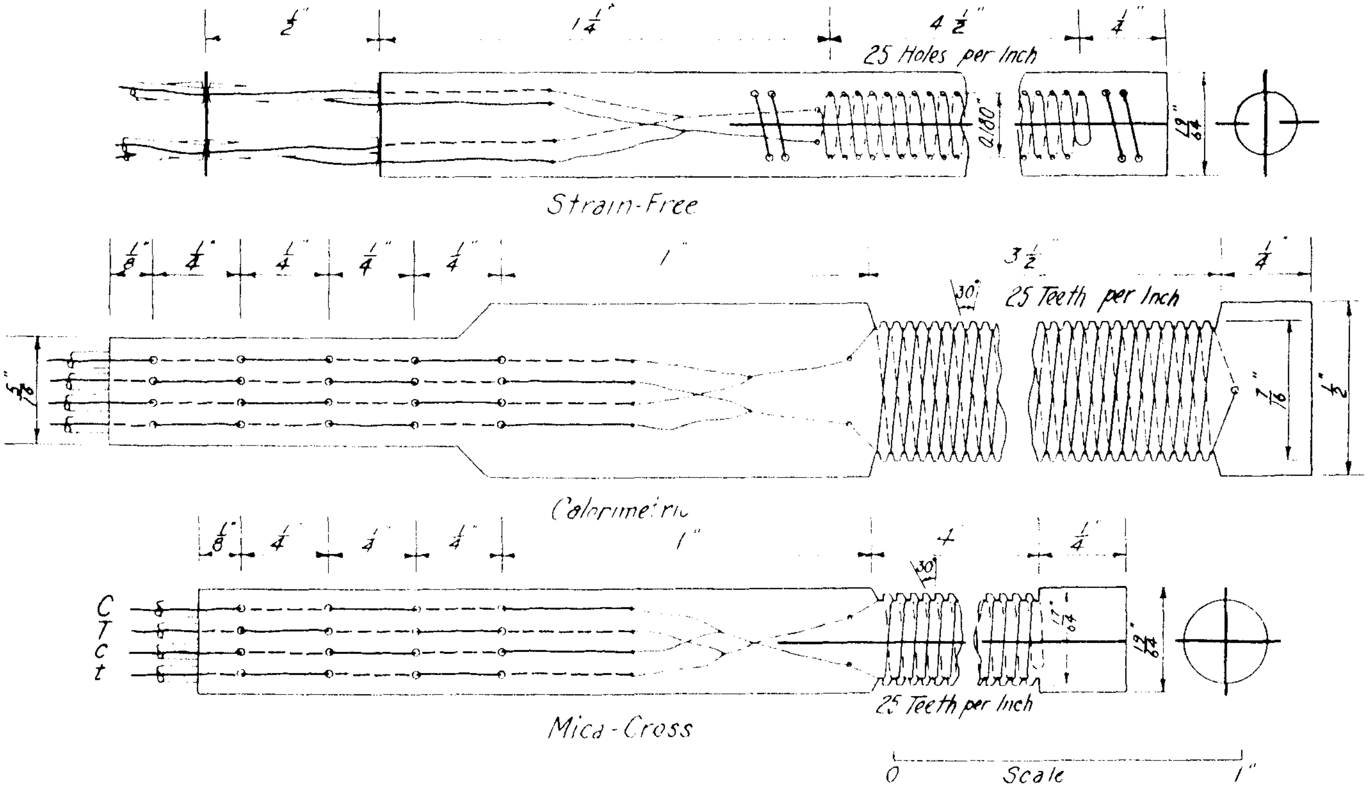

1/2"
1 1/4"
4 1/2"
1/4"
25 Holes per Inch
0.180"
19/64"
Strain-Free
1/8"
1/4"
1/4"
1/4"
1/4"
1"
3 1/2"
1/4"
30°
25 Teeth per Inch
7/16"
1/2"
5/16"
Calorimetric
1/8"
1/4"
1/4"
1/4"
1"
1"
1/4"
30°
17/64"
19/64"
C
T
C
t
25 Teeth per Inch
Mica-Cross
0
Scale
1"

FIGURE 2. Details of the mica forms.

The calorimetric type. The two halves of the templet were fastened together and turned to a diameter of 7/16 inch for a distance of 5¼ inches. Beginning ¾ inch from the bottom, a series of circular grooves were cut with the tool described in the preceding paragraph. For this templet however the axis of the tool was held perpendicular to the axis of the templet. The grooves were spaced 0.040 inch apart and were 0.027 inch deep. Two holes for the platinum wire and a series of holes for the gold lead wire were drilled in the top of the templet, spaced as shown in Figure 2.

The strain-free type. The first templet for this type of winding was made by Mr. Harold Benner of Crufts Laboratory, Harvard University, and our templet was modeled in part after that designed by him. The drilling of so many holes through steel with the fragile pivot drill was greatly facilitated by the use of a very sensitive yet simple drill press designed by Mr. Benner, the castings for which he kindly made available to us.

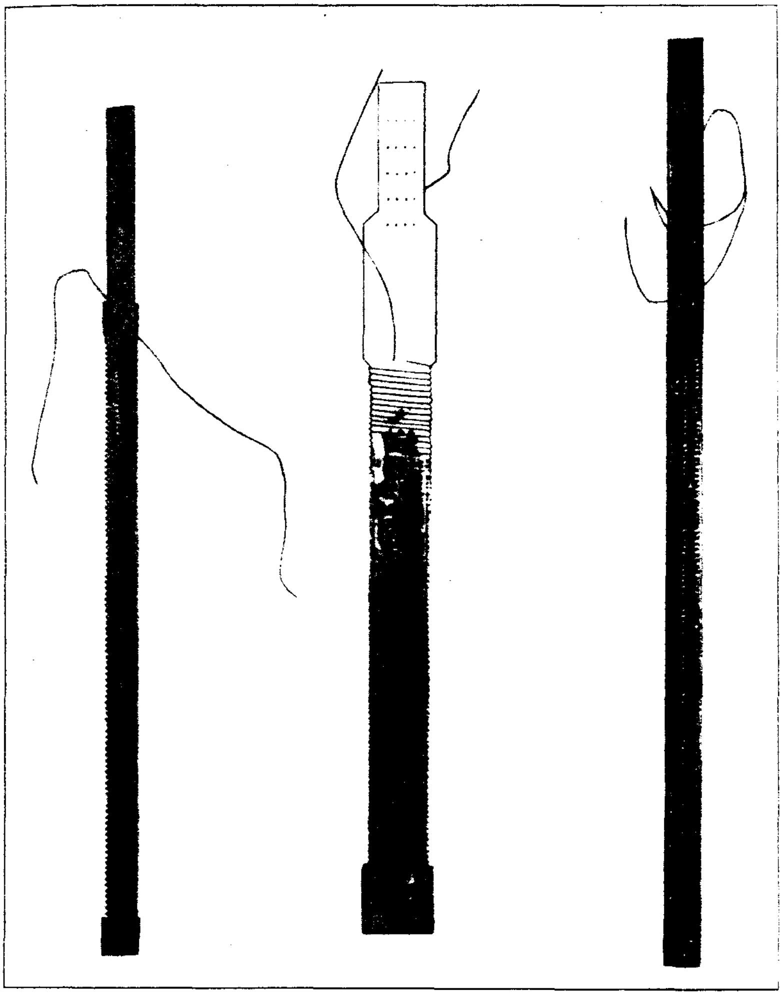
Two rows of holes each 5 inches long and 0.180 inch apart were drilled in the upper plate of the templet. The holes were spaced 0.040 inch apart and corresponding holes were directly opposite each other. They were first spotted in a small lathe, the spacing being obtained by use of the cross-feed, and then drilled to within 0.01 inch of the bottom of the plate with a No. 66 drill. The two halves of the templet were then fastened together and the 0.008 inch hole drilled through the upper plate and for a distance of 0.03 inch into the lower plate.

In the earlier thermometers of this type the gold lead wire was attached as shown in Figure 2, but in a later type a series of holes spaced as shown for the mica-cross type was used.

4. Construction and Winding of the Mica Forms.

The mica used was clear India mica free from iron stains. A piece of appropriate thickness was selected and the blank of the desired type accurately laid out on the strip by means of a needle. The mica was clamped between two pieces of brass, the edges of which were flush with the line along which the cut was to be made and the mica cut first from one side and then from the other with a safety razor blade.

The dimensions to which the mica for each type of thermometer was cut are given in Figure 2 and photographs of the completed windings are shown in Figure 3.



Mica-Cross Winding
Calorimetric Winding
Strain-Free Winding

FIGURE 3. Photograph of the Windings.

Mica-cross type. Two mica blanks each about 0.010 inch thick were prepared: one, which was to be the main strip, being cut to the dimensions given in Figure 2; and the other, the cross strip, being a duplicate of the bottom portion of the main strip but 4½ inches long. Each piece was placed between the halves of the templets and notched by means of a hack saw blade ground to exactly fit the groove, 100 notches being made in each edge. The blade was drawn across the mica in the opposite direction to that used in ordinary sawing.

In the main strip were drilled the holes for anchoring the last turn of the winding and for the gold lead-wire. In the center of the 4½ inch strip, and about 2¼ inches from the bottom of the main strip, a small hole was drilled and a thin slot about 0.01 inch wide cut along the center line of each strip, from the hole to the bottom of the main strip and from the hole to the top of the cross strip so that they could be fitted together to form a cross as shown in Figure 4.

The cross was placed in a quartered mandrel (as shown in Figure 4) which was made from four pieces of angle brass. A small glass capillary was laid in each of the four grooves of the mandrel and the platinum wire wound around the form. The capillaries were then removed and the thermometer wire pressed flat against the face of the mandrel, which was finally withdrawn from the winding. Thus even though the platinum wire was pulled rather tightly into the grooves of the mica form during the winding process, it was forced away from the bottom of the grooves when flattened against the face of the mandrel. This process accomplishes the same result as that proposed by H. C. Dickinson and mentioned in the paper by Sligh.[6]

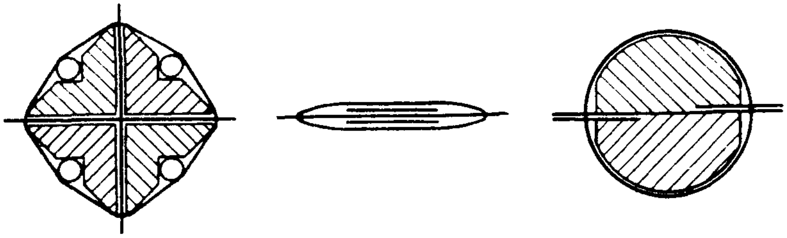
The calorimetric type. A piece of mica about 0.012 inch thick was cut to the appropriate dimensions. It was placed between the halves of the templet, notched as described in the previous paragraph, 86 notches being made on each edge, and the various holes drilled. Two insulating strips of mica each about 0.006 inch thick were cut to the same overall dimensions as the main strip, but no indentation was cut for notching.

A piece of mica about ¼ inch wide was placed at the front and one at the back of the main strip as shown in Figure 4, and the platinum wire wound in the grooves. The two mica strips were then removed and the wire pressed flat on the form thus forcing the wires away from the bottom of the grooves.

The insulating strips were placed, one on each side of the main strip, and attached to it by wires at the top and the bottom.

The strain-free type. Two pieces of mica one about 0.010 inch and the other about 0.006 inch thick were prepared, the former being cut to the dimensions given in Figure 2 and the latter about 1½ inches shorter. Each piece was placed between the halves of the templet and the two rows of holes drilled, 113 holes in each row. There were also drilled at the same time the holes for the supporting rings. In our later thermometers holes were drilled in the longer piece for the gold lead wire, these holes being spaced as shown for the mica-cross type in Figure 2.

From the shorter piece two strips were cut each ⅛ inch wide and each containing along its center line one of the rows of holes. The main strip and these two spacers were placed between the halves of the winding mandrel, one of the spacers being placed above and one below the main strip as shown in Figure 4. The mandrel was 0.175 inch in diameter by 5 inches long and the edges where the two halves fitted together were slightly flattened to permit easier removal from the winding. The platinum wire was wound around the form, being threaded through the holes every half turn. Care should be taken to allow a loop of sufficient length at the start of the coil to permit the spacer to be moved through 90° after removal of the mandrel.



Mica-Cross *Calorimetric* *Strain-Free*

FIGURE 4. Schematic details of the winding process. All views are of sections taken perpendicular to long axis of windings.

It was noticed that as the winding progressed the resistance of the wire per centimeter of length increased in a rather even manner. The resistance of the completed winding was in each case about 4% greater than that of the original wire. This is due no doubt to the scraping of fine threads of platinum from the wire as it was pulled through the holes.

When the winding was completed, two supporting rings of 0.3 mm. platinum wire were put into place and welded, the mandrel was removed, and the spacers moved through 90° and anchored in place by slightly bending the supporting rings.

5. THE CASINGS.

The mica cross and strain free types of windings were designed to fit into the same type of casing which was made of pyrex glass and was about 36 inches long. The head of the casing, see Figure 5, contained four short tubes through which the lead wires were drawn and which were plugged with De Khotinsky cement, and a central tubulature through which the desired gas was introduced into the thermometer and which was finally sealed off. For protection, the glass tube of the casing was usually enclosed, except when being calibrated, in a nickel sheath made from seamless tubing ½ inch outside diameter with 0.020 inch wall.

The casing for the calorimetric type winding was made from seamless nickel tubing $\frac{7}{16}$ inch outside diameter with 0.020 inch wall. A portion of the tube about 6½ inches long was flattened on a form by a method suggested by Dr. F. G. Keyes. The form, which was 0.093 inch thick by 0.525 inch wide and about 7 inches long, was somewhat tapered at one end as shown in Figure 6. The nickel tube was heated to bright redness, the tapered portion of the form inserted, and the tube squeezed on to the form between two jaws, shown in Figure 6, which were held in a vise. The form was driven further into the tube and the process repeated until the desired length of the tube had been flattened. The form was freed from the tube whenever desired by a slight pressure exerted along the rounded edge of the tube by use of a vise.

The head for this sheath was of glass and similar to that used for the other types of winding. A glass tube from the head extended into the nickel case for a distance of about 4 inches and was made tight by De Khotinsky cement.

6. ASSEMBLY.

Gold lead-wire. The resistances of the various completed windings were adjusted to about 25.5 ohms at 0° C. Four 0.3 mm. gold lead-wires were threaded loosely through the various holes provided for that purpose. To each one was welded a short piece of platinum wire. Two of these short pieces and one of the free ends of the winding were twisted together tightly, cut off square and welded to form a Y. The three platinum wires should leave the button of

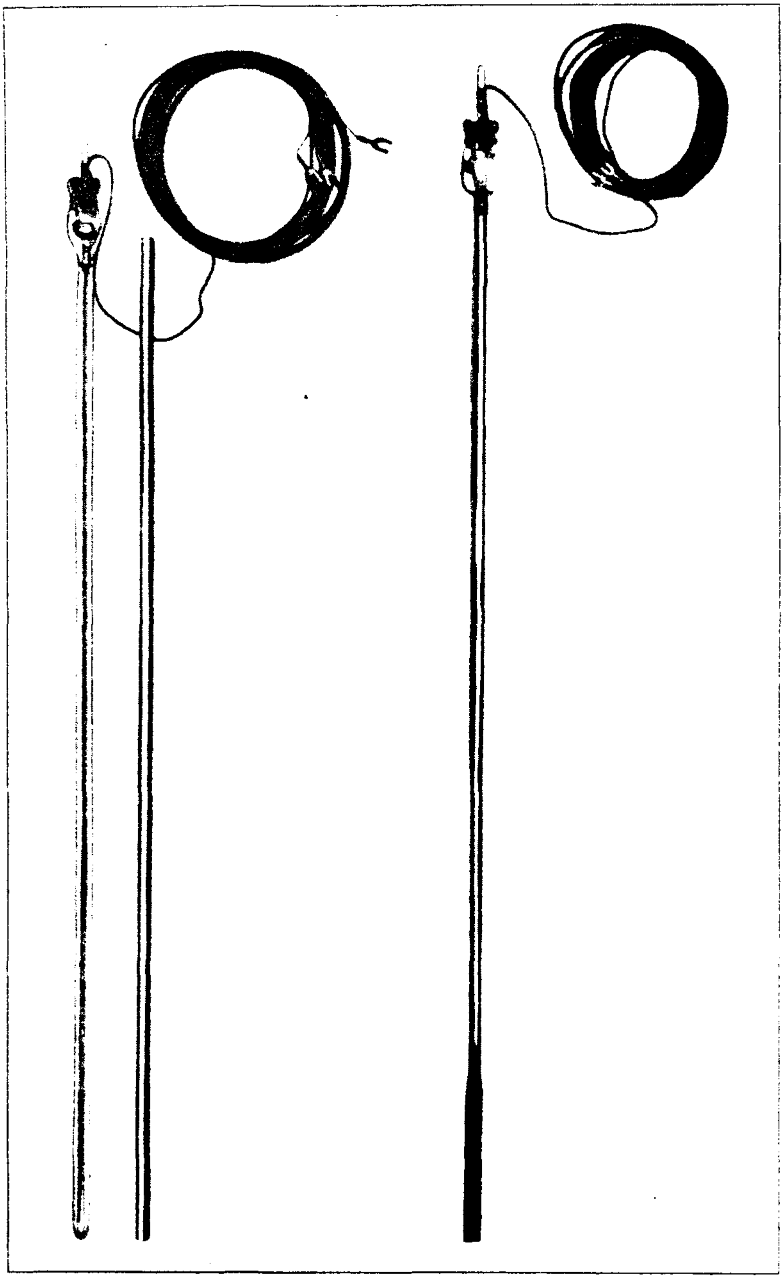


FIGURE 5. Photograph of the Thermometers.

platinum made in welding without additional contact with each other. After the welding was completed the gold wires were pulled through the holes to take up all slack.

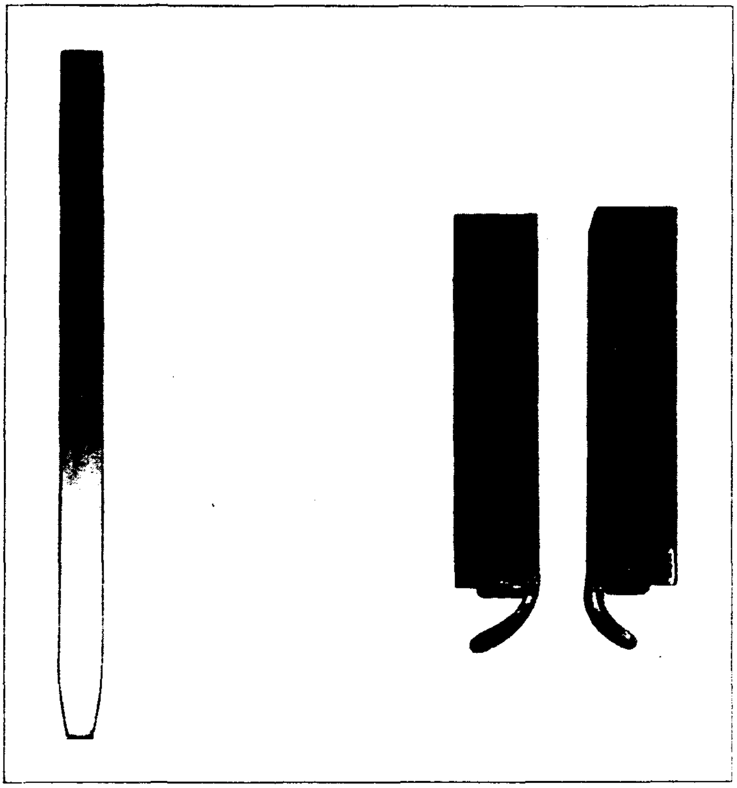


FIGURE 6. Photograph of the Forms for Flattening the Casing.

Insulation of lead-wires. For the mica-cross and strain-free types, a series of pyrex capillaries each ½ inch long were strung on the lead-wire. They were separated by mica spacing discs, which fitted the enclosing tube rather tightly. In this manner about twelve dead air spaces were formed which effectually prevent convection currents. Above these, long capillary tubes were used as shown in Figure 5. No. 24 copper wires were then welded to the upper end of the lead wire and the winding drawn into the glass case from the bottom. The gold-to-copper weld was just inside the head, the copper wire extending outside through the four tubes which were plugged with De Khotinsky cement. The bottom of the glass case was sealed off and the thermometer filled with the desired gas through the central tubulature, which was then sealed off.

For the calorimetric type, the winding was pulled into the nickel casing with the insulating glass capillaries, which were the length of the nickel casing, inserted over the gold wire. The copper wire was next welded to the gold and the glass head attached.

External lead-wire. The external copper lead-wire was soldered to the four copper wires which projected through the glass tubes of the thermometer head.

Washing. Just before the winding was drawn into the casing, it was washed several times with distilled water.

7. MATERIALS.

Thermometer wire. The platinum wire used was obtained from the Bureau of Standards and had an average temperature coefficient of resistance between 0° and 100° C. somewhat greater than 0.00391. The resistance at 0° C. should be about 25.5 ohms to produce a fundamental interval of 10 ohms, this corresponding to about 175 cm. of 0.1 mm. diameter wire, which size of wire is suitable for all work up to the sulphur boiling point.

Lead-wire. The gold lead-wires which extended from the winding to the head of the thermometer were made of 0.3 mm. diameter wire of 99.96% purity furnished by Baker and Company.

The external wire leading from the thermometer to the commutator was the same as that described by Sligh[6] and was purchased from Leeds and Northrup.

Gas. The majority of the thermometers were filled with commercial nitrogen which had been passed over hot copper and then through a liquid air trap.

Hydrogen was used in one thermometer but it seemed to react with the pyrex glass causing a darkening of the wall. One thermometer was also filled with helium and one with neon. Each of these gases has the advantage when used in a mica-cross or strain-free thermometer that the lag of the thermometers is considerably decreased. However, after several years these thermometers reacted to changes in temperature as slowly as the nitrogen filled ones, indicating without doubt that the greater part of the gas had escaped, probably by diffusion at the De Khotinsky seals.

The pressure of gas to which the thermometers was filled was about ⅔ atmospheres.

8. ANNEALING.

At first, it was attempted to anneal the thermometers by successive heating in an air furnace to about 600° C. and cooling in liquid air. Under this treatment the ice points of the flat type thermometer became constant but not those of either other type. The mica-cross and strain-free types were successfully annealed by successive heating in a mixed nitrate bath to about 480° C. and cooling in liquid air. It is believed that there existed a temperature gradient in the air baths which was more or less evenly smoothed by the metal case of the flat thermometers.

Annealing by "flashing" the winding, that is by connecting 110 volts across the terminals of a 25 ohm thermometer for a few seconds, did not yield good results in our hands.

9. THE COMMUTATOR.

A commutator for connecting any one of four potential-type resistance thermometers to a Mueller bridge has been described by Mueller.[7] In the present work it was desired to use more than four thermometers and for this purpose the commutator shown in Figures 7 and 8 was made. It permits any one of twenty-four resistance thermometers to be connected to the bridge and can be extended to any desired number. By use of the reversing switch, "direct" and "reverse" readings of resistance can be made, the arithmetic average of which give the true resistance of the thermometer coil from one Y weld to the other, the lead resistance being completely cancelled up to the reversing switch. The wire connecting the switch to the bridge was made as short as possible and of 10 gage copper.

The commutator consisted of a rack which holds the thermometer terminal pieces; a universal contact piece which can be rather quickly attached to any one of the terminal pieces and which completes the circuit from the thermometer to the reversing switch; and a reversing switch which interchanges the connections between the bridge and the thermometer leads in a manner which can readily be seen from Figure 7 in which the bridge is shown schematically.

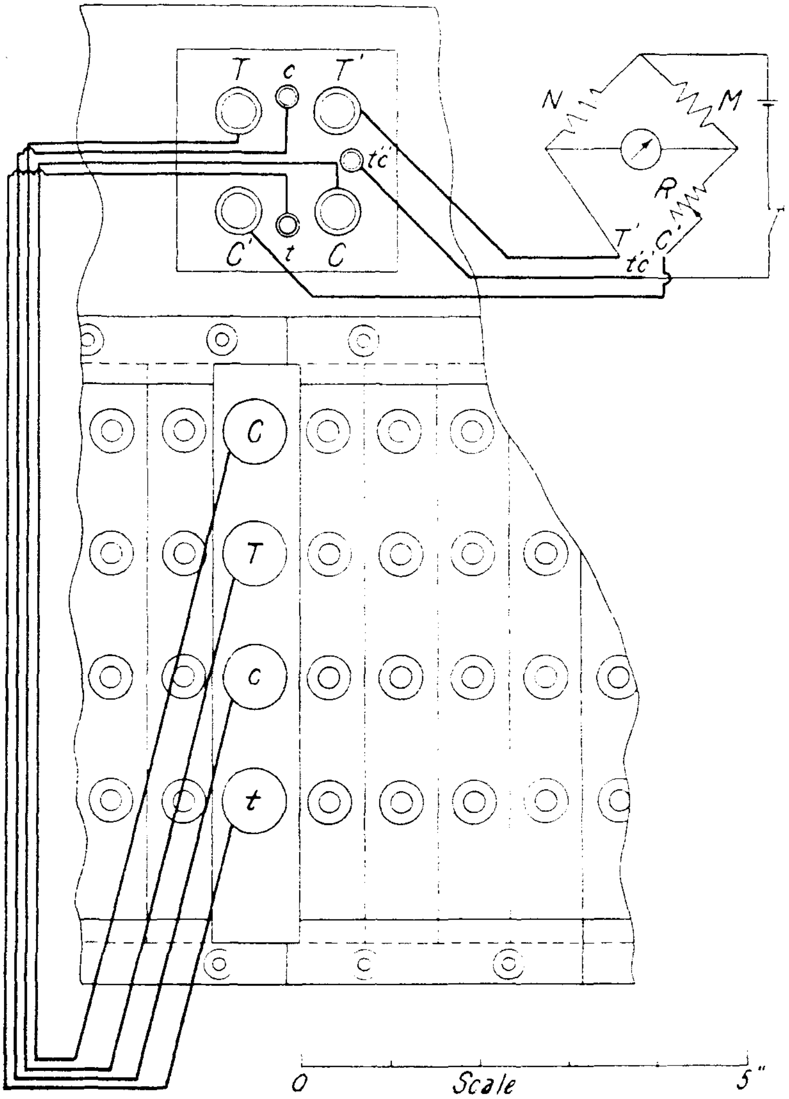

T
c
T'
t'c'
C'
t
C
N
M
R
T'
C'
t'c'
C
T
c
t
0
Scale
5"

FIGURE 7. Plan view of the commutator.

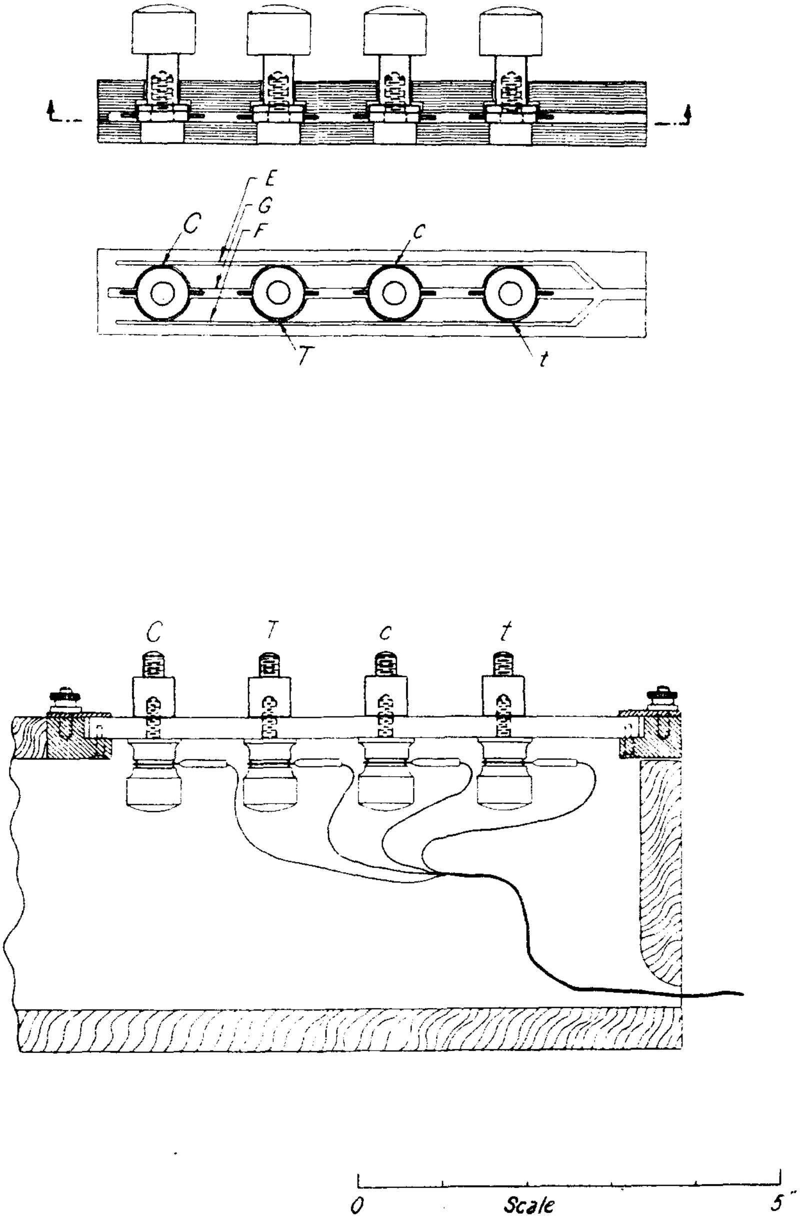

E
G
F
C
c
T
t
C
T
c
t
0
Scale
5"

FIGURE 8. Universal contact piece and thermometer terminal piece.

Thermometer terminal piece. For each resistance thermometer there was a terminal piece which consisted of a Bakelite strip $6\frac{1}{2}$ inches by $\frac{13}{16}$ inch by $\frac{1}{4}$ inch. In each strip were mounted four binding posts the bases of which were screwed into nickel-plated brass terminals as shown in Figure 8. It is advantageous to have the upper portions of these terminals threaded with a double thread.

Universal contact piece. The body of this piece was made of two strips of Bakelite which, when attached to each other by screws, held and centered the four contact pieces (see Figure 8). The grooves E and F permitted the lead wire to be conducted to the four contact pieces C, c, T and t, which consisted of nickel-plated brass discs each of which contained two small pins. These pins rested in the groove G and prevented the discs from turning. The hole in the center of each contact disc was large enough to slip readily over the thread of the thermometer terminal studs. Above each disc was a knob containing a threaded hole. When the universal contact piece was fitted over the thermometer terminal piece and the four knobs tightened, the brass discs were pressed tightly against the faces of the corresponding studs.

The discs in the universal contact piece had sufficient "play" in their sockets so that the connection to the terminal piece could be readily made.

Rack and reversing switch. The rack, a plan of which is shown in Figure 7 and a cross section in Figure 8, held the thermometer terminal pieces in place so that the universal contact piece could be readily connected to any one. This put the desired thermometer in direct contact with the bridge through the reversing switch shown in Figure 7 and described by Mueller.[7]

10. SUMMARY.

Details of the construction of 25 ohm platinum thermometers are described. This includes the construction of templets for drilling and notching the mica, the methods of winding the wire on the forms, the construction of the sheaths, and the assembly of the parts. These details are given for the mica-cross, the calorimeteric, and the strain-free type resistance thermometers.

A commutator for connecting any number of potential-terminal type resistance thermometers to a Mueller thermometer bridge is described.

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[1] Siemens, Proc. Roy. Soc. London, *19*, 443 (1871).

[2] See Burgess, Bureau of Standards Journal of Research, *1*, 635 (1928)

[3] Callendar, Phil. Mag., *32*, 104 (1891).

[4] Dickinson and Mueller, Bulletin of Bureau of Standards, *3*, 641 (1907); ibid., *9*, 483 (1913).

[5] Waidner and Burgess, Bulletin of Bureau of Standards, *6*, 149 (1909)

[6] Sligh, Scientific Papers of the Bureau of Standards, *17*, 49 (1921)

[7] Waidner, Dickinson, Mueller and Harper, Bulletin of Bureau of Standards *11*, 571 (1915); Mueller, ibid., *13*, 547 (1917).

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THE VOLUME OF EIGHTEEN LIQUIDS AS A FUNCTION OF PRESSURE AND TEMPERATURE.

By P. W. Bridgman.

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INTRODUCTION.

ONE of the first extended series of measurements of the effects of pressure that I attempted was on the volume of twelve liquids between 20° and 80° C. and up to 12000 kg/cm².[1] Since then I have made many determinations of the compressibility of solids, but have not returned to the question of liquids, in spite of the interest of the subject and the comparatively small number of liquids investigated. My reason for waiting so long before extending the early investigation has been in large part my desire to improve the method there used. That method was extremely simple: it consisted merely in measuring the motion of the piston by which pressure was produced. There were, however, several unsatisfactory features: the correction for the compressibility of the transmitting medium might be as large or larger than the effect being measured; the correction for the distortion of the cylinder, although small, had to be obtained by a calculatoin which is unsatisfactory because the metal is stressed beyond its elastic limit; there was considerable hysteretic difference between the readings with increasing and decreasing pressure with consequent possibility of error in the mean; and the thermal expansions under pressure were likely to be in error because the pressure measuring gauge was subject to the same changes of temperature as the liquid itself. This last point I felt to be of particular importance; for several speculations an exact knowledge of the limiting behavior of the thermal expansion at high pressures is important, so that it was particularly desirable to devise a method capable of giving this with greater accuracy. After a number of trials, extending over several years, and including work with the free piston piezometer,[2] which at first was very promising, but which had to be discarded because it could not give accurate enough values for the thermal expansion, such a method has been devised, so that the way is now open for the routine measurement of a large number of liquids.

In the following are presented the first of the data obtained in this projected campaign on the determination of the volume of a large number of liquids as a function of pressure and temperature. The twelve liquids chosen for the previous investigation were those which had been previously investigated by Amagat; the reason for this choice was to allow comparison with his results in the pressure range up to 3000 kg, and, particularly, to supplement the results at low pressures by Amagat's measurements, the previous method not being well adapted to give accurate values at low pressures. The liquids of Amagat were, however, in many cases of complex structure, and are not those which would naturally be chosen for their significance in theoretical speculations. In the following an effort has been made to select liquids more "normal" in structure, and therefore of greater theoretical interest. In the first place, a number of hydro-carbons are investigated: normal- and iso- pentane, n- hexane and its four isomers, and n- heptane, n- octane, and n- decane. Five liquids are measured which the theoretical investigations of Hildebrand have suggested as of interest, namely, C_6H_6, C_6H_5Cl, C_6H_5Br, CCl_4 and $CHBr_3$. The previous work has been supplemented by measurements of iso-propyl and n-butyl alcohols, the previous measurements having included n-propyl and iso-butyl alcohol. Finally, measurements, merely for check purposes, were made on ether and water and a mixture of glycerine and water.

The liquids chosen for this investigation often suffer from the disadvantage that the range open to measurement, particularly at the lower temperature, is small because of freezing under pressure. However, this limitation is unavoidable; many of the liquids of greatest theoretical interest are easily frozen by pressure.

The previous measurements were made at 20°, 40°, 60°, and 80° C. The temperature behavior proved to be very complicated, there being numerous small scale irregularities superposed on the large scale regularities. These small scale irregularities are, however, so complicated that they are not of much present significance for any theory, and in fact they have been entirely ignored in the theoretical speculations of various writers which have been based on these data. I felt therefore that in this new investigation the emphasis should be placed less on these small scale effects, and have accordingly increased the temperature range from 60° to 95°, but have diminished the number of temperatures from four to three, the new measurements being made at 0°, 50°, and 95°. In this way fewer of the small scale phenomena are disclosed, but greater confidence may be felt in the accuracy of the large scale effects, because of the increase of range.

It has also been possible to cover a greater range of material in the same time by decreasing the number of temperatures. A still wider range of temperature would obviously have been desirable, but this would have demanded essential changes and complications in the whole technique, with a corresponding great decrease in the number of substances that could be examined.

THE METHOD.

The fundamental idea of the method is very simple; the liquid to be measured is sealed into a metal "sylphon" (trade name for a flexible metal bellows), which is then exposed to external hydrostatic pressure. The sylphon shortens until the internal pressure is equal to the external, and from a measurement of the shortening, the change of volume of the liquid inside may be obtained. The advantages of this simple scheme are obvious; the separation of the liquid to be measured from the transmitting liquid is absolute, allowing no contamination; the effect measured is nearly the entire effect, for obviously the corrections are small, and furthermore may be computed with precision, for all the parts involved are subject to hydrostatic pressure and there is no distortion by exceeding the elastic limit; and finally, the pressure gauge may be placed in a vessel separate from that containing the sylphon, so that the temperature of the gauge may be kept independent of the varying temperature of the sylphon, thus eliminating this source of possible error in the previous method of measuring thermal expansion at high pressure.

The method is not entirely smooth sailing, however, but there are several matters, either of construction or function, that require examination. In the first place, there is the question of whether enough distortion can be obtained with the sylphon to permit convenient measurement of the rather large changes of volume involved, amounting to at least 30%. The ordinary sylphon of commerce does not have the requisite flexibility, and furthermore, at the time that this investigation was begun, was not made in sizes small enough to permit its use except by constructing an entire new high pressure plant capable of handling larger volumes. The construction of the necessary tools and jigs for the making of a single small sylphon of the commercial pattern, that is, by forming seamless from a single piece of metal, would have cost $10000, and was therefore prohibitive. The only course open was to build up a sylphon of the desired size by soldering together small sections; this was successfully accomplished, and will be described in detail later. In the second place, there was the fundamental question of whether the interior volume of such a sylphon may not be a complicated function of its length and the hydrostatic pressure. This happily proved not to be the case, but the sylphon has the property that it acts as if it had a fixed effective cross section independent of its extension, so that the change of internal volume may be obtained at once, after the necessary calibration has been made, in terms of its change of length. The check measurements by which this was established will be described in detail later.

The way in which the sylphon is built up in sections is indicated in Figure 1. The individual sections are formed by stamping into a die in a single operation, as indicated in Figure 2. After stamping, the edges of the section are trimmed with a sharp knife. A good many trials were made before the proper dimensions and material were found for the stampings. If the material is too thick, the resulting sylphon is too stiff; if it is too thin, it cannot be formed without tearing. The material finally adopted was "shim" brass, 0.0015 inches (0.0038 cm.) thick. It was cut into true discs in the lathe for forming, and then was carefully annealed to soften it. The forming operation restores it to just the right amount of springiness. Brass 0.002 inches thick was much too thick, and 0.0012 inches was too thin. Materials unsuccessfully tried were: another grade of brass specially manufactured and recommended for deep drawing, as well as grades of phosphor bronze and nickel silver also made for the same purpose, soft iron, silver, platinum, and copper.

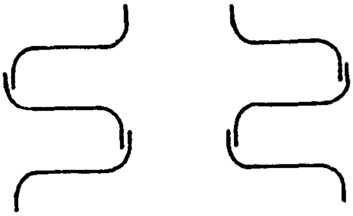

FIGURE 1. Shows the way in which the sylphon is built up in sections.

The sections are held together by first spinning over the edges and then by soldering. A specially built tool of readily suggested design for holding and rotating the sections as they are built up facilitates the assembly. The soldering must be above suspicion. It pays to brighten the edges of the seams with fine emery paper before applying the solder. Minute leaks proved the greatest difficulty in practise; these were fully as likely to be due to flaws in the metal as to imperfections in the soldering, and were often extremely difficult to detect at atmospheric pressure. The best method found for locating a leak at atmospheric pressure was to submerge the sylphon in alcohol while attached to a source of hydrogen at not more than half an atmosphere pressure. The hydrogen gets through fine holes much more easily than air, and alcohol has the advantage of water that it does not deposit on the brass fine bubbles of air from solution. The most satisfactory method of proving that the sylphon is tight is to fill it with a volatile liquid, such as ether or carbon bisulfide, sealing it when the sylphon is stretched so that the liquid will be under a slight excess pressure, and then finding whether the sylphon with its contents changes in weight over an interval of several days. No sylphon which passed this test ever proved defective under pressure.

Considerable skill is necessary in soldering the sections together so as not to put them in such a state of strain that they will snap from one position to another, like the bottom of an oil squirt, at some stage in the stretching. A sylphon which snaps in this way may as well be discarded at once, because the calibration will show jumps in the volume as a function of length, and it is therefore useless for accurate measurements. After some practice my mechanic, Mr. Charles Chase, developed much skill in the production of these sylphons, and is now able to complete one in about half a day; this paper would have been hardly possible without his skillful assistance.

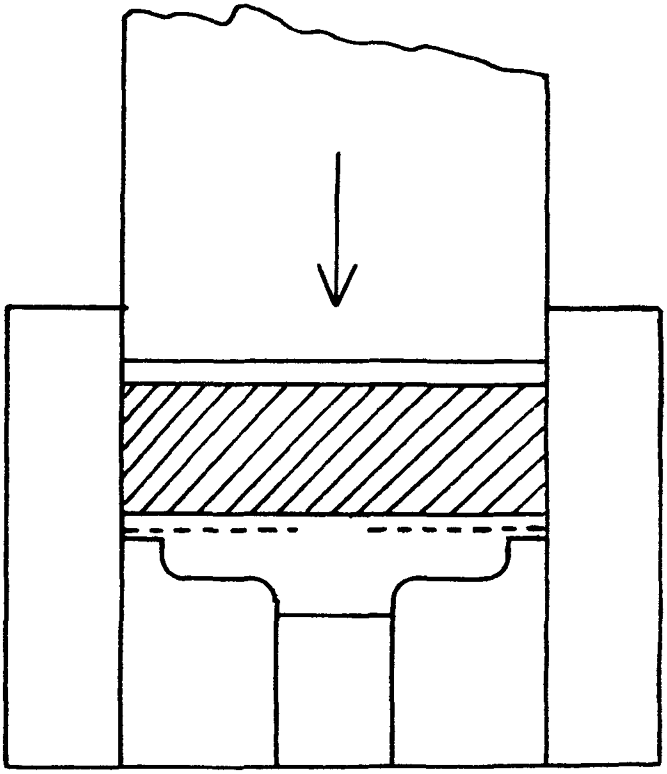

FIGURE 2. Shows the method of forming the sylphon sections by stamping. The thin disc to be formed is indicated by the dotted line. It is forced to the contour of the die by the lead disc, shaded, which is squeezed into the die by the action of the steel piston, indicated by the arrow.

The sylphon as finally constructed consists of nine double sections, of an unextended length of 1 inch (2.5 cm.). The flexibility allows a motion of 0.187 inch (0.475 cm.) in both directions, giving 0.375 inches altogether, or about 32% on the measured volume. The essential parts of the mounting of the sylphon are shown in Figure 3. End pieces are soldered to the sylphon bellows, and the combination is mounted in a brass sleeve which carries the arrangement by which the change of length is measured. This is the conventional sliding contact arrangement, consisting of a manganin wire sliding over an insulated contact. Measurements of the potential difference between the points *A* and *B* give the length of the segment *AB* and so the length of the sylphon to which the wire is attached.

Some sort of a guide has to be provided to constrain the sylphon to shorten and lengthen in the same straight line. This guide is provided by a piston and cylinder integral with the end pieces of the sylphon, playing one within the other, as sufficiently indicated by the drawing. At first the guide was made external to the sylphon, as would naturally suggest itself, but free motion was often interferred with by particles of dirt getting between the bearing surfaces, and the guides were finally put inside with complete success. Complete freedom of motion is obviously a prime requisite, for if the difference of pressure between the inside and the outside of the sylphon reaches much more than half an atmosphere the metal is permanently deformed, and the sylphon must be discarded. In order to avoid seizing of the guides, which often occurs when metal parts in relative motion are made of the same metal, the upper end piece, bearing the piston part of the guide, was made of brass, and the lower part, bearing the cylinder, of copper (not vice-versa because brass is more compressible than copper). To further prevent sticking if any particles of dirt should accidentally be in the sylphon initially, a spiral groove, not shown in the diagram, was cut on the brass piston, in which such particles might lodge. A further advantage gained by placing the guides inside the sylphon is that the internal volume available to the liquid is thereby cut down, thus decreasing the stroke necessary, which otherwise would have been uncomfortably close to the maximum allowable.

The sylphon was filled through the German silver tube at the lower end. During filling the sylphon is first stretched to a definite length and held there with a pin through appropriate holes in the brass sleeve. It is then inverted, a thistle-shape glass containing

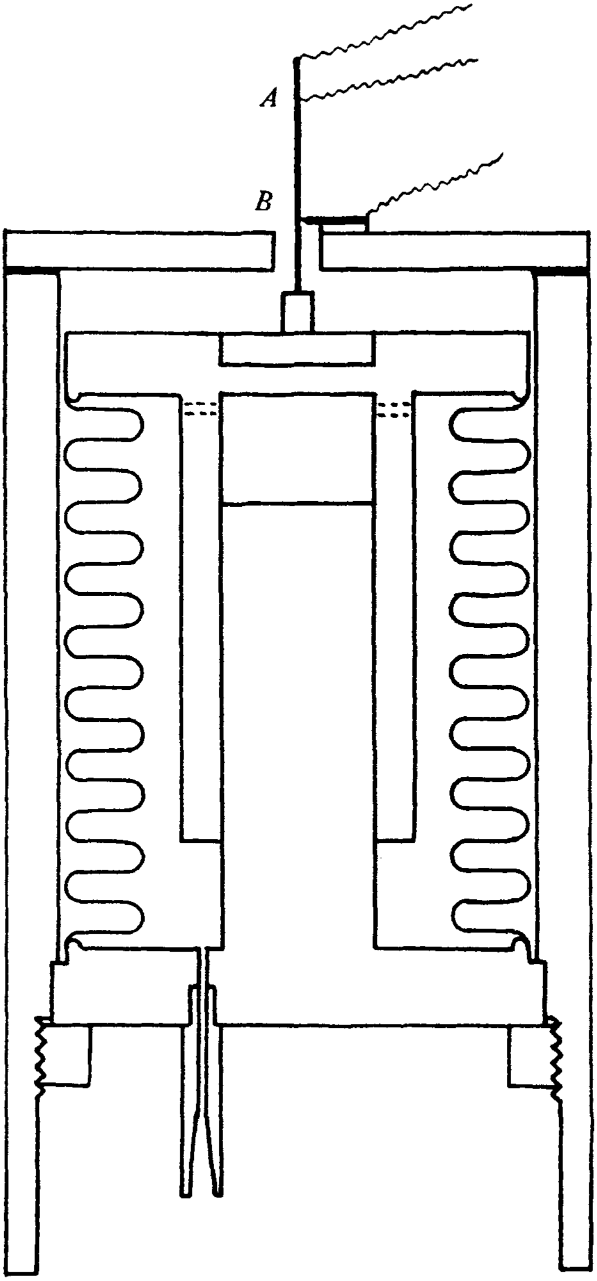

FIGURE 3. The sylphon assembled, with the internal guides and the electrical connections.

the liquid is attached to the German silver tube, and the whole is placed under the receiver of an air pump. The receiver is alternately exhausted and air readmitted until the sylphon is entirely full. The exhaustion also serves the purpose of removing dissolved air from the liquid. During the process of filling, the air must both be removed and readmitted with sufficient slowness to prevent the difference of pressure within and without the sylphon exceeding a safe value. After filling, the German silver tube is filled with a long tapering pin of German silver, and the amount of liquid in the sylphon determined by weighing. The total available volume of the sylphon was about 5 cm.^{3}, and weighings could be made consistent to a fraction of a milligram, so that no appreciable error attaches to the determination of the amount of liquid, After weighing, the sylphon was sealed by soldering into place the German silver pin, the outside of the tube being surrounded with water during the soldering to prevent transmission of heat to the liquid. German silver was used in order to make transmission of heat to the liquid through it as small as possible.

The sylphon thus filled was attached to an insulating three-terminal plug, the various necessary electrical connections were made by soldering, and the whole thing put as one assembly into the high pressure cylinder. This part of the process is so much like that which I have previously employed and described in detail[3] that it is not necessary to go into further detail here.

The first pressure run was almost always made at 0°, starting with the reading at atmospheric pressure, then at a few hundred kilograms, and then at increasing intervals to the maximum pressure which it was judged could be safely reached without freezing, or if there was no danger of freezing, to a maximum of 12000 kg. After reaching the maximum, readings were made with decreasing pressure back to about 500 kg, interspersing the readings with decreasing pressure between those with increasing pressure. The final result was a series of readings separated by intervals of the order of 300 kg at the lower end of the pressure range, and 1000 kg at the upper end. When the apparatus was functioning properly, there was never the slightest hysteresis between the readings with increasing and decreasing pressure, but both lay on the same smooth curve. This constitutes a great advantage over the piston displacement method. It was, however, sometimes necessary to season a new sylphon by a preliminary application of pressure; apparently the soldered seams as left by the soldering iron are capable of slight permanent deformation on the first application of pressure. The transmitting medium was pure petroleum ether; a liquid like kerosene becomes sufficiently viscous at high pressures to permanently deform the sylphon. Obviously excessive viscosity in the liquid within the sylphon may produce irregular results as well as viscosity in the external transmitting liquid. In fact there does seem a distinct tendency in the following for those liquids to give the most irregular results whose increase of viscosity under pressure is likely to be the greatest, that is, those liquids with the most complicated molecules.

In measuring those liquids whose freezing points under pressure were not known a number of sylphons were ruined by carrying the pressure up to the freezing point. The necessity for caution thus having been impressed, a small amount of freezing often being sufficient to do irreparable damage, the tendency in later work was perhaps to stay farther away from the freezing point than was necessary, and it would probably be possible to materially extend the range of data for a number of the following liquids without freezing. Freezing disclosed itself, not in the sudden decrease of volume that might perhaps at first be expected, but usually by a stationary reading, the guides freezing fast to each other, and the decrease of volume being taken up at the expense of permanent distortion of the sylphon. In all, eleven different sylphons were used in the following measurements.

After completion of the measurements at 0°, *temperature* was raised to 50° while pressure was maintained at about 500 kg. and a series of readings was made similar to those at 0°, with both increasing and decreasing pressure, except that the maximum pressure possible without freezing was now higher. After the 50° run, temperature was raised to 95°, and a similar series of readings made. In many cases the lowest pressure possible at 95° was somewhat higher than 500 kg., since the thermal expansion, if the pressure had been allowed to get lower, would have been sufficient to extend the sylphon beyond its limit. This accounts for the fact that in the following Tables many of the results given for 95° begin with the 1000 kg value instead of with that for 500 kg. After completion of a run at 95°, check measurements were always made again at 0° and 50° at some low pressure in the neighborhood of 500 kg, in order to be sure that the sylphon had experienced no permanent distortion. This test was practically always met within the errors common to all the readings.

In fact, the perfectly single valued character of the readings was a source of great satisfaction; it was possible to make runs at different temperatures on successive days without paying the slightest regard to the intermediate history of the apparatus. On several occasions a much severer test was successfully passed, in that the sylphon, sealed full with liquid, was removed from the holder, and the entire apparatus disassembled and then put together again, with perfect recovery of the original settings. This is all in striking contrast with the piston displacement method, in which a rigorous schedule of manipulation must be followed in order to reduce the hysteresis phenomena to reproducibility.

The completely single valued character of the readings made it possible to obtain accurate values for the thermal expansions from successive pressure runs at different temperatures, instead of being compelled to adopt the more direct procedure of changing temperature at constant pressure, which would have been much more inconvenient, because pressure is always easier to change than temperature. That the two procedures give identical results was established in a preliminary investigation.

In order to convert the measured changes of length of the sylphon into changes of volume, the effective cross section of the sylphon must be found by special calibration. To do this the sylphon was mounted in a special holder in which it could be shortened or lengthened by measured amounts with a micrometer screw, a graduated and calibrated glass capillary was attached to the German silver outlet tube, the sylphon was filled with kerosene, and the height of the kerosene in the capillary determined as a function of the length as given by the micrometer, making about 20 readings in all. The relation between length and change of volume turned out to be linear to better than 0.1 %, a result very gratifying and not expected. Furthermore, it was established by special trial that the volume is not perceptibly affected by pressure differences between the inside and outside of the sylphon up to as much as one-half atmosphere, which is close to the pressure of permanent deformation.

The manganin wire by which the length of the sylphon was determined was also specially calibrated. This again was done by moving it with a micrometer screw over a fixed contact in a specially constructed appliance, which need not be described in detail. Samples of wire were selected which were uniform to considerably better than 0.1%, so that no corrections had to be applied for lack of uniformity.

As indicated in Figure 3, the manganin wire was mounted detachably on the sylphon, so that the same wire could be used with several sylphons, considerably simplifying the manipulations and the calculations.

There are a number of small corrections which must be applied to the readings. The actual formulas for the corrections are somewhat complicated, and will not be written out in detail, since they are simple enough in general idea. The corrections are mostly for the compressibility of the various parts of the apparatus, and include the following.

(1) Correction for the compressibility of the brass sleeve which carries the contacts. Three different sleeves were made and used interchangeably at different times. These were all made from the same bar of brass. Its compressibility was determined by special experiment in the "lever" piezometer, and had the following values

$$\text{At } 30^\circ, \quad -\Delta V/V_0 = 9.207 \times 10^{-7}p - 6.42 \times 10^{-12}p^2.$$

$$\text{At } 75^\circ, \quad -\Delta V/V_0 = 9.309 \times 10^{-7}p - 5.79 \times 10^{-12}p^2.$$

(2) Correction for the compressibility of the brass part of the guide. This was made from the same bar as the sleeve, and therefore the correction has the value just given.

(3) Correction for the compressibility of the copper part of the guide. The compressibility of copper has been previously determined,[4] and the previous value was used.

(4) Correction for the compressibility of the manganin wire. This has also been previously determined, and the previous value was used.

(5) Correction for the compressibility of the sheet brass of which the sylphon is constructed. This was not independently measured, as it would have been difficult. I assumed that its compressibility is the same as that of the brass of the sleeve; the correction is in any event small, and commercial grades of brass do not differ by large amounts, and I believe the error made by the assumption to be inappreciable.

In addition to the various corrections for compressibility, there is a correction for the change of resistance of the manganin wire under pressure. This is the largest of the corrections, and it was determined by direct measurement on the identical piece of wire used at the three temperatures. The following results were found for the pressure coefficient of resistance: At 0°, 2.288×10^{-6}; at 50°, 2.311×10^{-6}; and at 95°, 2.289×10^{-6}.

There are also corrections for the thermal expansion of various parts of the apparatus on changing temperature. For these, values listed in various tables for brass, copper, and manganin were used, and are accurate enough. The temperature coefficient of resistance of the manganin wire at atmospheric pressure was also specially determined. The maximum variation in the constants of the apparatus due to changes of temperature in the range is about 1.5 parts in 1000. The maximum net correction for pure pressure effects on the runs at constant temperature was not much more than 2%, and the maximum net correction on the thermal expansion at constant pressure 5 or 6%. It is thus evident that as far as magnitude of corrections is concerned, the sylphon method is very greatly superior to the previous method of piston displacement.

The method of computation was first to plot the resistance readings as given by the potentiometer on a large scale on millimeter paper, 1000 kg on the scale of abscissas corresponding to 5 cm on the graph. Smooth curves were then drawn through the observed points, and the resistance read off at even intervals of 500 or 1000 kg. The changes of volume corresponding to these even intervals were then calculated with the aid of the various correction formulas. The results so obtained were the actual losses of volume in cm^3 reckoned from the volume at 0° and atmospheric pressure as fiducial. To convert this into fractional changes of volume in terms of the volume at 0° and atmospheric pressure as unity, the density at the fiducial point must be known. This could usually be taken from tables, and usually from International Critical Tables. The values thus used in the final reduction will be given in detail in the following for each individual liquid. The volumes at atmospheric pressure at 50° and 95° were also taken from the tables when possible; this was necessary because, as already explained, the sylphon readings often could not be made at atmospheric pressure at these two temperatures because the thermal expansion was too great, or else because the vapor pressure was above atmospheric.

There are several substances in the list, however, for which the density and thermal expansion at atmospheric pressure has apparently not been determined, notably the isomers of hexane. For these substances the density and thermal expansion at atmospheric pressure was determined by special experiment. For this purpose an alcohol thermometer was emptied, and used as densitometer and dilatometer. The bulb and stem were calibrated by filling with weighed amounts of mercury, and the stem was calibrated for uniformity with a moving length of mercury. Measurements of the volumes of the various liquids were made at three temperatures: 0°, a temperature about 10° below the boiling point, and a temperature half way between the maximum and 0°. Readings were made with both increasing and decreasing temperature as check; this check was always perfect to the limit of reading, which was 0.1° on the scale of the original alcohol thermometer. The results were put in the form of a second degree series in the temperature, and are given in the following. The density was determined by weighing the dilatometer when filled to a known mark at a definite temperature and when empty. The quantities of liquid used were of the order of 1 cm^3, and weighings were to 0.1 milligram, so that the density should be accurate to perhaps 1/30%. *I felt that there was no point in trying for greater precision* unless a much more exhaustive examination was made than was practical of the purity and reproducibility of the liquids.

If at any time in the future it should be found that better values for the densities or thermal expansions at atmospheric pressure are available, the relative volumes given in the following Tables can be readily corrected in accordance with the improved information. In the Tables the differences of volume from the fiducial volume at 0° and atmospheric pressure are the immediate result of the pressure measurements. If for instance, the future best value for the density at the fiducial point should turn out to be 1% less than the value assumed in the calculations, then the differences of volume reckoned from the fiducial point should all be decreased by 1%, increasing the actual volumes at any definite temperature and pressure. For example, a volume tabulated as 0.900 should be corrected to 0.901 in the hypothetical case.

The liquids measured in the following were obtained from various sources, selected for the greatest probable purity, and will be given in the detailed description. In general, the liquids were used as obtained, with no attempt at further purification; previous experience has shown that compressibility is insensitive to small amounts of impurity.

The detailed presentation of the data for the various liquids now follows.

DETAILED DATA.

Normal Pentane. The material was from Kahlbaum, their purest, provided in a sealed glass container. The runs at three temperatures and the return check readings were made without incident of any kind on three successive days. The average departure of a single one of the 46 observed points from the smooth curves was 0.11% of the maximum effect.

The final values for the volume are given in Table I. For the

TABLE I.

RELATIVE VOLUME OF N-PENTANE.

Pressure kg./cm.^{2}	Temperature 0°	50°	95°
0	1.0000	1.0837	1.1869
1,000	.9021	.9395	.9768
2,000	.8546	.8820	.9078
3,000	.8229	.8454	.8671
4,000	.7997	.8193	.8371
5,000	.7811	.7985	.8125
6,000	.7647	.7807	.7933
7,000	.7506	.7657	.7775
8,000	.7381	.7520	.7641
9,000	.7281	.7409	.7527
10,000	.7192	.7316	.7433

density at atmospheric pressure the following formula was used from International Critical Tables (abbreviated in the following as I. C. T.)

$$\text{Density} = 0.64539 - 0.9398 \times 10^{-3}t - 0.6243 \times 10^{-6}t^2 - 7.53 \times 10^{-9}t^3.$$

The two temperatures, 50° and 95°, listed in the Table are above the boiling point, which is 36°.2. The volumes given in Table I at atmospheric pressure at 50° and 95° were obtained by formal substitution in the formula for density, and must be used merely to get the approximate location of the 50° and 95° volumes at the lower pressure end of the range.

Iso-pentane. This material was from the Eastman Kodak Co., provided in a sealed glass container. Runs were made without incident at 0° and 50°, but at 95° there was an explosion at 10000 kg, so that readings with decreasing pressure were not made at this temperature, nor were the return check readings made. The average departure from a smooth curve of a single one of the 37 readings was 0.17% of the maximum effect.

An experimental point was obtained at 7200 kg at 0° that lay off the curve by three or four times as much as any other point, and was taken as indicating the beginning of freezing. Because of possible damage to the sylphon, the matter was not investigated further. Further consideration makes it very improbable, however, that this irregularity was actually due to freezing, since the melting temperature at atmospheric pressure of iso-pentane is about 28° lower than that of normal pentane, and normal pentane has been carried to 10000 kg at 0° without a trace of freezing. Of course it is not impossible that the melting curves should cross, and there are known examples of it, but certainly 7200 kg should not be accepted as the melting pressure at 0° of iso-pentane without further confirmation.

The values for the relative volumes are given in Table II. The volume of i-pentane as a function of temperature at atmospheric pressure does not seem to be recorded in the literature, and it was therefore specially determined by the method already described. The following results were found:

$$\text{Density} = \frac{.6406}{1 + .001527t + 3.21 \times 10^{-6}t^2}.$$

Because of the low boiling point, the temperature interval was only from 0° to 23°.5, so that the temperature coefficients in this formula have no great accuracy. The relative volumes are given in Table II. The volumes at 50° and 95° at atmospheric pressure were ob-

TABLE II.

RELATIVE VOLUME OF I-PENTANE.

Pressure	Temperature		
kg./cm.^{2}	0°	50°	95°
0	1.0000	1.0843	1.1741
500	.9383	.9883	
1,000	.9028	.9415	.9809
1,500	.8775	.9084	.9405
2,000	.8571	.8845	.9119
3,000	.8264	.8490	.8712
4,000	.8025	.8222	.8403
5,000	.7830	.8014	.8178
6,000	.7670	.7850	.8002
7,000	.7534	.7729	.7863
8,000		.7592	.7706
9,000		.7474	

tained by formal extrapolation of the formula above, and the remarks already made concerning n-pentane apply here also.

Normal Hexane. The material was obtained from the Eastman Kodak Co. Complete runs at three temperatures and the return check runs were made without incident on three successive days. Discarding two points, in which there were probably errors of recording, the average departure from a smooth curve of a single one of the remaining 42 readings was 0.22% of the maximum effect.

The maximum pressure at which readings were attempted at 0° was 6000 kg. From the atmospheric freezing point and a previous determination of the freezing point at 30° as 10600 kg. this was roughly estimated to be the highest safe pressure.

The relative volumes are given in Table III. For the density at atmospheric pressure the following formula from I. C. T. was assumed:

TABLE III.

RELATIVE VOLUME OF N-HEXANE

Pressure kg./cm.²	Temperature 0°	50°	95°
0	1.0000	1.0712	1.1535
1,000	.9191	.9567	
2,000	.8763	.9048	.9304
3,000	.8472	.8720	.8914
4,000	.8259	.8472	.8629
5,000	.8068	.8262	.8404
6,000	.7905	.8091	.8225
7,000		.7943	.8071
8,000		.7817	.7942
9,000		.7707	.7824
10,000		.7615	.7723
11,000		.7529	.7632

$$\text{Density} = 0.6769 - 0.8486 \times 10^{-3}t - 1.084 \times 10^{-6}t^2 + 0.164 \times 10^{-9}t^3$$

The boiling point is 69°, so that the legitimate use of the formula is restricted to temperatures lower than this; the volume at 95° at atmospheric pressure in the table was obtained by formal substitution in the formula.

2-methyl Pentane. For this isomer of hexane, as well as for the following three others, I am exceedingly indebted to Mr. T. A. Boyd of the Fuel Research Section of the General Motors Corporation. I am also indebted to Professor J. B. Conant for putting me in touch with Mr. Boyd.

The first attempt with this substance at 0° had to be discontinued at 4500 kg because of minor trouble with the insulation, which was speedily repaired. After reassembling, the three regular runs at three temperatures were made on three successive days. The return check readings were inadvertently omitted. The average departure from a smooth curve of a single one of the 42 readings was 0.20% of the maximum effect.

The maximum pressure attempted at 0° was 6000 kg. This was merely estimated as the safe maximum from relative boiling and melting points at atmospheric pressure as compared with n-hexane; no evidence of freezing under pressure was found during the experiment.

The relative volumes are given in Table IV. The density at atmospheric pressure, which had to be determined by special experiment as already mentioned, was:

$$\text{Density} = \frac{0.6720}{1 + 0.001297t + 3.52 \times 10^{-6}t^2}.$$

TABLE IV.

RELATIVE VOLUME OF 2-METHYL PENTANE.

Pressure	Temperature		
kg./cm.^{2}	0°	50°	95°
0	1.0000	1.0736	1.1549
500	.9460	.9930	
1,000	.9131	.9496	.9851
1,500	.8896	.9203	.9496
2,000	.8711	.8976	.9227
3,000	.8419	.8638	.8826
4,000	.8191	.8382	.8552
5,000	.8009	.8184	.8336
6,000	.7844	.8023	.8179
7,000		.7878	.8022
8,000		.7754	.7886
9,000		.7644	.7763
10,000		.7538	.7658
11,000		7450	.7564
12,000			.7484

The temperature range of the density measurements was 52°; the value given in the table at atmospheric pressure at 95° is merely a formal extrapolation.

3-methyl Pentane. This also was obtained from the General Motors Corporation. The only incident during the run was a leak in the connecting pipe, which developed at 50° at about 9000 kg. because the washers had been used too long without changing. Pressure was released, the apparatus reassembled, and the run continued. The part of the run below 9000 at 50° was repeated, in perfect agreement. The usual return check readings were made after the three runs at constant temperatures; all measurements were made on three successive days. The average departure from a smooth curve of a single one of the 55 readings was 0.19% of the maximum effect.

The maximum pressure attempted at 0° was much higher than for the two preceding isomers, 11000 against about 6000. That this was safe was suggested by the much lower melting point at atmospheric pressure; no sign of freezing was detected.

The relative volumes are given in Table V. The density at atmospheric pressure, as found by special experiment, was:

TABLE V.

RELATIVE VOLUME OF 3-METHYL PENTANE.

Pressure kg./cm.^{2}	Temperature 0°	50°	95°
0	1.0000	1.0727	1.1530
500	.9434	.9884	
1,000	.9120	.9471	.9815
1,500	.8879	.9189	.9462
2,000	.8689	.8971	.9205
3,000	.8408	.8630	.8824
4,000	.8200	.8379	.8553
5,000	.8022	.8189	.8342
6,000	.7866	.8026	.8158
7,000	.7723	.7888	.8006
8,000	.7592	.7761	.7876
9,000	.7476	.7647	.7753
10,000	.7372	.7544	.7649
11,000	.7276	.7455	.7550
12,000		.7373	.7452

$$\text{Density} = \frac{0.6835}{1 + 0.001285t + 3.43 \times 10^{-6}t^2}.$$

The volume given in the table at 95° at atmospheric pressure is a formal extrapolation.

One striking feature of the results with this substance is the way in which the isotherm for 50° is displaced away from the 0° isotherm and toward the 95° isotherm at the upper end of the pressure range. There can be little question but that this effect is genuine, the effect being much beyond the error of the measurements, and the irregularity in the points in the neighborhood being on the whole less than the average.

2–2-dimethyl Butane. This, like the two preceeding isomers, was obtained from General Motors. On the first attempt at 0° there was a leak in the manganin gauge plug at 3000, but this was successfully repaired, and the run continued, the points after reassembly checking those before. Three regular runs were made on three successive days. The return check readings were not made because after the run at 95° the pressure was inadvertently allowed to get so low that the sylphon blew up by boiler pressure. This was particularly unfortunate, because the quantity of this isomer available was smaller than that of any of the others, and there was not enough left for the dilatometer and density measurements. The average departure from a smooth curve of a single one of the 40 readings was 0.15%.

The freezing pressure of this isomer was so unexpectedly low that I was not successful in guessing its location, but freezing was encountered at each of the three temperatures. At 0° freezing was observed at 5400 kg., at 50° at 8000, and at 95° at 10800. It is not pretended that these give accurately the coördinates of the melting curve; it is merely certain that at each temperature the equilibrium pressure is lower than that given. Fortunately, no permanent damage was done to the sylphon by the amount of freezing that took place, as was proved by the perfect check between the points obtained with increasing and decreasing pressure.

The relative volumes are given in Table VI. These values are only approximate, because, as already explained, no measurements were made on the density or dilatation of this isomer. In calculating the values in the table, the density at 0° was assumed to be 0.6693. This value was estimated, on the assumption that the change of density on passing from 2–3 dimethyl butane to 2–2-dimethyl butane

TABLE VI.

RELATIVE VOLUME OF 2–2-DIMETHYL BUTANE.

Pressure	Temperature		
kg./cm.^{2}	0°	50°	95°
0	1.0000		
500	.9496	.9955	
1,000	.9154	.9517	.9824
1,500	.8919	.9232	.9478
2,000	.8737	.9016	.9218
3,000	.8473	.8703	.8855
4,000	.8260	.8464	.8594
5,000	.8083	.8269	.8392
6,000		.8106	.8223
7,000			.8079
8,000			.7958
9,000			.7850
10,000			.7756

would be the same as on passing from 3-methyl pentane to 2-methyl pentane. No attempt was made to estimate the dilation, and these places are left vacant in the table.

2–3-dimethyl Butane. This, the last of the isomers of hexane, was also obtained from General Motors. Several vicissitudes were encountered during the runs with this substance. At the termination of the 0° run short circuits developed in the manganin gauge and the sylphon circuits. These were repaired, the apparatus set up again, and perfect check readings obtained at 0°. The 50° run was now completed without incident. The readings from the beginning of the 95° run were unsteady. The matter finally grew so bad that the apparatus was dismantled, and the sylphon set up again in another holder, thus of course changing by a slight amount the zero reading. The 95° run was now completed without incident. The usual check readings at 50° and 0° were finally taken; the difference between these agreed with the value found in the previous holder, as it should. The difference between the 50° readings with the old and new holders gives the difference of zero readings between the two holders. The average departure from a smooth curve of a single one of the 45 readings available for locating the isotherms was 0.23%.

At 0° freezing was probably encountered at 6200 kg.; no permanent distortion in the sylphon seemed to be the result of this freezing. At the other temperatures there was no trace of freezing.

The relative volumes are given in Table VII. The density at atmospheric pressure, found by special experiment, was:

TABLE VII.

Relative Volume of 2-3-dimethyl butane.

Pressure kg./cm.²	Temperature 0°	50°	95°
0	1.0000	1.0722	1.1496
500	.9485	.9930	
1,000	.9147	.9503	.9841
1,500	.8909	.9202	.9478
2,000	.8695	.8960	.9198
3,000	.8395	.8633	.8836
4,000	.8180	.8383	.8562
5,000	.8004	.8186	.8340
6,000	.7855	.8028	.8162
7,000		.7881	.8008
8,000		.7747	.7874
9,000		.7624	.7757
10,000		.7509	.7645
11,000			.7546

$$\text{Density} = \frac{0.6808}{1 + .001311t + 2.67 \times 10^{-6}t^2}.$$

The volume at atmospheric pressure at 95° listed in the table is merely a formal extrapolation.

Normal Heptane. The material was obtained from the Eastman Kodak Co. The runs at three temperatures and the return check readings were made on three successive days without incident. This substance was one of the very few for which the return check readings were not satisfactory. The interval between the 0° and the 50° isotherms was satisfactorily checked, but there is some doubt about the relative separation of the 50° and the 95° isotherms. The values given in the table are those which the internal evidence suggests as most probable, but it is barely possible that all the volumes of the 95° isotherm should be decreased by 0.0019. The average departure from a smooth curve of a single one of the 41 readings was 0.12% of the maximum effect.

No trace of freezing was found. The maximum safe pressure was estimated from the atmospheric melting point and the known behavior of n-hexane.

The relative volumes are given in Table VIII. The value of density used in the calculations was that given in I. C. T. and is:

TABLE VIII

RELATIVE VOLUMES OF N-HEPTANE.

Pressure kg./cm.²	Temperature 0°	50°	95°
0	1.0000	1.0633	1.1350
500	.9535	.9972	
1,000	.9223	.9584	.9919
2,000	.8813	.9083	.9337
3,000	.8531	.8753	.8954
4,000	.8315	.8508	.8676
5,000	.8145	.8312	.8457
6,000		.8146	.8276
7,000		.8001	.8125
8,000		.7875	.7992
9,000		.7763	.7870
10,000		.7659	.7762
11,000			.7660

$$\text{Density} = 0.70048 - 0.8476 \times 10^{-3}t + 0.1880 \times 10^{-6}t^2 - 5.23 \times 10^{-9}t^3.$$

The normal boiling point of heptane is 98°.4, so that the 95° value given in the table at atmospheric pressure is in the range of actual values.

Normal Octane. This material was obtained from the Eastman Kodak Co. The measurements on this were made early in my experience, with a new sylphon, and before the necessity for seasoning a new sylphon to the highest pressure was recognized. There were considerable irregularities on the first applications, so that it was desirable to repeat some of the runs. Two runs were made at 0°, two at 50°, and one at 95°. The first run at 0° was the most irregular, and was not used in the calculations. However, the mean of the points of this discarded run lay on the curve obtained later, and for this reason the return check readings were thought to be unnecessary. The first run at 50° showed less irregularity than the first at 0°, the sylphon by now having been fairly well seasoned, although the maximum pressure had not been reached, and all except two of the points of the first 50° run as well as all of the second were used in the final calculation. The highest point at 95° was also irregular and probably for the same reason. Except for the discarded points just mentioned in detail, the average departure from a smooth curve of a single one of the remaining 50 points was 0.24% of the maximum effect.

No irregularity was found at the highest pressures of the second runs at 0° and 50°. The freezing pressures at 30° and 75° have been previously determined, so that there were data at hand to make it possible to stay within the maximum safe pressure.

The relative volumes are given in Table IX. The density assumed in the calculation is that given in I. C. T.:

TABLE IX.

Relative Volumes of n-octane.

Pressure	Temperature		
kg./cm.²	0°	50°	95°
0	1.0000	1.0595	1.1230
500	.9572	1.0005	
1,000	.9311	.9654	.9943
2,000	.8924	.9200	.9422
3,000	.8640	.8882	.9068
4,000		.8639	.8802
5,000		.8428	.8592
6,000		.8251	.8416
7,000		.8103	.8267
8,000			.8134
9,000			.8014
10,000			.7915

$$\text{Density} = 0.71848 - 0.8239 \times 10^{-3}t + 0.4459 \times 10^{-6}t^2 - 5.293 \times 10^{-9}t^3.$$

Normal Decane. This was obtained from the Eastman Kodak Co. The initial attempt at 50° was not good because of trouble with the insulation of the manganin gauge. The check readings were made several days after the completion of the regular runs, with the same filling of the sylphon, but with a different holder. The average departure from a smooth curve of a single one of the 34 readings was 0.29% of the maximum effect. This irregularity is apparently a little greater than usual; it is really not so, but the appearance is due to the smaller range possible with this material.

Freezing was not encountered on any of the runs. The safe limits were estimated from the known freezing point at atmospheric pressure and the behavior under pressure of the other hydro-carbons.

The relative volumes are given in Table X. In calculating the results the following formula for the density given in I. C. T. was used:

TABLE X.

RELATIVE VOLUMES OF N-DECANE

Pressure kg./cm.^{2}	Temperature 0°	50°	95°
0	1.0000	1.0530	1.1083
500	.9646	1.0029	
1,000	.9383	.9683	.9952
1,500		.9454	.9664
2,000		.9263	.9466
3,000		.8952	.9146
4,000		.8714	.8880
5,000			.8675
6,000			.8481
7,000			.8341
8,000			.8215

$$\text{Density} = 0.7455 - 0.7293 \times 10^{-3}t - 0.371 \times 10^{-6}t^2.$$

Benzene. This was Kahlbaum's purest grade, prepared for molecular weight determinations. The pressure range open to investigation with this liquid is very small because of the high freezing point; however the importance of the liquid in various theoretical speculations justified its measurement. The melting curve had been previously exactly determined, so that the maximum safe pressure was known without any guess work. Runs were made at 50° and 95°, and a rather larger number than usual of return check readings were made at 50° without incident. The average departure of a single one of the 25 readings from smooth curves was 0.25% of the maximum effect.

The relative volumes are shown in Table XI. For the density at atmospheric pressure the formula of I. C. T. was used:

$$\text{Density} = 0.90005 - 1.0636 \times 10^{-3}t - 0.0376 \times 10^{-6}t^2 - 2.213 \times 10^{-9}t^3.$$

In Table XI unit volume is taken as the volume at 0°, obtained by extrapolating with the density formula from the melting point down to 0°. In the pressure measurements the volume was not actually measured at atmospheric pressure at 50°, but was obtained by graphical extrapolation from the lowest pressure reading, which was at 370 kg. Such an extrapolation is not very certain, so that it is possible that there may be a slight error in the table in the volume difference between 0 and 500 kg at 50°. The relative volumes in the table at 50° from 500 kg on are based on direct measurement without extrapolation, and these should be relatively exact in terms of the volume at 500 kg and 50° (1.0160), independent of any error in the extrapolated interval.

TABLE XI.

Relative Volumes of Benzene.

Pressure kg./cm.^{2}	Temperature 50°	95°
0	1.0630	1.1295
500	1.0160	
1,000	.9841	1.0201
1,500	.9591	.9916
2,000		.9684
2,500		.9494
3,000		.9325
3,500		.9177

The volume of benzene has been previously measured by Essex at a number of temperatures between 0° and 100° up to 3000 kg. At the higher temperature the agreement between his results and mine is usually good to one in the third decimal place. At 50° the disagreement is greater, being 3 or 4 in the third place at 500, 1000, and 1500 kg, and 1 in the second place at atmospheric pressure. His volumes are consistently lower than mine.

C_6H_5Cl. This was Kahlbaum's purest. Regular runs at the three temperatures and the return check readings were made on two successive days with no incident. The average departure from smooth curves of a single one of the 39 readings was 0.09%, the measurements on this material being among the most regular of those obtained.

Freezing was not encountered at any temperature.

The relative volumes are given in Table XII. In the calculation

TABLE XII.

RELATIVE VOLUME OF CHLORO-BENZENE.

Pressure kg./cm.^{2}	Temperature 0°	50°	95°
0	1.0000	1.0502	1.1013
500	.9737	1.0170	
1,000	.9541	.9882	1.0215
1,500	.9380	.9673	.9955
2,000	.9244	.9511	.9755
2,500	.9127	.9384	.9595
3,000		.9268	.9463
4,000		.9068	.9247
5,000		.8903	.9072
6,000		.8762	.8924
7,000		.8652	.8794
8,000			.8675
9,000			.8580
10,000			.8487
11,000			.8406

the density given in I. C. T. was used:

$$\text{Density} = 1.12782 - 1.0664 \times 10^{-3}t \qquad \times 10^{-9}t^3$$
$$- 0.2463 \times 10^{-6}t^2 - 0.53 \times 10^{-9}t^3.$$

C_6H_5Br. This was Kahlbaum's purest. The 0° and the 50° measurements were made without incident. The 95° run, however, as only partially completed, and then measurements were interrupted for nearly three weeks by the illness of my assistant. After this time the 95° run was resumed, repeating the points formerly obtained, and also making the return check readings at 50° and 0°, all in perfect consistency with the first measurements. This constitutes a rather severe test of the reproducibility and single valuedness of the measurements.

Freezing was not encountered at any temperature.

The relative volumes are given in Table XIII. The density used in the calculation was that given in I. C. T.:

$$\text{Density} = 1.5223 - 1.345 \times 10^{-3}t$$
$$- 0.24 \times 10^{-6}t^2 + 0.76 \times 10^{-9}t^3.$$

The range of application of this formula given in I. C. T. is 0° to 80°. The value at 95° at atmospheric pressure must therefore be regarded

TABLE XIII.

RELATIVE VOLUMES OF BROMO-BENZENE.

Pressure kg/cm.^{2}	Temperature 0°	50°	95°
0	1.0000	1.0467	1.0940
500	.9763	1.0125	
1,000	.9570	.9891	1.0169
1,500	.9409	.9697	.9953
2,000		.9527	.9765
3,000		.9261	.9460
4,000		.9047	.9229
5,000		.8860	.9033
6,000			.8868
7,000			.8726
8,000			.8604
9,000			.8501

in the nature of an extrapolation, although not nearly so hazardous an extrapolation as one beyond the boiling point.

CCl_4. This was Kahlbaum's purest. The region open to measurement is very small because of the unusually steep melting curve. No run was attempted at 0°, because, although the normal melting temperature is − 23°, the freezing pressure at 0° is only about 500 kg. Regular readings at 50° and 95° and the return check readings were made without incident in one day. The average departure from smooth curves of a single one of the 21 readings was 0.15%. If allowance is made for the smallness of the range, the regularity is rather better than usual.

The relative volumes are given in Table XIV. The density at atmospheric pressure is given by I. C. T., as:

TABLE XIV

RELATIVE VOLUMES OF CARBON TETRACHLORIDE.

Pressure kg./cm.^{2}	Temperature 50°	95°
0	1.0000	
500	.9519	.9928
1,000	.9192	.9540
1,500	.8962	.9262
2,000		.9049
2,500		.8872
3,000		.8726
3,500		.8603

$$\text{Density} = 1.63255 - 1.9110 \times 10^{-3}t - 0.690 \times 10^{-6}t^2,$$

restricted to the range 0° to 40°. However, I used the formula by extrapolation to 50°, and the values in the table are based on the density so obtained. The actual volume was not measured at atmospheric pressure at 50°, but was obtained by extrapolation from the lowest pressure, 130 kg. The volume interval between 0 and 500 kg at 50° in the table is therefore subject to slight uncertainty, and the remarks made with regard to benzene apply.

Bromoform. This was obtained from Kahlbaum. The normal melting point is at +7°.7, so that the possible pressure range was very narrow. A regular run was made at 50°, and on the same day at 95° good points with increasing pressure were obtained, but the decreasing pressure points lay further and further from the curve, and on taking the apparatus apart it was found that the sylphon had developed a leak. On allowing it to stand for several days in the room, the sylphon fell completely to pieces, the bromoform exerting some corrosive action. I am informed by my colleagues among the chemists that this is to be expected. No attempt was made to repeat the run. The average departure from a smooth curve of a single one of the 24 readings up to the maximum pressure at 95° was 0.74% of the maximum effect. This large irregularity is mostly due to the smallness of the range; if the range had been normal, the irregularity would have been only 0.25%, not much larger than usual. There is no reason to think that the results obtained before the leak developed are not essentially correct.

No trace of freezing was encountered.

The relative volumes are given in Table XV. The specific volume

TABLE XV.

Relative Volumes of Bromoform

Pressure kg./cm.²	Temperature 50°	95°
0	1.0000	1.0427
500	.9628	.9993
1,000	.9369	.9662
1,500	.9158	.9418
2,000		.9225
2,500		.9063
3,000		.8915
3,500		.8782

as a function of temperature at atmospheric pressure is given in a paper by A. and J. Shuman[6] as:

$$V = 0.34204[1 + 9.0411 \times 10^{-4}(t - 7.7) + 6.766 \times 10^{-7}(t - 7.7)^2].$$

Their range was 7°.7 to 50°. I used the formula by extrapolation to 95°. At 50° the volume was not measured at pressure lower than 300 kg, and the volume interval between 0 and 500 kg at 50° given in the table is therefore based on an extrapolation and may be somewhat uncertain. The remarks already made concerning benzene and CCl_4 apply.

Iso-propyl Alcohol. This was originally obtained a number of years ago from Kahlbaum. It had been standing in a cork stoppered bottle and may have absorbed moisture in the intervening time. It was therefore redistilled twice in a fractionating column immediately before the compressibility measurements. As already stated, the reason for including this substance was that measurements had already been made on n-propyl alcohol in the preceding series. The first run at 0° gave highly irregular readings, doubtless because this was a fresh sylphon, not previously seasoned. The runs at 50° and 95° were of normal regularity. The run at 0° was then repeated, with regular readings except at the maximum, 8000 kg, where there was a slight irregularity that had the appearance of freezing. Pressure was not pushed higher for fear of loosing the points already obtained; the points with decreasing pressure lay on a smooth curve with the increasing pressure points. It is somewhat puzzling to understand why on the first application of pressure 10000 was reached at 0° with no appearance of freezing; there is, of course, the possibility of a sub-cooling effect, which may be very capricious. The succession of the melting points of the alcohols at atmospheric pressure would not make it surprising to find freezing at 0° below 12000 kg.

Discarding the first run at 0°, the average departure from smooth curves of a single one of the remaining 42 readings was 0.17%.

The relative volumes are given in Table XVI. The calculations are based on the formula for density given in I. C. T.:

$$\text{Density} = 0.8014 - 0.809 \times 10^{-3}t - 0.27 \times 10^{-6}t^2.$$

Normal-butyl Alcohol. The material was recently obtained from Kahlbaum, and was used without further purification. Considerable trouble was found with this material because of unusual freezing phenomena, and five runs in all were made at 0°, before satisfactory

TABLE XVI.

RELATIVE VOLUMES OF I-PROPYL ALCOHOL.

Pressure kg./cm.^{2}	Temperature 0°	50°	95°
0	1.0000	1.0540	1.1097
500	.9630	1.0056	
1,000	.9387	.9725	1.0112
1,500	.9200	.9485	.9807
2,000	.9040	.9296	.9562
3,000	.8769	.8995	.9218
4,000	.8571	.8765	.8961
5,000	.8398	.8579	.8750
6,000	.8244	.8415	.8572
7,000	.8116	.8279	.8419
8,000	.8002	.8159	.8289
9,000	.7901	.8052	.8168
10,000	.7813	.7948	.8062
11,000		.7861	.7971
12,000		.7784	.7894

results were obtained, the first two with a different filling of the apparatus from the others. I had not anticipated any trouble from the freezing of this alcohol, so that pressure was pushed to 12000 on the first run at 0°. But the reading at 12000 was the same as at 10200, showing that freezing had taken place somewhere in the interval. On the next application the pressure was kept below 10000, but an irregularity at the maximum, 9700, showed that freezing had again taken place. On the next two applications of pressure freezing phenomena were successively found at 7900 and probably at 7000 kg, and it was only on the fifth attempt, when pressure was not pushed higher than 5000, that no freezing was encountered. The successive lowering of the freezing pressure may have been due to a subcooling effect, there being nuclei of the solid phase left in the liquid, making the new formation of the solid easier at every successive attempt, as might be suggested by my previous experience with ice VI, or it may be that with each successive freezing there was increasing purification of part of the alcohol, complete remixing and diffusion of the segregated impurity being prevented by the high viscosity under pressure and the complex geometrical shape of the sylphon.

The two other runs at 50° and 95° and the return check readings were made without further incident. The average departure from a smooth curve of a single one of the 44 readings of the good runs was 0.27%. This is somewhat higher than usual, and is probably an effect of the freezing, although not the slightest damage to the sylphon was visible to the eye.

The relative volumes are given in Table XVII. The calculations are based on the formula for density given in I. C. T.:

TABLE XVII.

RELATIVE VOLUMES OF N-BUTYL ALCOHOL.

Pressure kg./cm.^{2}	Temperature 0°	50°	95°
0	1.0000	1.0455	1.0907
500	.9697		
1,000	.9459	.9779	
1,500	.9264	.9557	.9801
2,000	.9111	.9372	.9595
3,000	.8874	.9087	.9276
4,000	.8681	.8867	.9033
5,000	.8512	.8684	.8834
6,000		.8530	.8667
7,000		.8390	.8519
8,000		.8269	.8388
9,000		.8153	.8277
10,000		.8076	.8174
11,000			.8082
12,000			.7999

$$\text{Density} = 0.82390 - 0.699 \times 10^{-3}t - 0.32 \times 10^{-6}t^2.$$

Normal-hexyl Alcohol. This was obtained from Kahlbaum; it was evidently not entirely pure, because it was of a slight yellow color. On the first set-up, runs were attempted at 0° and 50°. On the first run at 0° freezing was encountered, but the amount was so slight that I hoped the sylphon had not been damaged. The results with decreasing pressure at 50°, however, strongly suggested a leak, and on taking the apparatus apart a crease was found in one of the folds of the sylphon which was doubtless the result of the freezing at 0°. A second set-up was therefore made with a new sylphon, and three successful runs at 0°, 50°, and 95°, and the return check readings were obtained without incident. The average departure from smooth curves of a single one of the 39 readings was 0.50% of the maximum effect, considerably larger than usual, but in large part accounted for by the smaller range.

The relative volumes are given in Table XVIII. In making the calculations the density at atmospheric pressure at 0° was assumed to be 0.8204, this being the figure given in Landolt and Börnstein. The thermal expansion has apparently not been recorded in the literature, and no attempt is made to give values for it in the table.

TABLE XVIII.

RELATIVE VOLUMES OF N-HEXYL ALCOHOL

Pressure	Temperature		
kg./cm.^{2}	0°	50°	95°
0	1.0000		
500	.9719	1.0078	
1,000	.9488	.9791	1.0052
1,500	.9316	.9583	.9813
2,000	.9183	.9420	.9619
2,500	.9075	.9278	.9467
3,000		.9159	.9337
4,000		.8960	.9118
5,000		.8787	.8931
6,000			.8773
7,000			.8637

In addition to these results on new liquids, certain measurements were made on other liquids during the preliminary testing of the apparatus which are worth recording.

Ether. This material was from Mallinckrodt, reagent quality, but it had been standing in a cork stoppered bottle for some time and may have absorbed moisture. The runs with this material were the first successful ones in which the sylphon was carried through a complete cycle without leak. Ether was chosen as the liquid because previous experience had shown that among easily obtainable liquids this, or CS_2, most readily disclosed a leak. In order to check further the reliability of the method, the compressibility of ether was determined numerically, and compared with the results of the previous method. The temperatures for this comparison were chosen as 30° and 75°, which are within the range of the previous work, whereas 0° and 95° both lie outside the previous range. Since the purpose was only comparative, pressure at 75° was not lowered below 2400 kg, the manipulations in this way being somewhat easier. In this preliminary exploration special attention was also given to the method proposed for getting the thermal expansion, the temperature being changed back and forth several times between 30° and 75°. The check here was perfect; it was possible to return exactly to the same isotherm after leaving it at any pressure in the range. The average departure from smooth curves of a single one of the 32 readings was 0.08% of the maximum effect.

In calculating the results the formula given in I. C. T. for the density was used:

$$\text{Density} = 0.73629 - 1.1138 \times 10^{-3}t - 1.237 \times 10^{-6}t^2.$$

The relative volumes are given in Table XIX, and in parallel columns in italics the values previously found. The agreement is not by any means as good as could be desired; the maximum discrepancy in the volume is about 0.006 in the neighborhood of 3000 kg. At the maximum pressure the difference becomes much less. It is important to notice that the thermal expansions of the two series differ from the mean by only a few per cent, so that the fundamental question

TABLE XIX.

RELATIVE VOLUMES OF ETHER.

Pressure	Temperature			
	30°		75°	
kg./cm.^{2}	new	previous	new	previous
0	1.0495	*1.0492*		
500	.9761	*.9790*		
1,000	.9364	*.9445*		
1,500	.9085	*.9153*		
2,000	.8858	*.8980*		
2,500	.8671	*.8734*	.8909	*.8970*
3,000	.8511	*.8576*	.8726	*.8788*
4,000	.8255	*.8318*	.8446	*.8496*
5,000	.8055	*.8111*	.8225	*.8268*
6,000	.7888	*.7954*	.8038	*.8098*
7,000	.7742	*.7806*	.7884	*.7940*
8,000	.7616	*.7675*	.7747	*.7800*
9,000	.7504	*.7554*	.7629	*.7672*
10,000	.7399	*.7444*	.7519	*.7560*
11,000	.7305	*.7335*	.7418	*.7457*
12,000	.7225	*.7237*	.7329	*.7353*

about the functioning of the new apparatus was thereby answered satisfactorily.

These preliminary measurements could be regarded as satisfactory in establishing most of the questionable points with regard to the new method, but in view of the uncertainty introduced by the unknown purity of the ether, did not answer the question as to the consistency of the absolute values obtained by the two methods. Subsequently another attempt was made to check the absolute values by using a liquid easily obtained in sufficient purity. For this purpose water was chosen. From one point of view the test with water was particularly severe, because it is one of the least compressible of liquids.

Water. This was distilled water of ordinary laboratory purity; it would have been useless to have started with conductivity water in view of the fact that it was to be placed in contact immediately with a metal. The three regular runs at 0° 50°, and 95° and the return check readings were made without incident. Except for one discarded point, which was so far from the curve that a blunder in reading is indicated, the average departure from smooth curves of a single one of the remaining 44 points was 0.15%.

The relative volumes are given in Table XX, with the previous values at 0° and 50° for comparison in italics; the previous range

TABLE XX.

RELATIVE VOLUMES OF WATER.

Pressure	Temperature				
	0°		50°		95°
kg./cm.^{2}	new	previous	new	previous	new
0	1.0000				
500	.9771				
1,000	.9567	*.9578*	.9741	*.9743*	.9984
1,500	.9396	*.9424*	.9582	*.9586*	.9812
2,000	.9248	*.9260*	.9439	*.9445*	.9661
3,000	.8996	*.9015*	.9201	*.9205*	.9409
4,000	.8795	*.8807*	.8997	*.8996*	.9194
5,000	.8626	*.8632*	.8824	*.8818*	.9009
6,000			.8668	*.8662*	.8849
7,000			.8530	*.8524*	.8705
8,000			.8407	*.8399*	.8577
9,000			.8296	*.8288*	.8461
10,000			.8192	*.8188*	.8352
11,000					.8256

extended only to 80° so that comparison at 95° is not possible. Because of danger of freezing, no measurements were made at atmospheric pressure at 0°, and in making the calculations the fiducial volume was chosen as that at 500 kg and 0°, using the value 0.9771 obtained in the previous measurements. The agreement is seen to be much closer than for ether, and in particular the agreement at 50° is as good as could be desired. In view of the fact that the new data cover a wider range than the old I think that perhaps the new values should be given the preference, although this is a difficult question.

Finally, during the preliminary examination of the method, an attempt was made to find whether consistent results for compressibility could be obtained irrespective of the amount of liquid in the sylphon. If the cross section is truly independent of the extension of the sylphon, then the compressibility obtained with it should be independent of the amount of liquid in the sylphon, that is, independent of the initial extension of the sylphon. In order to make this test it is obviously necessary to choose a relatively incompressible liquid, because the compression of the more compressible liquids is so great as to use all available stroke of the sylphon. An incompressible liquid has the disadvantage of giving relatively small effects, and so enhancing the error from slight irregularities; there seemed, however, no way of avoiding this source of error.

Glycerine and water. The liquid chosen for this test was a mixture of equal parts by volume of water and glycerine, the glycerine being Kahlbaum's purest. Pure glycerine is much less compressible than water, but its viscosity increases so much under pressure that fear was felt for the safety of the sylphon had it been used pure. The comparison with two fillings of the sylphon was made only at 30°. The compressibilities and dimensions of the apparatus were such that a difference in the two initial amounts of about 25% was possible. Two runs were made with the larger filling, the initial points of the first being irregular, perhaps because the changes of pressure were made too rapidly for this rather viscous liquid. The decreasing points of this run and the second run gave regular points. A single run with the second filling was sufficient, the points lying smoothly. The average departure from a smooth curve of a single one of the 22 readings with the larger filling was 0.44%, and of the 11 readings with the smaller filling 0.25%.

The ratio of the changes of volume obtained with the two fillings should be a constant, independent of pressure. The following values were found for the ratio at successive intervals of 1000 kg, from 1000 to 11000 kg respectively: 1.236, 1.223, 1.230, 1.238, 1.250, 1.259, 1.265, 1.265, 1.265, 1.268, 1.268, 1.266. The ratio is therefore effectively constant beyond 6000 kg. Below 6000, greater deviation would be expected because the effects are absolutely smaller and the chance for relative error greater. In fact, on making a detailed examination of the experimental points, it will be seen that the deviation at the lower pressures is not more than can be accounted for by the irregularity of the experimental points, so that these measurements may be considered to give satisfactory proof of the fact that even under pressure the effective cross section of the sylphon is independent of its extension.

The actual values of the compressibility of the glycerine-water mixture are of some interest, and are given in Table XXI. The density of the mixture was not specially measured, so that the results are best given as loss of volume in cm.^{3} per gm. of mixture. The average of the two fillings is given.

TABLE XXI.

Compression of 50 per cent Glycerine-water Mixture at 30° C.

Pressure kg./cm.^{2}	Loss of volume in cm.^{3} per gm.
1,000	.0291
2,000	.0521
3,000	.0715
4,000	.0886
5,000	.1033
6,000	.1159
7,000	.1279
8,000	.1385
9,000	.1479
10,000	.1569
11,000	.1651

Discussion.

This discussion will touch only some of the main qualitative features of the situation; no attempt will be made to extend the elaborate discussion of the former paper on twelve liquids to these new liquids. All the information to be obtained from these new measurements is implicitly contained in the tables of volumes, from which any one may extricate for himself any information important for his particular purposes.

The new data of this paper allow a very considerable extension in one respect of the information of the previous paper, that is, with regard to the behavior of isomers at high pressures. In the previous list of 12 liquids there were only two isomeric compounds, that is, compounds with the same chemical formula, namely ether and butyl alcohol ($C_4H_{10}O$). These are, however, so different in structure that the chemist is not usually inclined to think of them as isomers. The fact previously found was that the volume of ether and butyl alcohol tends to become more nearly the same at high pressures, the ratio of the volumes of the same number of grams, that is, of the same number of molecules, being 1.106 at 500 kg and 1.038 at 12000 kg at 50. This seems the natural behavior, and it was my expectation that the new data would act in the same way. The number of isomers between which comparison may be made is now considerably increased, there being normal- and iso- propyl alcohols, normal- and iso- butyl alcohols, normal- and iso- pentane, and the 5 isomeric hexanes. These substances are more nearly what the chemist usually thinks of as isomers, the structural differences not being so great as between ether and butyl alcohol.

It turns out that the rule found for ether and butyl alcohol is not followed by these new isomers, but the volume differences between equal weights of the various pairs or sequences of isomers follows no rule. The relations are brought out in the following tables, in which the volumes of equal weights of different isomers are compared at a temperature of 50°. In Table XXII the relative volumes of the

TABLE XXII.

Volumes of Equal Weights of Isomeric Alcohols at 50° C.

Pressure	Propyl Alcohols			Butyl Alcohols		
	Volume of .8179 gm.			Volume of .8239 gm.		
kg./cm.^{2}	normal-	iso-	Ratio	normal	iso-	Ratio
0	1.0493	1.0756	1.0250	1.0455	1.0508	1.0051
2,000	.9293	.9487	1.0209	.9372	.9336	.9962
4,000	.8813	.8945	1.0149	.8867	.8791	.9914
6,000	.8494	.8588	1.0111	.8530	.8445	.9900
8,000	.8250	.8327	1.0093	.8269	.8197	.9913
10,000	.8060	.8114	1.0067	.8076	.7996	.9901
12,000	.7905	.7944	1.0049		.7821	

propyl and butyl alcohols are shown. The ratio of the volumes of the two forms of propyl alcohol approaches more closely to unity at high pressure than at atmospheric pressure, according to expectation, but the reverse is true for the butyl alcohols. The iso- form of propyl alcohol has a larger volume than the normal form over the entire pressure range, whereas the iso- form of butyl alcohol has a larger volume than the normal form at atmospheric pressure, but at high pressures has the smaller volume. The normal form of both alcohols is seen to have a smaller compressibility than the iso- form. In Table XXIII the volumes of equal weights of normal and iso- pentane are compared. Iso-pentane has throughout the greater volume, and the difference becomes greater at high pressures than at atmospheric pressure. This means that the normal form of pentane has greater average compressibility than the iso-form, unlike the alcohols. In Table XXIV the volumes of 1 gm. of the isomeric hexanes are given,

TABLE XXIII.

VOLUMES OF EQUAL WEIGHTS OF PENTANES AT 50°

Pressure	Volume of .6454 gm.		
kg./cm.^{2}	normal-	iso-	Ratio
0	1.0837	1.0924	1.0080
2,000	.8820	.8911	1.0103
4,000	.8193	.8283	1.0110
6,000	.7807	.7909	1.0131
8,000	.7520	.7649	1.0172
9,000	.7409	.7530	1.0163

TABLE XXIV.

VOLUMES OF 1 GM. OF ISOMERIC HEXANES AT 50°.

Pressure	Volumes					Ratio to n-hexane			
kg/cm.^{2}	Normal hexane	2-methyl pentane	3-methyl pentane	2.2-dimethyl butane	2.3-dimethyl butane	2-methyl pentane	3-methyl pentane	2.2-dimethyl butane	2.3-dimethyl butane
0	1.5827	1.5973	1.5697		1.5750	1.0092	.9918		.9951
2,000	1.3363	1.3357	1.3125	1.3470	1.3160	.9996	.9897	1.0080	.9851
4,000	1.2517	1.2470	1.2260	1.2647	1.2313	.9962	.9794	1.0104	.9837
6,000	1.1953	1.1940	1.1743	1.2113	1.1793	.9989	.9824	1.0134	.9866
8,000	1.1550	1.1537	1.1357		1.1380	.9989	.9833		.9853
10,000	1.1250	1.1217	1.1037		1.1030	.9971	.9810		.9804
12,000	1.1008*		1.0790				.9802		

* Extrapolated.

and the ratios to the volume of n-hexane. With the exception of 2-methyl pentane, the volumes of these isomers are more different from the normal form at high pressures than at atmospheric pressure, again contrary to expectation. In general, the compressibility of those compounds is greatest in which the CH_2 radical occupies a 3-position.

In contrast to the previous example of ether and butyl alcohol, the phenomena shown by these isomers are on a much smaller scale, and the volumes are always much more nearly equal. In view of this, I do not believe that the fact that the volumes of these new isomers do not in general tend toward greater equality at high pressures need alter the conclusion previously drawn, namely that because of the compressibility of the molecules there is a tendency to equality of volumes at high pressures. It is obvious however that the comparatively small structural differences of these new isomers persist, and may even become more prominent up to pressures of 12000 kg. Other lines of evidence have already suggested that there are conditions under which liquids may approach a greater semblance of order at high pressures, a fact which is not inconsistent with the discovery by X rays of structure in liquids under ordinary conditions.

Another instructive comparison is afforded by the volumes of the series of normal hydro-carbons, comparing the volumes per gm. molecule. In Table XXV are shown the volumes at 50° of 1 gm. molecule of the normal hydro-carbons as a function of pressure, together with the successive differences, in italics, on passing from one member of the series to another. In the first place, there are very distinct differences between the odd and even members of the series, the increase of volume when a CH_2 group is added on passing from an odd to an even member being greater than the increase on passing from an even to an odd member. The only exception to this is at atmospheric pressure between pentane and hexane; here there is a

TABLE XXV.

Volume of 1 gm. Molecule of Normal Hydro-carbons at 50°.

Pressure kg./cm.²	Pentane		Hexane		Heptane		Octane		Decane
0	121.03	*15.24*	136.27	*15.63*	151.90	*16.37*	168.27	*32.40*	200.67
2,000	98.51	*16.54*	115.05	*14.72*	129.77	*16.33*	146.10	*30.50*	176.60
4,000	91.52	*16.25*	107.77	*13.80*	121.57	*15.63*	137.20	*28.93*	166.13
6,000	87.20	*15.73*	102.93	*13.44*	116.37	*14.70*	131.07		
8,000	83.99	*15.46*	99.45	*13.08*	112.53				
10,000	81.72	*15.15*	96.87	*12.58*	109.45				

special effect. 50° is above the boiling point of pentane, and the volume listed was obtained by extrapolation of the volumes at considerably lower temperatures; the volume so obtained accentuates too much the larger volume of the approaching vapor phase in comparison with the other hydro-carbons, so that the difference between pentane and hexane is too small. The difference between odd and even steps in the series becomes less in going to higher members of the series, but becomes greater on going to higher pressures. This again is not what might at first be expected, and reminds one of the behavior of the isomers. It means that the first effect of high pressure is to accentuate the effect of structural differences in the molecules, in virtue of which they are perhaps squeezed into some semblance to an orderly arrangement. It is already known from X-ray investigations that there are differences between the volumes occupied in the solid crystal form by the odd and even members of the series, and Müller (7) has been able to give a detailed account of this in terms of the shapes of the molecules.

Pass now from a consideration of the volume to the compressibility. The most striking fact found for the previous twelve liquids was the very great decrease of compressibility with increasing pressure, the compressibility at 12000 kg being on the average about 1/15th of the compressibility at atmospheric pressure. This continues to be the fact for these new liquids, and what is more, the details of behavior are the same, in that the compressibility has dropped to about half its initial value in the first thousand kilograms, the rate of decrease becoming slower, until at 6000 pressure must be raised another 6000, that is, from 6000 to 12000, to reduce the compressibility again by a factor of 2.

The compressibilities do not vary through a very wide range, as may be seen by plotting the relative volumes of the various liquids in a single diagram. Of the new liquids, n-pentane is the most compressible. The compressibility in general decreases in a series of compounds on passing in the direction of increasing molecular weight. Thus in the series of normal hydro-carbons, the compressibility of decane is of the order of two-thirds that of pentane; in the series C_6H_6, C_6H_5Cl, C_6H_5Br, the compressibility decreases in the order given. Of the organic liquids previously examined, glycerine stood in a class by itself, with the smallest compressibility. Then came water, also in a class by itself, with a compressibility of the order of twice that of glycerine, and then the twelve liquids, with a compressibility on the average perhaps 50% greater than that of water, although the initial compressibilities were much more than 50% greater. Most of the new liquids fall in the same group as the preceding twelve, but there are two, C_6H_5Cl and C_6H_5Br, which fall in the same class as water; in fact the curves of relative volume of these three liquids very nearly coincide. The compressibility of C_6H_6, on the other hand, is about 60% greater than that of its two substitution products. This very large difference seems surprising. The difference is without doubt due in part to the effect so often discussed by Professor T. W. Richards:[8] the cohesive forces in C_6H_5Cl and C_6H_5Br must be fairly high, as shown by a consideration of the molecular volumes, for the increase of volume on passing from C_6H_6 to C_6H_5Cl and C_6H_5Br is of the order of half the molecular volume of free (solid) Cl or Br, and this means a high internal pressure and small compressibility. But the matter is not as simple as this, because the compressibilities of C_6H_5Cl and C_6H_5Br are very nearly the same, whereas the atomic compression on passing from C_6H_6 to C_6H_5Cl is materially less than on passing to C_6H_5Br.

It is evident that there is an interesting field for future investigation in seeking out organic compounds with abnormally small compressibility; the unique position of glycerine has as yet received no explanation.

The thermal expansion next concerns us. In the previous work on twelve liquids two important facts stood out with regard to thermal expansion; the thermal expansion at 12000 kg is almost always 4 or 5 times smaller than at atmospheric pressure, and at a pressure of a few thousand kilograms there is a reversal of sign of $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$, that is, at low pressures the thermal expansion increases with increasing temperature, but at high pressures it decreases with increasing temperature. The new data of this paper are not sufficient to give as much detail about thermal expansion as the preceding data because of the smaller number of temperature intervals, but they are adequate on these two important points. The decrease of thermal expansion with increasing pressure is larger on the average in the series of hydrocarbons than for the preceding twelve liquids; thus for pentane the decrease is by a factor of 7, and for n-hexane by 8. Many of the new data do not extend over as wide a pressure range as the preceding because of freezing, so that a complete comparison cannot be made; the two new isomeric alcohols fall in line with the previous results. The ratio for C_6H_5Cl and C_6H_5Br is smaller than the previous average.

The second important previous feature, the reversal of sign of $\left(\frac{\partial^2 v}{\partial \tau^2}\right)_p$, is also shown by all of these new liquids in which the pressure range attainable at 0° was extensive enough to provide the requisite data. The pressure at which the reversal takes place is also roughly the same as before, that is, in the neighborhood of 2000 or 3000 kg. In the series of hydro-carbons the pressure of reversal tends to become less as the molecular weight increases. Among the isomers of hexane there are some striking examples of strong reversal. 3-methyl pentane is perhaps the most striking: at atmospheric pressure the mean expansion 50°–100° (obtained by increasing the tabulated difference 50°–95° by 11%) is 22.7% greater than the mean expansion between 0° and 50°, whereas at 11000 kg it is 40.7% less. The fact that this reversal of sign is now found again with this fundamentally different apparatus affords welcome presumptive evidence of the essential correctness of the results obtained by both methods.

The suggestion has been made in a previous publication[9] that the probable reason for this reversal is that at high pressures and low temperatures the molecules are crowded so close together that the departures from linearity of the forces between molecules must be greater than at higher temperatures at the same pressure (that is, at larger volumes), since it is known that a large thermal expansion goes with a large departure from linearity, the thermal expansion of a substance with a linear law of force being zero.

Considerable importance attaches to the "pressure coefficient" $\left(\frac{\partial p}{\partial \tau}\right)_v$; the suggestion has been made a number of times that this should be a function of volume only, or an equivalent statement is that at constant volume, pressure is a linear function of temperature. In the previous paper on 12 liquids I showed that this does not agree with the experimental facts, and detailed calculations were presented for CS_2 from which it appeared that $\left(\frac{\partial p}{\partial \tau}\right)_v$ was a complicated function of temperature at constant volume. The theoretical reasons for thinking $\left(\frac{\partial p}{\partial \tau}\right)_v$ to be a volume function only have, however, appealed to some writers as strong enough to justify them in ascribing the complicated variations there found to experimental irregularities, and in continuing to assume that the theoretical relation is correct.

The new data of this paper allow a more decisive inquiry as to the validity of this relation, because the temperature intervals are fewer and wider, thus smoothing out small scale irregularities. The question may at once be answered from the graphs showing volume as

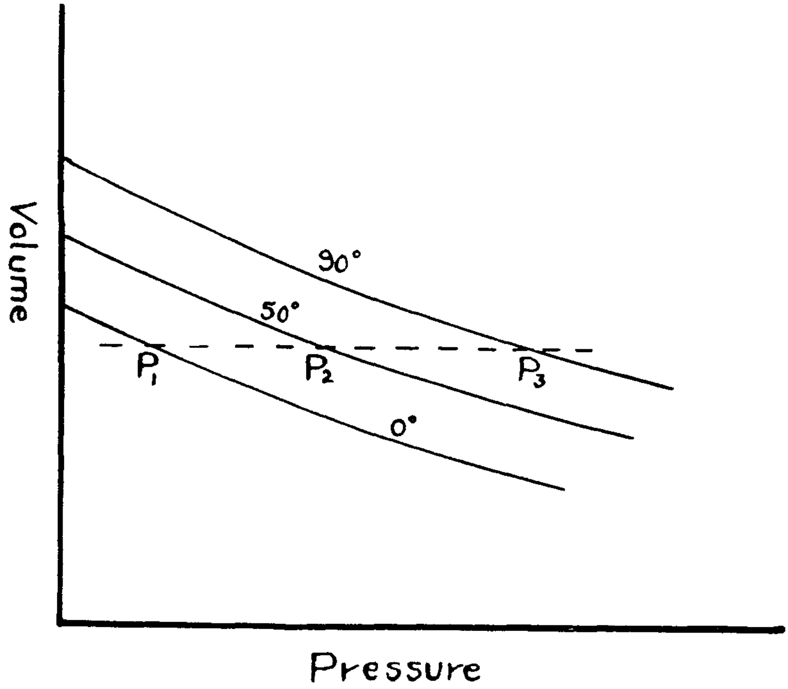

FIGURE 4. Graphical construction for finding whether $\left(\frac{\partial p}{\partial \tau}\right)_v$ is a function of volume only.

a function of pressure at 0°, 50°, and 95°. If $\left(\frac{\partial p}{\partial \tau}\right)_v$ is constant on a line of constant volume then $p_3 - p_2$ should be to $p_2 - p_1$ as 45 to 50, that is $\frac{p_3 - p_2}{p_2 - p_1}$ should equal 0.90 (Fig. 4). This ratio has been calculated at a number of points for many of the liquids of this paper, and the results are shown in Table XXVI. In making the calcula-

TABLE XXVI.

TEST OF THE INDEPENDENCE OF $\left(\frac{\partial p}{\partial \tau}\right)_v$ OF TEMPERATURE AT CONSTANT VOLUME.

Substance	Value of ratio $\left(\frac{p_3 - p_2}{p_2 - p_1}\right)_v$					
	Rel. vol.	ratio	Rel. vol.	ratio	Rel. vol.	ratio
n-pentane	.92	.84 ±	.75	.98		
i-pentane	.94	.905	.80	.972		
n-hexane	.92	.95 ±	.79	.774		
2-methyl pentane	.94	.897	.80	.910		
3-methyl pentane	.94	.865	.80	.817	.75	.597
2-2-dimethyl butane	.92	.743	.82	.68		
2-3-dimethyl butane	.95	.895	.80	.747		
n-heptane	.99	.914	.82	.832		
n-octane	.99	.806	.87	.767		
n-decane	.94	.837				
C_6H_5Cl	.95	.893				
C_6H_5Br	.95	.870				
i-propyl alcohol	.96	.947	.86	.925	.79	.900
n-butyl alcohol	.96	.837	.86	.850		
n-hexyl alcohol			.80	.845		

tions, the figures given in the tables of relative volume were used by linear interpolation; this accounts for the irregular volumes used, which were so chosen as to be as near as possible to the integral pressures of the tables, thus minimizing the small errors from linear interpolation. Table XXVI shows many deviations of the ratio from 0.90 which are far beyond any experimental error. The ratio usually starts with a value fairly near 0.90 at volumes near 1, that is, at low pressures, but as volume decreases, the ratio nearly always decreases, which means that at high pressures $\left(\frac{\partial p}{\partial \tau}\right)_v$ decreases with increasing temperature at constant volume. This was also the general nature of the results previously found for CS_2. There are, however, a few exceptions to this rule, n- and i- pentane being especially conspicuous for the large increase of the ratio on passing from larger to smaller volumes.

It is worth while to examine a little further the implications in assuming $\left(\frac{\partial p}{\partial \tau}\right)_v$ is a volume function only. If this is the case, integration gives at once

$$p = \tau f(v) + \varphi(v),$$

where f and φ are two arbitrary volume functions. The physical meaning of this equation is easy to see; it states that the pressure exerted by a fluid can be regarded as arising from two mechanisms acting independently of each other. One part, $\varphi(v)$, is a pure volume effect, in virtue of which a fluid exerts as much pressure at any temperature as it would at 0° Abs. at the same volume. This part of the pressure may be thought of as arising from the attractive or repulsive forces between the molecules. The second part is a kinetic part, proportional to absolute temperature, and arising from the change of momentum in unit time of the molecules as they collide with the walls. In a perfect gas, elementary considerations show that the momentum change in unit time is proportional to the kinetic energy, that is, to the temperature, and the volume function, f, becomes very simple. *If the molecules have a finite size,* f becomes more complicated. The nature of the two terms is clearly shown in van der Waals' equation, which satisfies the demand that $\left(\frac{\partial p}{\partial \tau}\right)_v$ be a function of volume only, and which may be written in the form $p = \tau \frac{R}{v - b} - \frac{a}{v^2}$, where the volume function $\frac{R}{v - b}$ arises from the finite size of the molecules, and $\frac{a}{v^2}$ arises from the mutual attractions. But van der Waals' equation certainly is not correct at high pressures, in that it omits, among other things, the very important factor of the compressibility of the molecules. In fact, one can expect in general that the hypothesis that there are two mechanisms, each acting independently of the other with no interaction effects, can be valid only over a narrow range in which the forces are approximately linear in the changes of volume, and in particular cannot be expected to hold at high pressures where there are certainly departures from linearity. That such is the case may be shown in detail in one very simple case. In the preceding paper on 12 liquids is worked out in detail the characteristic equation of the simplest conceivable substance, consisting of a single molecule. This molecule was, however, deformable, so that during collision with the walls it exerts pressure through an elastic mechanism which could be completely specified. Any ordinary liquid must have features in its mechanism closely corresponding to this. Within a certain range, which alone need concern us here, the characteristic equation of this substance consisting of a single molecule had the form

$$p\{v - b + a\tau^{1/2}\} = 2\tau,$$

where b gives the undeformed size of the molecule, and a depends on its elasticity. This equation is not of the general form given above, and gives at once:

$$\left(\frac{\partial p}{\partial \tau}\right)_v = \frac{2(v-b) + a\tau^{1/2}}{[(v-b) + a\tau^{1/2}]^2}.$$

This is evidently not a function of volume only, and in fact becomes smaller at higher temperature at constant volume. This corresponds to the fact brought out in Table XXVI that in general the ratio $\frac{p_3 - p_2}{p_2 - p_1}$ is less than 0.9. I believe that this feature of the mechanism must be present to a greater or less extent in any actual liquid, and that we are not to expect $\left(\frac{\partial p}{\partial \tau}\right)_v$ to be a volume function only, particularly at high pressures.

Another closely connected topic discussed in the former paper was concerned with the connection between internal energy and volume. Normally, as volume decreases with increasing pressure at constant temperature the internal energy decreases, but eventually passes through a minimum at a pressure equal to $\tau\left(\frac{\partial p}{\partial \tau}\right)_v$, and at higher pressures increases. The interpretation is that at ordinary volumes and temperatures the intermolecular forces are on the average attractive, and at small volumes (high pressures) repulsive; the volume at which $\left(\frac{\partial E}{\partial p}\right)_\tau = 0$ is the volume at which the attractive and the repulsive forces balance. Now at 0° Abs at atmospheric pressure the two sets of forces are in equilibrium, so that if the parts played by the various mechanisms are additive, the volume at which $\left(\frac{\partial E}{\partial p}\right)_\tau$ becomes 0 at room temperature would be expected to be equal to the volume at 0° Abs at atmospheric pressure. I have already examined this question for the solid metals[10] and found, as might be expected, that the critical volume at room temperature is not equal to that at 0° Abs at atmospheric pressure, but in general is less, so that the mechanisms cannot be additive. Recently Hildebrand[11] has applied the same idea to liquids, using in his discussion the data which I had obtained for ether. Most of the others of the twelve liquids were highly polar, and might, therefore, be expected to have complications.

Hildebrand found that the volume at which $\left(\frac{\partial E}{\partial p}\right)_\tau$ becomes 0 for ether at room temperature is very nearly, as well as can be judged from other data, its volume at 0° Abs at atmospheric pressure. Hildebrand also found the same state of affairs approximately for mercury. The liquids of this paper are normal enough to justify an examination of this point for them. In the case of n-pentane, $\left(\frac{\partial E}{\partial p}\right)_\tau = 0$ when relative volume is equal 0.77. The volume at 0° Abs at atmospheric pressure, calculated from the density of the solid[12] by the same argument that Hildebrand used for ether, is 0.80.

It should be noted that these considerations can be good only to the extent that $\left(\frac{\partial p}{\partial \tau}\right)_v$ is a pure volume function, and that therefore they cannot be expected to apply exactly to the liquids of this paper.

SUMMARY.

A new method of measuring compressibility and thermal expansions of liquids has been developed, in which the liquid is enclosed in a sylphon, which is then exposed to external hydrostatic pressure, and the volume change determined from the change of length of the sylphon. This method has been applied to 18 liquids at 0°, 50°, and 95° up to a pressure of 12000 kg, or to the freezing pressure, and the results are collected into extensive tables giving the volume as a function of pressure aand temperature over this range. In the discussion it is shown that small scale differences in the volumes of various isomers persist to high pressures, and there is no simple connection between the relative densities at atmospheric pressure and at high pressure. The compressibility falls off rapidly with rising pressure, as was found in a preceding investigation. Two liquids are found to have the abnormally low compressibility of water. Thermal expansion also drops off by a large factor with increasing pressure, but not as much as the compressibility, as was also found before. The "pressure coefficient" $\left(\frac{\partial p}{\partial \tau}\right)_v$, is not a function of volume only, as has often been supposed, and suggestions are made as to the theoretical significance of this.

I am much indebted to my assistant Mr. W. A. Zisman for making the readings, and to my mechanic Mr. Charles Chase for the con-

struction of the sylphon. I am also indebted to the Milton Fund of Harvard University and to the Rumford Fund of the American Academy of Arts and Sciences for generous financial assistance.

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THE PROBLEMS OF LAGRANGE AND MAYER WITH VARIABLE END POINTS.

BY MARSTON MORSE AND SUMNER BYRON MYERS.

THE PROBLEMS OF LAGRANGE AND MAYER WITH VARIABLE END POINTS.

By Marston Morse and Sumner Byron Myers.

1. **Introduction.** In developing sufficient conditions[1] in this problem, Morse found it convenient to give the finite conditions on the end points of the curves by representing these end points as functions of parameters rather than by expressing these conditions in the usual implicit form. This eliminated much of the algebra involved in the second variation and brought to the fore the essentials of the problem.

From the point of view of simplicity and elegance this was compatible only with a treatment of the Euler equations and the transversality conditions in corresponding form.

This was the first objective of the present paper. Incidentally the transversality condition is given a form that has apparently never appeared in the general problem, a form that reduces to the classic condition in the ordinary problem.

The paper also contains a new criterion for "normalcy" for the case of variable end points. This criterion has the advantage that it is near to the criterion given in the fixed end point problem, and has an immediate geometric interpretation which leads to various new theorems.

The form of the problem is an extension of the one used by Bolza.[2] The expression to be minimized is the sum of an integral and a function of a set of parameters which determine the end points, but which are not themselves in general uniquely determined by the end points. It is more general than any form hitherto treated at length, unless one resorts to devices such as introducing new differential conditions or terminal conditions. Such devices often change normal extremals to anormal extremals, or vice versa, and since sufficient conditions usually require normalcy, such changes lead to new difficulties. Furthermore, the generality of the form of the problem used here corresponds to an actual need in the treatment of the second variation.

The authors have made extensive use of the lectures of Bliss.[3]

2. **The problem.** In the space of the variables x and

$$(y) = (y_1, \cdots, y_n)$$

let there be given a curve E

$$y_i = \bar{y}_i(x) \qquad a^1 \leqq x \leqq a^2 \qquad (i = 1, \cdots, n) \tag{2.1}$$

of class D'*. Points neighboring the initial and final end points of E will be denoted respectively by

$$(x^s, y_1^s, \cdots, y_n^s) = (x^s, y^s), \tag{2.2}$$

where $s = 2$ at the final end point and $s = 1$ at the initial end point.

We consider curves of class D' neighboring E. Such curves will be called *differentially admissible* if they satisfy m differential equations of the form

$$\phi_\beta(x, y, y') = 0 \qquad (\beta = 1, \cdots, m). \qquad m < n† \tag{2.3}$$

We suppose that E is differentially admissible and that along E the functional matrix of the functions (2.3) with respect to the variables y_i' is of rank m.

A curve neighboring E will be said to be *terminally admissible* if its end points are given for some value of (α) by the functions

$$x^s = x^s(\alpha_1, \cdots, \alpha_r) \qquad y_i^s = y_i^s(\alpha_1, \cdots, \alpha_r), \tag{2.4}$$

where these functions of (α) are defined for (α) near (0), and reduce to the end points of E for $(\alpha) = (0)$.

A curve that is both differentially and terminally admissible will be called *admissible*. A set (α) neighboring $(\alpha) = (0)$ will be called *admissible* if it determines through (2.4) the end points of some admissible curve. We note that infinitely many sets (α) might determine the same admissible curve.

We seek conditions under which E and the set $(\alpha) = (0)$ afford a minimum for the expression

$$J = \int_{x^1}^{x^2} f(x, y, y')dx + \theta(\alpha) \tag{2.5}$$

among admissible sets (α) and corresponding admissible curves of class D'. The functions f and ϕ_β are to be of class C'' while the function $\theta(\alpha)$ and the end point functions in (2.4) need be of no more than class C'.

We note that $\theta(\alpha)$ may contain variables which do not explicitly occur in the terminal conditions (2.4).

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* See Bolza, "Vorlesungen über Variationsrechnung," p. 63.

† Here m could equal n as well, and the following developments would hold. One would not have to "extend" the differential equations $\phi_\beta = 0$ nor prove any of the "multipliers" zero.

3. **The variations.** Suppose that we have a family of differentially admissible curves

$$y_i = y_i(x, t) \qquad x_0^1(t) \leqq x \leqq x_0^2(t) \qquad (i = 1, \cdots, n) \tag{3.1}$$

containing E for $t = 0$. The *variations* of the family (3.1) along E with respect to t are defined to be the following derivatives:

$$\gamma^s = x_{0t}^s(0) \qquad \eta_i(x) = y_{it}(x, 0) \tag{3.2}$$

where the subscript t denotes differentiation with respect to t. These variations satisfy the *differential equations of variation**

$$\Phi_\beta \equiv \phi_{\beta y_i}\eta_i + \phi_{\beta y_i'}\eta_i' = 0 \tag{3.3}$$

obtained by differentiating (2.3) with respect to t and setting $t = 0$. Here (x, y, y') are to be taken on E.

A set (α) in (2.4) determines a set of end points. Let us take a set of points (α) of the form

$$\alpha_h = \alpha_h(t) \qquad \alpha_h(0) = 0, \qquad (h = 1, \cdots, r) \tag{3.4}$$

where the functions $\alpha_h(t)$ are of class C' for t near 0. Suppose that the family (3.1) takes on the end points determined by (α) in (3.4). That is, we suppose we have, subject to (3.4), the following identities in t:

$$\begin{aligned} x_0^s(t) &= x^s(\alpha) \\ y_i[x_0^s(t), t] &= y_i^s(\alpha). \end{aligned} \tag{3.5}$$

The curves of the family (3.1) are now also terminally admissible, and we include among the variations the following derivatives:

$$u_h = \alpha_h'(0).$$

These, together with the variations (3.2) satisfy what we shall call the *terminal equations of variation*

$$\begin{aligned} \gamma^s &= x_h^s u_h \qquad (s = 1, 2;\ h = 1, \cdots, r;\ i = 1, \cdots, n) \\ \eta_i^s &= (y_{ih}^s - \bar{y}_i'^s x_h^s)u_h \qquad\qquad u_h = \alpha_h'(0) \end{aligned} \tag{3.6}$$

obtained by differentiation of (3.5) with respect to t and setting $t = 0$. In (3.6) the superscript s means evaluation at the corresponding end of E and the subscript h attached to x^s or y_i^s means differentiation with respect to α_h.

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* The summation convention of tensor analysis is used throughout.

A set γ^s, $\eta_i(x)$, where the set γ^s consists of two arbitrary constants and the functions $\eta_i(x)$ are n functions of class D' which satisfy (3.3), will be called a set of *differentially admissible variations*.

A set of numbers γ^s, η_i^s which satisfy (3.6) for some set $(u) = (u_1 \cdots, u_r)$ will be called a set of *terminally admissible variations*, and will be said to be determined by the set (u).

With these definitions, we can now prove the following theorem:

THEOREM 1. *For every set of differentially admissible variations* γ^s, $\eta_i(x)$ *there exists a one-parameter family*

$$(3.7) \qquad y_i = y_i(x, t) \qquad x_0^1(t) \leqq x \leqq x_0^2(t)$$

of differentially admissible arcs containing E *for* $t = 0$ *and having* γ^s, $\eta_i(x)$ *as its variations along* E. *The functions* $y_i(x, t)$ *are continuous and have continuous derivatives with respect to* t *for all values* (x, t) *near those defining* E, *and the derivatives* $y_{ix}(x, t)$ *have the same properties except possibly at the values of* x *defining corners of* E, *while the functions* $x_0^s(t)$ *are of class* C' *for* t *near* 0.

This theorem and its proof are due to Bliss.* For purposes of reference we will reproduce the essentials of the proof here.

First we enlarge the system $\phi_\beta = 0$ to the extended system of differential equations

$$(3.8) \qquad \phi_\beta = 0 \qquad \phi_\tau = z_\tau(x) \qquad (\beta = 1, \cdots, m;\ \tau = m + 1, \cdots, n)$$

where ϕ_τ are new functions of (x, y, y') such that the functional determinant of the extended system with respect to the variables y_i' is different from 0 along E† and the functions $z_\tau(x)$ are defined by (3.8) upon substituting in ϕ_τ the functions $\bar{y}_i(x)$ defining E.

From this extended system we obtain an extended system of differential equations of variation

$$(3.9) \qquad \Phi_\beta = 0 \qquad \Phi_\tau = \zeta_\tau(x) \qquad (\beta = 1, \cdots, m;\ \tau = m + 1, \cdots, n)$$

along E where the functions $\zeta_\tau(x)$ are defined by these equations corresponding to a set of functions $\eta_i(x)$ of any set of differentially admissible variations.

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* See Bliss,[3] "The Problem of Lagrange in the Calculus of Variations," loc. cit., p. 4–6, and p. 20.

† For a proof of the possibility of this enlargement see Bliss, "The Problem of Mayer with Variable End Points," Transactions of the American Mathematical Society, vol. 19 (1918), p. 312.

Let the set γ^s, $\eta_i(x)$ be a set of differentially admissible variations for E, and $\zeta_\tau(x)$ the corresponding set defined by (3.9). The existence theorems for differential equations applied to the system

$$\phi_\beta = 0 \qquad \phi_\tau = z_\tau(x) + t\zeta_\tau(x) \tag{3.10}$$

furnish a one-parameter family of continuous solutions

$$y_i = y_i(x, t) \tag{3.11}$$

where

$$x_0^1(t) \leqq x \leqq x_0^2(t) \qquad x_0^s(t) = a^s + t\gamma^s$$

and where the initial values are given by

$$y_i(a^1, t) = \bar{y}_i(a^1) + t\eta_i(a^1)$$

The solutions are required to be continuous at the corner values of x. This family contains the arc E for $t = 0$, and has the prescribed variations γ^s. Its η variations have the initial values $\eta_i(a^1)$ and satisfy (3.9) with the functions $\zeta_\tau(x)$, and hence coincide with the prescribed functions $\eta_i(x)$.

In a similar manner the following corollary can be proved:

For a matrix

$$\begin{vmatrix} \gamma_1^1 & \cdots\cdots & \gamma_q^1 \\ \gamma_1^2 & \cdots\cdots & \gamma_q^2 \\ \eta_{11} & \cdots\cdots & \eta_{1q} \\ \cdots & \cdots\cdots & \cdots \\ \eta_{n1} & \cdots\cdots & \eta_{nq} \end{vmatrix}$$

where each column is a set of differentially admissible variations, there exists a q-parameter family of differentially admissible arcs

$$y_i = y_i(x, e_1, \cdots, e_q) \quad x_0^1(e_1, \cdots, e_q) \leqq x \leqq x_0^2(e_1, \cdots, e_q) \tag{3.12}$$

containing the arc E for $(e_1, \cdots, e_q) = (0, \cdots 0)$ and having γ_p^s, η_{ip} as its variations along E with respect to e_p. The continuity properties are similar to those of the preceding theorem.

4. **The Euler equations and the transversality condition.** Let us substitute in the integral in J the functions (3.7) defining a one-parameter family of differentially admissible arcs containing E for $t = 0$. If we take the parameters (α) thus,

$$\alpha_h = tu_h$$

where u_h is a set of r arbitrary constants, and substitute them in $\theta(\alpha)$, then J becomes a function of t defined by the formula

$$J(t) = \int_{x_0^1(t)}^{x_0^2(t)} f[x, y(x, t), y'(x, t)]dx + \theta(tu). \tag{4.1}$$

The derivative of this function with respect to t at the value $t = 0$ has the value*

$$V(\gamma, \eta_i, u) = \int_{a^1}^{a^2} (f_{y_i}\eta_i + f_{y_i'}\eta_i')dx + (f^s\gamma^s)_1^2 + \theta_h u_h \tag{4.2}$$

where f^1 and f^2 are the values of f at the respective end points of E, and the arguments of the derivatives are those defining E.

Let us obtain a more useful form of $V(\gamma, \eta_i, u)$. Let F be defined by the equation

$$F(x, y, y', \lambda) = \lambda_0 f + \lambda_1\phi_1 + \cdots + \lambda_n\phi_n$$

where λ_0 is a constant and $\lambda_i(x)$ are functions of x in the interval $(x^1\ x^2)$. The value of $\lambda_0 V(\gamma, \eta_i, u)$ is not changed if we add the sum

$$\lambda_\beta\Phi_\beta + \lambda_\tau(\Phi_\tau - \zeta_\tau)$$

to its integrand, since the variations $\eta_i(x)$ satisfy (3.9). We obtain the equation

$$\lambda_0 V(\gamma, \eta_i, u) = \int_{a^1}^{a^2} (F_{y_i}\eta_i + F_{y_i'}\eta_i' - \lambda_\tau\zeta_\tau)dx + (\lambda_0 f^s\gamma^s)_1^2 + \lambda_0\theta_h u_h. \tag{4.3}$$

So far the functions $\lambda_i(x)$ have been arbitrary. With the admissible curve E we now associate a constant λ_0 and multipliers $\lambda_i(x)$ so that the equations

$$F_{y_i'} = \int_{a^1}^{x} F_{y_i}dx + c_i \qquad (i = 1, \cdots, n) \tag{4.4}$$

are satisfied for an arbitrarily selected set of constants λ_0, c_i and (x, y, y') on E. This is possible since if we introduce the new variables.

$$v_i = F_{y_i'} = \lambda_0 f_{y_i'} + \lambda_1\phi_{1y_i'} + \cdots + \lambda_n\phi_{ny_i'} \qquad (i = 1, \cdots, n) \tag{4.4a}$$

---

* Here $(\ )_1^2$ means the value of $(\ )$ for $s = 2$, minus its value for $s = 1$.

equations (4.4) are equivalent to the following equations:

$$\frac{dv_i}{dx} = F_{y_i} = A_{i1}v_1 + \cdots + A_{in}v_n + B_i \quad (i = 1, \cdots, n) \tag{4.4b}$$
$$v_i(a^1) = c_i$$

The coefficients A_{ij}, B_i are determined by solving (4.4a) for $\lambda_1, \cdots, \lambda_n$ and substituting in F_{y_i}, and they are continuous except possibly at the values of x defining corners of E. Hence equations (4.4b) have unique continuous solutions $v_i(x)$ which are of class C' except possibly at the corner values of x. Equations (4.4a) then determine the functions $\lambda_i(x)$, continuous except possibly at the corner values of x.

By means of equations (4.3), after integrating by parts we find that

$$\lambda_0 V(\gamma, \eta, u) = -\int_{a^1}^{a^2} \lambda_\tau \zeta_\tau dx + (\lambda_0 f^s \gamma^s + \eta_i{}^s F^s{}_{y_i'})_1^2 + \lambda_0 \theta_h u_h. \tag{4.5}$$

This equation holds for arbitrary values of λ_0 and $F_{y_i'}{}^1$, for any set of differentially admissible variations γ^s, $\eta_i(x)$, and for any set of numbers u_h.

Consider the family (3.12) of the corollary of § 3, letting $q = 2n + 3$. Let

$$u_{hj} \quad (h = 1, \cdots, r; j = 1, \cdots, 2n + 3)$$

be $2n + 3$ sets of arbitrary constants. Take the parameters α_h of (2.4) as follows:

$$\alpha_h = e_j u_{hj} \quad (h = 1, \cdots, r; j = 1, \cdots, 2n + 3). \tag{4.6}$$

If we substitute the functions defining the family (3.12) in the integral in J, and the parameters (4.6) in $\theta(\alpha)$, then J becomes a function $J(e_1, \cdots, e_{2n+3})$, which when $(e_1, \cdots, e_{2n+3}) = (0, \cdots, 0)$ takes on the value J_0, its value along E. The $2n + 3$ equations

$$J(e_1, \cdots, e_{2n+3}) - J_0 = w$$
$$x_0{}^s(e_1, \cdots, e_{2n+3}) - x^s(e_j u_{1j}, \cdots, e_j u_{rj}) = 0 \tag{4.7}$$
$$y_i[x_0{}^s(e_1, \cdots, e_{2n+3}), e_1, \cdots, e_{2n+3}] - y_i{}^s(e_j u_{1j}, \cdots, e_j u_{rj}) = 0$$

all except the first of which are conditions for terminal admissibility, have the initial solution

$$(w, e_1, \cdots, e_{2n+3}) = (0, 0, \cdots, 0).$$

If the jacobian of the left-hand sides of (4.7) with respect to $e_1, \cdots, e_{2n+3}$ were different from zero at this solution, then the implicit function theorems show that (4.7) would have solutions $(e_1, \cdots, e_{2n+3})$ neighboring $(0, \cdots, 0)$ for all values of w in the neighborhood of $w = 0$. In that case, for negative values of w there would be admissible arcs in the family (3.12) giving J a value smaller than J_0, which is impossible if E is a minimizing arc.

Hence, if E is a minimizing arc, we conclude that the jacobian must be zero.

Let us set

$$\frac{\partial J}{\partial e_j} = V_j \qquad (e_j = 0;\ j = 1, \cdots, 2n+3)$$

where V_j is $V(\gamma, \eta, u)$ evaluated for the variations (γ, η) arising from the family (3.12) for variation of e_j, and (u) stands for the set u_{hj}, $(h = 1, \cdots, r)$. Then the jacobian in question has as the jth of its $2n + 3$ columns:

$$\begin{array}{llll} & V_j & \text{1 row} & \\ (4.8) & \gamma_j^s - x_h^s u_{hj} & \text{2 rows} & (s = 1, 2) \\ & \bar{y}_i'^s \gamma_j^s + \eta^s_{ij} - y^s_{ih} u_{hj} & 2n \text{ rows} & (s = 1, 2;\ i = 1, \cdots, n). \end{array}$$

Let us use the notation $\bar{\gamma}_j^s$, $\bar{\eta}_{ij}^s$ for the jth set of terminally admissible variations determined, in the sense of § 3, by the jth set of constants u_{hj}, in contradistinction to the notation γ_j^s, η_{ij}^s for the set of end values of the jth set of differentially admissible variations. Then by means of (3.6), the column (4.8) may be written as

$$\begin{array}{lll} & V_j & \text{1 row} \\ (4.8a) & \gamma_j^s - \bar{\gamma}_j^s & \text{2 rows} \\ & \bar{y}_i'^s(\gamma_j^s - \bar{\gamma}_j^s) + (\eta^s_{ij} - \bar{\eta}^s_{ij}). & 2n \text{ rows} \end{array}$$

We can simplify these columns by multiplying the second and third rows respectively by $\bar{y}_i'^s$ and substracting from the last $2n$ rows.

We thus obtain the necessary condition

$$
(4.9)\qquad
\begin{vmatrix}
\Gamma_2 \cdots\cdots\cdots\cdots\cdots\cdots \Gamma_{2n+3} \\
\gamma_1{}^1 - \bar\gamma_1{}^1 \cdots\cdots \gamma^1{}_{2n+3} - \bar\gamma^1{}_{2n+3} \\
\gamma_1{}^2 - \bar\gamma_1{}^2 \cdots\cdots \gamma^2{}_{2n+3} - \bar\gamma^2{}_{2n+3} \\
\eta_i{}^1{}_{11} - \eta_i{}^1{}_{11} \cdots\cdots \eta_i{}^1{}_{1\,2n+3} - \bar\eta_i{}^1{}_{1\,2n+3} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\eta_i{}^1{}_{n1} - \bar\eta_i{}^1{}_{n1} \cdots\cdots \eta_i{}^1{}_{n\,2n+3} - \bar\eta_i{}^1{}_{n\,2n+3} \\
\eta_i{}^2{}_{11} - \bar\eta_i{}^2{}_{11} \cdots\cdots \eta_i{}^2{}_{1\,2n+3} - \bar\eta_i{}^2{}_{1\,2n+3} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\eta_i{}^2{}_{n1} - \bar\eta_i{}^2{}_{n1} \cdots\cdots \eta_i{}^2{}_{n\,2n+3} - \bar\eta_i{}^2{}_{n\,2n+3}
\end{vmatrix} = 0.
$$

Here $\gamma_j{}^s$, $\eta_{ij}{}^s$ are the end values of a set of differentially admissible variations, otherwise unrestricted, and $\bar\gamma_j{}^s$, $\bar\eta_{ij}{}^s$ is any set of terminally admissible variations.

Hence for any particular choice of $2n + 3$ sets of differentially admissible variations and of $2n + 3$ sets of constants u_{hj} $(j = 1, \cdots, 2n + 3)$, there must exist $2n + 3$ constants $(\lambda_0, b^s, k_i{}^s)$ not all zero such that the equation

$$(4.10)\qquad \lambda_0\Gamma(\gamma, \eta_i, u) + b^s(\gamma^s - \bar\gamma^s) + k_i{}^s(\eta_i{}^s - \bar\eta_i{}^s) = 0$$

is satisfied for every set γ^s, $\eta_i(x)$ of the chosen sets of differentially admissible variations, and for every set u_h of the chosen sets of constants u_{hj}. Here $\bar\gamma^s$, $\bar\eta_i{}^s$ is the set of terminally admissible variations determined by u_h.

In fact, if we choose the $2n + 3$ constants $(\lambda_0, b^s, k_i{}^s)$ corresponding to the choice of $\gamma_j{}^s$, $\eta_{ij}(x)$, u_{hj} which gives the determinant (4.9) its highest rank ρ, then (4.10) must be satisfied for any differentially admissible variations and any set of numbers u_h whatever, $\bar\gamma^s$, $\bar\eta_i{}^s$ being the terminally admissible variations determined by u_h. For otherwise one of the columns of (4.9) could be replaced by another making the determinant have the rank $\rho + 1$.

(a) *We shall now obtain (4.17) and (4.19) as consequences of (4.10). We call attention to the fact that the derivation of (4.17) and (4.19) from (4.10) will hold even if E is not a minimizing arc, in fact if E is simply an arbitrary admissible arc for which (4.10) holds as stated.*

If we substitute in (4.10) the value of $\lambda_0\Gamma(\gamma, \eta, u)$ given by (4.5), we obtain the equation

$$(4.11)\qquad -\int_{a^1}^{a^2} \lambda_r \zeta_r dx + (\lambda_0 f^s \gamma^s + \eta_i{}^s F_{y_i'}{}^s)_1^2 + \lambda_0 \theta_h u_h + b^s(\gamma^s - \bar{\gamma}^s) + k_i{}^s(\eta_i{}^s - \bar{\eta}_i{}^s) = 0.$$

This equation must be satisfied for every choice of differentially admissible variations γ^s, $\eta_i(x)$; that is, for arbitrary γ^s, ζ_r, and $\eta_i{}^2$. The values of $F_{y_i'}{}^1$ are also arbitrary. Furthermore, u_h is any set of r numbers, and $\bar{\gamma}^s$, $\bar{\eta}_i{}^s$ is the set of terminally admissible variations determined by u_h.

We can rewrite (4.11) as

$$(4.12)\qquad -\int_{a^1}^{a^2} \lambda_r \zeta_r dx + \gamma^s[(-1)^s\lambda_0 f^s + b^s] + \eta_i{}^s[(-1)^s F_{y_i'}{}^s + k_i{}^s] - b^s\bar{\gamma}^s - k_i{}^s\bar{\eta}_i{}^s + \lambda_0\theta_h u_h = 0.$$

Since the value of $F_{y'_i}{}^1$ is arbitrary, we can choose

$$(4.13)\qquad F_{y_i'}{}^1 = k_i{}^1$$

If we choose

$$\gamma^s = \eta_i{}^2 = u_h = 0,$$

thus determining $\bar{\gamma}^s$ and $\bar{\eta}_i{}^s$ as zero, we find that since ζ_r is arbitrary, we must have the identities $\lambda_{m+1} \equiv \cdots \equiv \lambda_n \equiv 0$. Since γ^2, γ^1, and $\eta_i{}^2$ are arbitrary, we find that

$$(4.14)\qquad b^2 = -\lambda_0 f^2 = -F^2 \qquad b^1 = \lambda_0 f^1 = F^1 \qquad k_i{}^2 = -F_{y_i'}{}^2.$$

Substituting these values for b^s, $k_i{}^s$ and setting $\lambda_r = 0$, (4.12) becomes

$$(4.15)\qquad (F^s\bar{\gamma}^s + F_{y_i'}{}^s\bar{\eta}_i{}^s)_1^2 + \lambda_0\theta_h u_h = 0.$$

Recalling (3.6), we can write

$$\bar{\gamma}^s = dx^s \qquad \bar{\eta}_i{}^s = dy_i{}^s - \bar{y}_i{}'^s dx^s \qquad \theta_h u_h = d\theta$$

From (4.15) we now obtain the "transversality" condition* in the form of the following identity in the differentials $d\alpha_h$:

$$(4.16)\qquad [F^s dx^s + F_{y_i'}{}^s(dy_i{}^s - \bar{y}_i{}'^s dx^s)]_1^2 + \lambda_0 d\theta = 0$$

---

* The transversality condition given by Bliss in his lectures, loc. cit. p. 21, can be simply derived from this condition, but this condition, if derivable at all from the Bliss form, would clearly be so derivable only after complicated transformations. This is partly due to the presence of θ, and partly to the greater generality of the end conditions.

where dx^s, dy_i^s, and $d\theta$ are understood as evaluated for $(\alpha) = (0)$, and expressed in terms of the differentials $d\alpha_h$.

The multipliers λ_0, $\lambda_\beta(x)$ cannot all vanish at any one point of (a^1a^2). For if $\lambda_0 = 0$, the differential equations which the v's satisfy are homogeneous, and hence if the v's are zero for any one value of x, they are identically zero on the interval between corners containing that value of x. Thus they are identically zero on (a^1a^2). But this makes the constants λ_0, b^s, k_i^s all zero, as follows from (4.13) and (4.14), and so is impossible.

A set of multipliers λ_0, $\lambda_\beta(x)$, where λ_0 is a constant and $\lambda_1(x)$, $\cdots$, $\lambda_m(x)$ are functions of x which are continuous except possibly at the corners, and where λ_0, $\lambda_\beta(x)$ are not all zero at any one point of (a^1a^2), will be called an *admissible* set of multipliers.

The results just attained may be summarized in the following theorem:

THEOREM 2. *If E affords a minimum in the problem, there exists a set of constants $c_1, \cdots, c_n$ and an admissible set of multipliers λ_0, $\lambda_1(x)$, $\cdots$, $\lambda_m(x)$ such that the equations*

$$F_{y_i'} = \int_{a^1}^{x} F_{y_i}dx + c_i \tag{4.17} \qquad A$$

are satisfied at every point of E, where

$$F(x, y, y', \lambda) = \lambda_0 f + \lambda_\beta \phi_\beta. \tag{4.18}$$

and such that the following identity

$$[(F^s - \bar{y}_i'^s F_{y_i'}{}^s)dx^s + F_{y_i'}{}^s dy_i{}^s]_1^2 + \lambda_0 d\theta = 0 \tag{4.19} \qquad B$$

in $d\alpha_h$ is satisfied when dx^s, dy_i^s, and $d\theta$ are evaluated for $(\alpha) = (0)$ and expressed in terms of the differentials $d\alpha_h$.

An immediate consequence of this theorem is the so-called Euler-Lagrange Multiplier Rule:

On every sub-arc between corners of a minimizing arc the equations

$$\frac{d}{dx}F_{y_i'} = F_{y_i} \qquad \phi_\beta(x, y, y') = 0 \tag{4.20}$$

must be satisfied, where F is the function (4.18).

5. **Relative normalcy. A new criterion.** An admissible arc E shall be said to be *normal relative to conditions A and B of Theorem*

2, or, more shortly, *normal* (AB),* if it does not satisfy these conditions with any admissible set of multipliers in which $\lambda_0 = 0$.

A differentially admissible arc with an admissible set of multipliers is called an *extremaloid* if it satisfies equations (4.17) with some set of constants $c_1, \cdots, c_n$.

THEOREM 3. *For an extremaloid E which is normal (AB) and which satisfies A and B with an admissible set of multipliers, there always exist admissible multipliers in the form $\lambda_0 = 1, \lambda_\beta(x)$ with which E satisfies conditions A and B. In this form the multipliers are unique.*

We have multipliers in the form $\lambda_0, \lambda_\beta(x)$ where $\lambda_0 \neq 0$, and we can evidently divide them by λ_0 and obtain a set with $\lambda_0 = 1$. If we had two sets with $\lambda_0 = 1$, then the difference of the sets would be a set of multipliers with $\lambda_0 = 0$. This is impossible unless the two sets are identical, if the extremaloid is normal (AB). Thus the multipliers $\lambda_0 = 1, \lambda_\beta(x)$ are unique in this form.

Let us now consider the determinant

$$\begin{vmatrix} \gamma_1^1 - \bar{\gamma}_1^1 \cdots\cdots\cdots \gamma^1_{2n+2} - \bar{\gamma}^1_{2n+2} \\ \gamma_1^2 - \bar{\gamma}_1^2 \cdots\cdots\cdots \gamma^2_{2n+2} - \bar{\gamma}^2_{2n+2} \\ \eta^1_{11} - \bar{\eta}^1_{11} \cdots\cdots \eta^1_{1\,2n+2} - \bar{\eta}^1_{1\,2n+2} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \eta^1_{n1} - \bar{\eta}^1_{n1} \cdots\cdots \eta^1_{n\,2n+2} - \bar{\eta}^1_{n\,2n+2} \\ \eta^2_{11} - \bar{\eta}^2_{11} \cdots\cdots \eta^2_{1\,2n+2} - \bar{\eta}^2_{1\,2n+2} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \eta^2_{n1} - \bar{\eta}^2_{n1} \cdots\cdots \eta^2_{n\,2n+2} - \bar{\eta}^2_{n\,2n+2} \end{vmatrix} \tag{5.1}$$

formed from $2n + 2$ sets of differentially admissible variations

$$\gamma_j^s, \eta_{ij}(x) \qquad (i = 1, \cdots, n) \quad (j = 1, \cdots, 2n + 2; s = 1, 2)$$

and $2n + 2$ sets of terminally admissable variations

$$\bar{\gamma}_j^s, \bar{\eta}^s_{ij} \qquad (i = 1, \cdots, n; j = 1, \cdots, 2n + 2; s = 1, 2)$$

---

* We say *normal* (AB) rather than simply *normal* for the following reason. Morse has shown that Theorem 2 holds with the following condition C added at each corner $x = c$,

$$[F - y_i' F_{y_i'}]_{c+}^{c-} = 0 \qquad C$$

and that moreover the multipliers in this new theorem do not always agree with those in the old theorem. He has shown that an admissible arc may be normal (ABC)—that is, relative to conditions A, B, and C—while anormal (AB).

and the determinant

$$
(5.2)\qquad
\left|\begin{array}{l}
\eta^1_{11} - \bar{\eta}^1_{11} \cdots \eta^1_{1\,2n} - \bar{\eta}^1_{1\,2n} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\eta^1_{n1} - \bar{\eta}^1_{n1} \cdots \eta^1_{n\,2n} - \bar{\eta}^1_{n\,2n} \\
\eta^2_{11} - \bar{\eta}^2_{11} \cdots \eta^2_{1\,2n} - \bar{\eta}^2_{1\,2n} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\eta^2_{n1} - \bar{\eta}^2_{n1} \cdots \eta^2_{n\,2n} - \bar{\eta}^2_{n\,2n}
\end{array}\right|
$$

formed from $2n$ sets of differentially admissible variations

$$\gamma_k{}^s,\ \eta_{ik}(x) \qquad (i = 2, \cdots, n;\ k = 1, \cdots, 2n;\ s = 1, 2)$$

and $2n$ sets of terminally admissible variations

$$\bar{\gamma}_k{}^s,\ \bar{\eta}^s{}_{ik} \qquad (i = 1, \cdots, n;\ k = 1, \cdots, 2n;\ s = 1, 2).$$

We will now prove the following lemma:

LEMMA. *A necessary and sufficient condition that there exist a determinant of form* (5.1) *different from zero is that there exist a determinant of form* (5.2) *different from zero.*

The sufficiency of this condition is proved by showing that if there exists a determinant of form (5.2) different from zero, then we can construct a determinant (5.1) different from zero.

In fact, if a_{pq} is the general element of the determinant (5.2), by choosing the arbitrary constants $\gamma_j{}^s$ of (5.1) suitably we obtain a determinant of the form

$$
(5.3)\qquad
\left|\begin{array}{c|cc}
0\cdots\cdots 0 & 0 & a \\
0\cdots\cdots 0 & b & 0 \\
\hline
 & & \\
a_{pq} & & \\
 & &
\end{array}\right|
\qquad a \neq 0,\ b \neq 0
$$

different from zero.

Conversely, if there exists a determinant of form (5.1) not zero, then determinants of form (5.2) obviously cannot all be zero. Thus the condition is also necessary.

The following theorem relates to the differential equations (3.3) alone, and is independent of whether E is or is not a minimizing arc.

THEOREM 4. *A necessary and sufficient condition that an admissible arc E be normal (AB) is that there exist for it a determinant of form* (5.1) *different from zero.*

If all determinants of form (5.1) vanished, there would clearly be a set of constants $\lambda_0 = 0$, b^r, k_i^s satisfying (4.10) for any sets of differentially and terminally admissible variations. But as pointed out in statement (a) of § 4, conditions A and B on E would then follow as consequences of (4.10), λ_0 being zero in this case. Thus E is anormal (AB) if all determinants of form (5.1) vanish, and the condition is proved necessary.

Now suppose that the arc E were anormal (AB). We shall show that there exists no determinant of form (5.1) different from zero.

Suppose that there were a determinant of form (5.1) *different from zero.* Then there would also be a determinant of form (5.2) different from zero. By the definition of normalcy, conditions A and B would be satisfied by E with a set of multipliers in which $\lambda_0 = 0$. The function F in (4.17) and (4.19) would then have the form

$$F = \lambda_\beta \phi_\beta \tag{5.4}$$

Every set of differentially admissible variations would satisfy the equations

$$0 = \int_{a^1}^{a^2} \lambda_\beta \Phi_\beta dx = \int_{a^1}^{a^2} (F_{y_i}\eta_i + F_{y_i'}\eta_i')dx. \tag{5.5}$$

Upon integration by parts, and application of (4.17), we have

$$(F_{y_i'}^{s}\eta_i^{s})_1^2 = 0. \tag{5.6}$$

Since we are assuming $\lambda_0 = 0$, and since $\phi_\beta = 0$ along E, we have $F^s = 0$. We then see from the transversality condition (4.16) that

$$(F_{y_i'}^{s}\bar{\eta}_i^{s})_1^2 = 0 \tag{5.7}$$

and this would hold for any terminally admissible variations $\bar{\eta}_i^s$. Combining (5.6) and (5.7), we would obtain the equation

$$[F_{y_i'}^{s}(\eta_i^s - \bar{\eta}_i^s)]_1^2 = 0 \tag{5.8}$$

But we have a determinant of form (5.2) different from zero, so that from (5.8) we deduce the equations

$$F_{y_i'}^{s} = 0 \qquad (s = 1, 2;\ i = 1, \cdots, n). \tag{5.9}$$

But equations (5.9) would imply that $\lambda_\beta(a^s) = 0$, which is contrary to our hypothesis that $\lambda_0, \lambda_\beta(x)$ form an admissible set of multipliers. Thus there cannot be a determinant of form (5.1) different from zero if E is anormal (AB). This proves the sufficiency of the condition.

COROLLARY. *A necessary and sufficient condition that an admissible arc E be normal (AB) is that there exist for it a determinant of form (5.2) different from zero.*

THEOREM 5. *If a set of differentially admissible variations $\gamma^s, \eta_i(x)$ for an admissible arc E which is normal (AB) satisfies the terminal equations of variation with a set (u), then there exists a one-parameter family of admissible arcs of the form (3.7) containing E for the parameter value $t = 0$ and having $\gamma^s, \eta_i(x), u_h$ as the variations of x^s, y_i, and α_h.*

According to the preceding corollary we have a determinant of form (5.1) different from zero, with its $2n + 2$ sets of differentially admissible variations

$$\gamma_j^s, \eta_{ij} \qquad (j = 1, \cdots, 2n + 2)$$

and its $2n + 2$ sets of terminally admissible variations

$$\bar{\gamma}_j^s, \bar{\eta}_{ij} \qquad (j = 1, \cdots, 2n + 2) \tag{5.10}$$

Let

$$u_{hj} \qquad (h = 1, \cdots, r; j = 1, \cdots, 2n + 2)$$

be constants of which the jth set determines the jth set in (5.10), in the sense of § 3.

There exists a $(2n + 3)$-parameter family of differentially admissible arcs

$$y_i = y_i(x, t, e) \qquad x_0^1(t, e) \leqq x \leqq x_0^2(t, e) \tag{5.11}$$

with parameters t and $(e) = (e_1, \cdots, e_{2n+2})$, containing E for the parameter values $(t, e) = (0, 0)$ and having the sets $\gamma^s, \eta_i(x)$ and $\gamma_j^s, \eta_{ij}(x)$ as its variations along E with respect to t and e_j respectively.

Let us take the parameters α_h of (2.4) as follows:

$$\alpha_h = tu_h + e_j u_{hj} \qquad (h = 1, \cdots, r; j = 1, \cdots, 2n + 2).$$

Then the equations

$$\begin{aligned} x_0^s(t, e) - x^s(tu_1 + e_j u_{1j}, \cdots, tu_r + e_j u_{rj}) &= 0 \\ y_i[x_0^s(t, e), t, e] - y_i^s(tu_1 + e_j u_{1j}, \cdots, tu_r + e_j u_{rj}) &= 0, \end{aligned} \tag{5.12}$$

which are conditions for terminal admissibility, have the initial solution $(t, e) = (0, 0)$, at which the jacobian with respect to (e) is the non-vanishing determinant (5.1), as is seen by comparison with (4.7), (4.8) and (4.9). Hence equations (5.12) have solutions

$$e_j = e_j(t) \qquad (j = 1, \cdots, 2n + 2) \tag{5.13}$$

with initial values $e_j(0) = 0$. The one-parameter family

$$y_i = y_i[x, t, e(t)] \qquad x_0^1[t, e(t)] \leqq x \leqq x_0^2[t, e(t)] \tag{5.14}$$

consists of admissible arcs, and contains E for $t = 0$.

Let us compute the derivatives $e_j'(0)$. From the choice of γ^s, $\eta_i(x)$ and u_h we see that the partial derivatives of the left hand sides of (5.12) with respect to t are zero for $t = 0$, and therefore $e_j'(0) = 0$. We see then that the variations of the family (5.14) with respect to t are γ^s, $\eta_i(x)$, u_h, as was to be proved.

6. **Geometric criteria for normalcy (AB).** Let γ^s, η_i^s, be the end-values of a set of differentially admissible variations. Then we will consider the $2n + 2$ numbers γ^s, η_i^s, as the coördinates of a point in $(2n + 2)$-dimensional space. Such a point will be called a *differentially admissible point*. Similarly, consider any set of terminally admissible variations $\bar{\gamma}^s, \bar{\eta}_i^s$ as the coördinates of a point in $(2n + 2)$-space. Such a point will be called a *terminally admissible point*. The following theorem furnishes a useful geometrical interpretation of normalcy (AB).

THEOREM 6. *A necessary and sufficient condition that an admissible arc E be normal (AB) is that the family of $(2n + 2)$-dimensional points linearly dependent on the differentially admissible points and the terminally admissible points fill out the whole of $(2n + 2)$-space.*

For if an admissible arc is normal (AB) there exists a determinant of form (5.1) different from zero. The columns of this determinant are $2n + 2$ linearly independent points of the family; hence the family fills out $(2n + 2)$-space.

Now suppose that the family fills out $(2n + 2)$-space. Then there exist $2n + 2$ linearly independent points in the family. But any point of the family may be expressed as the difference of a differentially admissible point and a terminally admissible point because of the linear homogeneity of the differential and terminal equations of variation. Therefore there exists a determinant of form (5.1) not zero, and the arc E is normal AB.

In a similar manner the following theorem can be proved:

THEOREM 7. *A necessary and sufficient condition that an admissible arc be normal (AB) is that the family of 2n-dimensional points linearly dependent on the points in 2n-space with coördinates* η_i^s *and the points in 2n-space with coördinates* $\bar{\eta}_i^s$ *fill out the whole of 2n-space. Here the set* η_i^s *consists of the end values of differentially admissible variations* η_i, *and the set* $\bar{\eta}_i^s$ *consists of terminally admissible variations.*

The following test for normalcy (AB) is useful:

THEOREM 8. *Let* ρ *be the highest rank attained by any determinant consisting of* $2n + 2$ *columns of differentially admissible points in* $(2n + 2)$*-space. The number of parameters* (α) *in the terminal conditions* (2.4) *cannot be less than* $2n + 2 - \rho$ *without E being anormal (AB). One can, however, always choose terminal conditions of the form* (2.4) *with no more than* $2n + 2 - \rho$ *parameters such that E is normal (AB).*

The differentially admissible points fill out a ρ-plane, and the terminally admissible points must fill out at least the $(2n+2-\rho)$-plane orthogonal to the ρ-plane at the origin if E is to be normal (AB). This latter plane is certainly not filled out if the number of parameters is less than $2n + 2 - \rho$. This proves the first part of the theorem.

Now the $(2n + 2 - \rho)$-plane which must be filled out by the differentially admissible points if the arc E is to be normal (AB) may be represented in the following form, using $(\bar{\gamma}^s, \bar{\eta}_i^s)$ as coördinates in $(2n + 2)$-space:

$$\begin{aligned} \bar{\gamma}^s &= c_h^s u_h \\ \bar{\eta}_i^s &= c_{ih}^s u_h. \end{aligned} \tag{6.1}$$

The matrix $\left\| \begin{matrix} c_h^s \\ c_{ih}^s \end{matrix} \right\|$ is to be of rank $2n + 2 - \rho$. A possible choice of terminal conditions which will produce (6.1) as the terminally admissible points is the following: (cf. (3.6))

$$\begin{aligned} x^s - a^s &= c_h^s \alpha_h \qquad (h = 1, \cdots, 2n + 2 - \rho) \\ y_i^s - b_i^s &= (\bar{y}_i'^{s} c_h^s + c^s_{ih})\alpha_h \end{aligned} \tag{6.2}$$

where (a^s, b_i^s) are the coördinates of the end points of E.

A theorem similar to the last one can be obtained by replacing $2n + 2 - \rho$ by $2n - \sigma$, where σ is the highest rank attained by any determinant of the form

$$(6.3)\qquad \begin{vmatrix} \gamma^1_{11} \cdots \cdots \gamma^1_{1\,2n} \\ \cdots\cdots\cdots\cdots \\ \gamma^1_{n1} \cdots \cdots \gamma^1_{n\,2n} \\ \gamma^2_{11} \cdots \cdots \gamma^2_{1\,2n} \\ \cdots\cdots\cdots\cdots \\ \gamma^2_{n1} \cdots \cdots \gamma^2_{n\,2n} \end{vmatrix} .$$

A differentially admissible arc shall be said to be *normal* (A) if it does not satisfy condition A of Theorem 2 with any admissible set of multipliers in which $\lambda_0 = 0$.

THEOREM 10.* *If a differentially admissible arc E is normal (A), there exists a family of differentially admissible arcs joining the points (x^1, y^1) and (x^2, y^2) neighboring the end points of E and representable in the form*

$$(6.4)\qquad y_i = Y_i(x, x^1, y^1, x^2, y^2)$$

where the functions Y_i are of class C'.

Since E is normal (A), it is normal (AB) if we choose the end-conditions so as to make the end points coincide with those of E. Hence, according to the corollary to Theorem 4, there exists a determinant of end values of differentially admissible variations of the form

$$(6.5)\qquad |\gamma^s_{ik}| \qquad (i = 1, \cdots, n;\ k = 1, \cdots, 2n)$$

different from zero.

Now let us use the corollary of § 3 to set up a 2n-parameter family of differentially admissible arcs

$$(6.6)\qquad y_i = y_i(x, e) \qquad (e) = (e_1, \cdots, e_{2n})$$

whose variations with respect to e_k are precisely the variations in the k-th column of (6.5) and which contains the arc E for $(e) = (0)$. We then seek to satisfy the conditions

$$(6.7)\qquad y_i^s = y_i(x^s, e).$$

These conditions are satisfied by the end points of E and $(e) = (0)$, and the jacobian of the right hand members with respect to (e) is the determinant (6.5), different from zero. We can accordingly solve

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* cf. Bliss, Lectures, loc. cit. p. 50.

(6.7) for (c), substitute the result in (6.6), and obtain the desired family.

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COMPRESSIBILITY AND PRESSURE COEFFICIENT OF RESISTANCE, INCLUDING SINGLE CRYSTAL MAGNESIUM.

BY P. W. BRIDGMAN.

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INTRODUCTION.

DURING the last year measurements of compressibility have been extended to ten new materials; the effect of pressure on the electrical resistance of two of these has also been measured. The methods are the same as those already described in sufficient detail.[1] All of the materials described in this paper, except metallic magnesium, crystallize in the cubic system, so that measurements on crystalline aggregates give as much information as measurements on single crystals. Magnesium on the other hand, crystallizes in the hexagonal system; single crystals of this were prepared, the compressibility was measured parallel and perpendicular to the axis, and the specific resistance in the same directions was determined.

DETAILED DATA.

NaF. The source of this material was powdered NaF from Kahlbaum. In order to get it into shape for the compressibility measurements it was compressed into a massive slug at a temperature of 580° C in an arbor press to a pressure of perhaps 10000 or 15000 kg/cm². The specimen was only 0.28 cm. long, so that the relative accuracy of the results was less than usual. Regular measurements were made at 30° and 75°; the average arithmetical deviation of a single one of the 24 observed points from a smooth curve was 1.14% and 1.55% at 30° and 75° respectively. The following are the results calculated for the volume change from the measured change of length:

$$\left.\begin{aligned}&\text{At } 30°, \quad -\Delta V/V_0 = 20.7 \times 10^{-7}p - 17.7 \times 10^{-12}p^2\\&\text{At } 75°, \quad -\Delta V/V_0 = 20.8 \times 10^{-7}p - 18.1 \times 10^{-12}p^2\end{aligned}\right\} p \text{ in kg/cm}^2.$$

BaF_2. This was prepared in the same way as NaF, by compressing Kahlbaum's powdered BaF_2 at a temperature of 510° C in an arbor press. Attempts to melt it resulted in decomposition. The length of the slug so produced was about 2.1 cm. After the two compressibility runs the slug was found to be darkly discolored throughout the entire interior, and there was a permanent decrease of length of about 0.0025 cm. Whatever the nature of these effects, they were probably too small to materially affect the accuracy of the compressibility, which is known not to be sensitive to impurity.

At 30° the average departure from a smooth curve of a single one of the 14 readings was 0.64% of the maximum effect, and at 75° the average departure of a single one of the 13 readings, making one discard, was 0.49%. The following results for volume compressibility were calculated from the change of length:

$$\text{At } 30°, \quad -\Delta V/V_0 = 19.3 \times 10^{-7}p - 14.8 \times 10^{-12}p^2,$$

$$\text{At } 75°, \quad -\Delta V/V_0 = 19.6 \times 10^{-7}p - 14.6 \times 10^{-12}p^2.$$

SrF_2. This was prepared from Kahlbaum's powdered material by compressing at 580° in the arbor press. Two cylindrical pieces were made, 0.62 and 0.86 cm. long, which were piled together for measurement. There was no detectible change, either of length or appearance, produced in these specimens by the application of hydrostatic pressure. The average deviations from smooth curves of single readings at 30° and 75° were 0.70% and 0.83% on 14 and 12 readings respectively. The results for change of volume are:

At 30°, $-\Delta V/V_0 = 15.8 \times 10^{-7}p - 10.3 \times 10^{-12}p^2$,

At 75°, $-\Delta V/V_0 = 16.1 \times 10^{-7}p - 10.8 \times 10^{-12}p^2$.

CdF_2. The material was powdered CdF_2 from Kahlbaum, compressed at 560° in the arbor press. It was, however, much more difficult to compact this into a uniform coherent mass, and a number of attempts had to be made. In order to get some idea of the possible error from imperfectly massive specimens, measurements were made on two different specimens. One of these was quite friable, and could easily be scratched to a powder with the point of a knife. The other was harder, and suggested no difficulties of a mechanical character. The length of the specimens was about 0.4 cm.; neither showed any appreciable change of dimensions after exposure to pressure. However, during the measurements on the softer specimen, there was an explosion due to the stem of the insulating plug blowing out, and after this there were large irregularities, so that apart from the softness of the specimen, the measurements with it could not be expected to be as good as those with the harder specimen. The compressibility calculated from the data obtained with the softer specimen was about 2.5% less than that with the harder specimen. The average deviations from smooth curves of single readings on the harder specimen were 1.2% and 0.9% at 30° and 75° respectively, on totals of 14 and 13 readings. The change of volume of the harder specimen was:

At 30°, $-\Delta V/V_0 = 11.02 \times 10^{-7}p - 8.5 \times 10^{-12}p^2$,

At 75°, $-\Delta V/V_0 = 10.96 \times 10^{-7}p - 8.4 \times 10^{-12}p^2$.

The temperature coefficient shown by these results is very uncertain. The results for either temperature by itself would not justify the four significant figures retained, which are given only to permit an estimate of the order of magnitude of the temperature effect.

AlSb. This intermetallic compound has been shown to be cubic by the measurements of Owen and Preston and of Goldschmidt.[2] The specimen was prepared for the compressibility measurements by fusing together in vacuum in a closed iron container weighed amounts of the pure metals, the antimony being Kahlbaum's purest, and the aluminum some specially pure material obtained from the Aluminum Co. of America which I had used in previous work.[3] In order to make the product as homogeneous as possible, it was heated and cooled several times. The specimen was about 1.5 cm. long. The grinding operation by which the specimen was fitted to the compressibility apparatus was not as successful as usual, there probably being some rocking on the end pieces. In consequence the specimen could assume either one of two positions, so that the resulting points lay on two curves instead of one. This sort of behavior has been found before. There need be no error in the final results if enough measurements are taken to give a considerable number of points on both of the curves. At 30° 8 such points were obtained, and at 75° 12 pertaining to the same curve; the average departures of a single one of these points from a smooth curve was 0.64% and 0.48% respectively. The results for volume compressibility were as follows:

$$\text{At } 30°, \quad -\Delta V/V_0 = 18.02 \times 10^{-7}p - 19.0 \times 10^{-12}p^2,$$

$$\text{At } 75°, \quad -\Delta V/V_0 = 20.53 \times 10^{-7}p - 26.9 \times 10^{-12}p^2.$$

CdTe. This has also been shown by Zachariasen[4] to crystallize in the cubic system. It was prepared by melting together in a closed quartz tube weighed amounts of Kahlbaum's Cd and Te, the latter further purified by my method of making single crystals. The specimen, which was about 0.8 cm. long, showed, when mounted in the compressibility apparatus, to a slight extent the same positional instability as the AlSb, it being necessary to discard four of the observed points at 30°; at 75° none were discarded. There was a permanent decrease of length of 0.006 cm. after exposure to hydrostatic pressure; however, a seasoning application was made before the regular runs, so that no error should have been introduced by this effect. At 30° the average departure from a smooth curve of a single one of the 7 readings that were retained was 0.43% of the maximum effect, and at 75° the corresponding departure was 1.2% on 10 readings. The change of volume calculated from the results was:

$$\text{At } 30°, \quad -\Delta V/V_0 = 23.3 \times 10^{-7}p - 12.2 \times 10^{-12}p^2,$$

$$\text{At } 75°, \quad -\Delta V/V_0 = 23.5 \times 10^{-7}p - 11.6 \times 10^{-12}p^2.$$

HgTe. The measurements of Goldschmidt[5] have established that this also is cubic. It was prepared by heating together in a closed quartz tube weighed amounts of Kahlbaum's Te (the same as that used in preparing CdTe), and pure redistilled Hg from the laboratory stock. The specimen was about 1.1 cm. long; there was no perceptible change of length after exposure to pressure. The measurements on this material were more irregular than for any other substance, and in fact were so irregular that it was useless to try to get from them the temperature coefficient of compressibility, but the observations at both temperatures were treated together to get from them the best single smooth curve. The average deviation from a smooth curve of a single one of the 26 observations was 2.1%. The following results were found for the volume compressibility:

$$\text{Mean, } 30^\circ \text{ and } 75^\circ, \quad -\Delta V/V_0 = 19.8 \times 10^{-7}p - 25 \times 10^{-12}p^2.$$

TiN. This material I owe to the kindness of Dr. van Arkel of the Philips Lamp Works, Eindhoven, Holland. It was prepared in the form of a slender rod about 1.0 mm. in diameter by the method of vacuum deposition which he has described.[6] The specimen was not a single grain, but the material is cubic, so that a single grain was not necessary.

Compressibility. The compressibility sample was 1.6 cm. long. Measurements were made as usual at 30° and 75°. This material is less compressible than iron, so that the slope of the curve was abnormal, and furthermore the effect is so small that no departure of the observed points from linearity could be established. At each temperature 13 observations were made; at 30° the average departure of a single point from a smooth curve was 1.2% and at 75° 1.1%. The final results are:

$$\text{At } 30^\circ, \quad -\Delta V/V_0 = 3.32 \times 10^{-7}p - 2.13 \times 10^{-12}p^2,$$

$$\text{At } 75^\circ, \quad -\Delta V/V_0 = 3.51 \times 10^{-7}p - 2.13 \times 10^{-12}p^2.$$

The second degree term in these expressions is contributed by the absolute compressibility of the iron, there being no appreciable departure of the differential compressibility from linearity.

Pressure Coefficient of Resistance. The same specimen was used for the resistance and the compressibility measurements. The potentiometer method was used, connections being made by spring clips, since it is not possible to solder this material. In addition to measurements under pressure, the specific resistance and temperature coefficient of resistance were measured at atmospheric pressure. At 30° at atmospheric pressure the specific resistance was 28.1×10^{-6} ohms per cm. cube. The temperature coefficient at 0° C at atmospheric pressure, obtained by linear extrapolation of measurements at 30° and 75°, was 0.00221.

Pressure measurements were made as usual at 30° and 75°. The effect is unusually small; this, combined with the smallness of the absolute resistance of the specimen, made the percentage irregularity in the results larger than usual. At 30° the average deviation from a straight line of a single one of the 11 readings was 1.2%, and at 75° the corresponding figure was 2.2% on 14 readings. The total displacement of the potentiometer slider for 12000 kg was 1 cm., so that an irregularity of 1% corresponds to 0.1 mm. setting of the slider, indicating that the readings were about as regular as the construction of the apparatus would allow. There was no appreciable departure from linearity at either temperature. The results follow:

Average coefficient to 12000 kg at 30°, -9.35×10^{-7}.

Average coefficient to 12000 kg at 75°, -9.97×10^{-7}.

The most important feature of the results is that the sign is negative, as it is for the majority of the pure metals.

TiC. This also I owe to the kindness of Dr. van Arkel. It was prepared by the same method as TiN; it also is cubic so that a single crystal was not necessary.

Compressibility. The sample was 2.5 cm. long, and 1.5 mm. in diameter; no perceptible alteration of dimensions as produced by pressure. This, like TiN, is less compressible than iron, and again it was not possible to detect any deviation from linearity of the observed differential compressibility. The average deviation from a straight line of a single one of the 13 observed points at 30° was 0.32%, and at 75° 0.41% for 11 points. The results are:

$$\text{At } 30^\circ, \quad -\Delta V/V_0 = 4.72 \times 10^{-7}p - 2.16 \times 10^{-12}p^2,$$

$$\text{At } 75^\circ, \quad -\Delta V/V_0 = 4.78 \times 10^{-7}p - 2.19 \times 10^{-12}p^2.$$

Pressure Coefficient of Resistance. The various details in measuring the resistance were the same as with TiN. The specific resistance at atmospheric pressure at 30° was found to be 158.4×10^{-6}. The temperature coefficient at atmospheric pressure at 0° C, by linear extrapolation of measurements at 30° and 75°, was + 0.000235.

The pressure measurements showed no appreciable departure from linearity. At 30° the average departure from a smooth curve of a single one of the 15 readings was 1.3%, and at 75° the corresponding deviation was 0.7% on 14 readings. The results are:

Average coefficient to 12000 at 30°, -1.375×10^{-6},

Average coefficient to 12000 at 75°, -1.35×10^{-6}.

Again the coefficient is negative and about 50% larger numerically than for TiC. This is to be contrasted with a specific resistance 5.6 fold larger.

Mg. This material crystallizes in the hexagonal system, so that measurements, to be significant, must be made on single crystals. As far as I can find in the literature, single crystals of magnesium have not been hitherto studied. It proved, however, extremely easy to make this in single crystal form by my method of slow lowering of the molten metal from a furnace.[7] For this purpose a special vacuum furnace was constructed, in which a vacuum of the order of 0.01 mm. was maintained during operation. After many trials, iron crucibles were found to be by far the best.

Compressibility. For making the compressibility samples the crucible was about 2 cm. inside diameter and 7.5 cm. long, and was provided with a lid tightly clamped into position. The center of the lid was pierced with a fine steel tube, long enough to reach from the furnace to the colder parts of the enclosure; in this way the interior of the crucible could be exhausted, and at the same time the magnesium prevented from distilling out of the crucible, an effect which may be very annoying. The crucible was lowered at a rate of about 2 cm. per hour from a maximum temperature about 50° above the melting point. The magnesium with which the crucible was loaded was cut from a rod 4 cm. in diameter of commercially pure magnesium, which is usually very pure. The dimensions of the magnesium after casting into single crystal form were 2 cm. diameter and 5 cm. long. The crystal axis was located by optical examination of the reflection pattern This was developed with unusual perfection, all 20 faces being present. These composed a band of 18 faces about the equator, 6 on the equator itself and 6 in a band on either side like the tropics of Cancer and Capricorn, and in addition 2 faces, one at each pole, perpendicular to the hexagonal axis. The crystal axis proved to make an angle of 51° with the length of the casting. Specimens for the compressibility measurements were cut from the casting with a jewellers saw, parallel and perpendicular to the axis. The following results were obtained.

Parallel to the axis. The length of the specimen was 2.0 cm.; there was no appreciable permanent deformation as a result of exposure to hydrostatic pressure. At 30° the average departure from a smooth curve of a single one of the 14 readings was 0.18% and at 75° 0.04% on the same number of readings. The greater regularity of these readings compared with that of the other materials of this paper is what might be expected in view both of the greater geometrical perfection and more perfect homogeneity of the specimens. The numerical results were as follows:

$$\text{At } 30°, \quad -\Delta l/l_0 = 9.842 \times 10^{-7}p - 6.51 \times 10^{-12}p^2,$$

$$\text{At } 75°, \quad -\Delta l/l_0 = 10.154 \times 10^{-7}p - 7.78 \times 10^{-12}p^2.$$

Perpendicular to the axis. The length of this specimen also was 2.0 cm. and it showed no permanent distortion after exposure. At 30° the average departure from a smooth curve of a single one of the 14 readings was 0.08%, and at 75° 0.15% on the same number. The numerical results are:

$$\text{At } 30°, \quad -\Delta l/l_0 = 9.845 \times 10^{-7}p - 9.19 \times 10^{-12}p^2,$$

$$\text{At } 75°, \quad -\Delta l/l_0 = 9.659 \times 10^{-7}p - 6.95 \times 10^{-12}p^2.$$

By combining these results for the linear changes, the following may be found for the volume compressibility:

$$\text{At } 30°, \quad -\Delta V/V_0 = 29.53 \times 10^{-7}p - 28.3 \times 10^{-12}p^2,$$

$$\text{At } 75°, \quad -\Delta V/V_0 = 29.47 \times 10^{-7}p - 24.7 \times 10^{-12}p^2.$$

I have previously measured the volume compressibility of a multicrystalline extruded sample.[8] The results for this were: at 30°, $29.60 \times 10^{-7}p - 20.3 \times 10^{-12}p^2$, and at 75°, $29.97 \times 10^{-7}p - 18.0 \times 10^{-12}p^2$. The agreement is as good as could be expected in view of the indefinite character of the former sample.

Electrical Resistance. It was necessary to make special castings for the resistance measurements, since it would not have been possible without undue distortion to machine sufficiently slender rods out of the massive casting from which the compressibility samples were cut. The material was specially purified magnesium, which I owe to the kindness of Mr. R. S. Archer of the Research Laboratory of the Aluminum Co. of America. The new material was cast in the form of slender rods about 3 mm. in diameter and 5 cm. long in cylindrical iron molds; these molds were tightly closed at both ends. The rate of lowering from the furnace was about the same as for the larger casting. The mold was removed after casting by slitting with a fine saw in the milling machine. It was usually not possible to remove the mold without some slight bending of the specimen. However, the error so introduced was presumably small, as the resistance of this material does not seem sensitive to slight distortions. The orientation of the rods was determined optically; the reflection patterns were not as fully developed as in the larger casting, but enough faces were always present to allow certain identification. Magnesium proved unexpectedly easy to get into single crystal form; none of the rods were multi-crystalline.

The specific resistance at 22.5° was obtained from potentiometer measurements of the resistance, combined with measurements of the dimensions. The results are shown with sufficient accuracy in Figure 1, in which specific resistance is plotted against $\cos^2\theta$, where θ is the angle between the length of the rod and the hexagonal axis. Since there is no reason to suspect the correctness of the theoretical result that resistance is a linear function of $\cos^2\theta$, the straight line drawn in

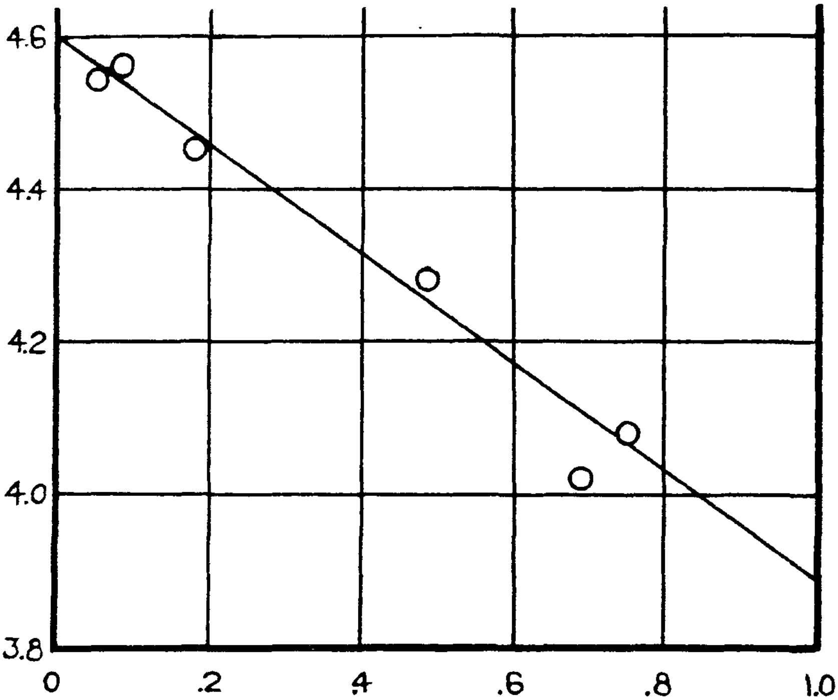

FIGURE 1. The specific resistance at 22.5° C of single crystal magnesium plotted against the square of the cosine of the angle between the hexagonal axis and the direction of current flow.

the figure may be taken as giving the most probable value of resistance. This means that the specific resistance perpendicular to the hexagonal axis is 4.60×10^{-6} and 3.89×10^{-6} parallel to it. The average specific resistance to be expected in a multi-crystalline casting is therefore 4.36×10^{-6}, or 4.32×10^{-6} corrected to 20°. This may be compared with the value 4.46×10^{-6} at 20° given in International Critical Tables for the polycrystalline metal. The new value is thus somewhat lower; part of this difference is probably to be explained by greater purity of the material, and part by the fact that it is in single crystal form.

The effect of pressure on the resistance up to 12000 kg/cm^2 at 30° and 75°, and the temperature coefficient of resistance at atmospheric pressure, was determined on three samples, with the axes inclined at 73°, 30°, and 29° to the length. The measurements on the 30° sample were irregular and could be used only to confirm in a general way the results on the 29° sample. There was considerable difficulty in obtaining satisfactory readings, both because of the smallness of the resistance and of the difficulty of making perfect electrical contact. The total resistance of the specimens was about 2×10^{-4}, and the maximum change produced by pressure about 5% of this. Consistency in the pressure readings to 1% demanded therefore resistance measurements correct to 10^{-7} ohms. Electrical connections were made with springs of 0.010 inch piano wire snapped into fine grooves girdling the rods. Since the potentiometer method was used, no error was introduced by resistance at the contacts, but in order to allow high enough currents for measurement, the goodness of the contacts had to be restored at intervals with a high tension current from a small bell ringing magneto.

The following results were found for the effect of pressure:

Hexagonal axis 73° to the length,

$$\text{At } 30^\circ \text{ C}, \quad -\frac{\Delta R}{R(0,\,30^\circ)} = 5.61 \times 10^{-6}p - 8.4 \times 10^{-11}p^2.$$

$$\text{Average coefficient to } 12000 = -4.60 \times 10^{-6}.$$

$$\text{At } 75^\circ \text{ C}, \quad -\frac{\Delta R}{R(0,\,75^\circ)} = 5.67 \times 10^{-6}p - 8.35 \times 10^{-11}p^2.$$

$$\text{Average coefficient to } 12000 = -4.65 \times 10^{-6}.$$

Hexagonal axis 29° to the length,

$$\text{At } 30^\circ \text{ C}, \quad -\frac{\Delta R}{R(0, 30^\circ)} = 5.48 \times 10^{-6} - 7.8 \times 10^{-11} p^2.$$

Average coefficient to 12000 = -4.55×10^{-6}.

$$\text{At } 75^\circ \text{ C}, \quad -\frac{\Delta R}{R(0, 75^\circ)} = 5.99 \times 10^{-6} - 11.8 \times 10^{-11} p^2.$$

Average coefficient to 12000 = -4.58×10^{-6}.

The average departures of single readings from smooth curves were 1.2 and 1.5% for the 73° specimen, and 1.4 and 1.6% for the 29° specimen at 30° C and 75° C respectively. In comparing results for different specimens more emphasis should be placed on the average coefficients to 12000 than on the coefficients in the power series expansion.

The average pressure coefficient to 12000 of the polycrystalline metal to be calculated from the above, assuming the pressure coefficient is linear in $\cos^2\theta$, is 4.58×10^{-6}. This may be compared with my best previous value of 4.07×10^{-6} for magnesium of doubtless inferior purity. The direction of variation of pressure coefficient with increasing purity is the same as that always found hitherto, without exception.

The temperature coefficient at atmospheric pressure was obtained by linear extrapolation of the resistance at 30° and 75°. The following are the results:

Hexagonal axis 73° to the length,

Mean coefficient 0° to 100° C, 0.00439.

Hexagonal axis 29° to the length,

Mean coefficient 0° to 100° C, 0.00420.

The mean coefficient for polycrystalline metal, assuming the temperature coefficient to be linear in $\cos^2\theta$, demanded by these values, is 0.00432. The best previous observed value is apparently 0.00412.

DISCUSSION.

NaF was investigated chiefly because it was one of the substances which Slater[10] did not measure in his investigation of the compressibility of the alkali halides, all in single crystal form. The specimen measured here was not a single crystal, but for a cubic crystal this is not important. The value for the compressibility of NaF found here fits smoothly into the series of values found by Slater. This is shown

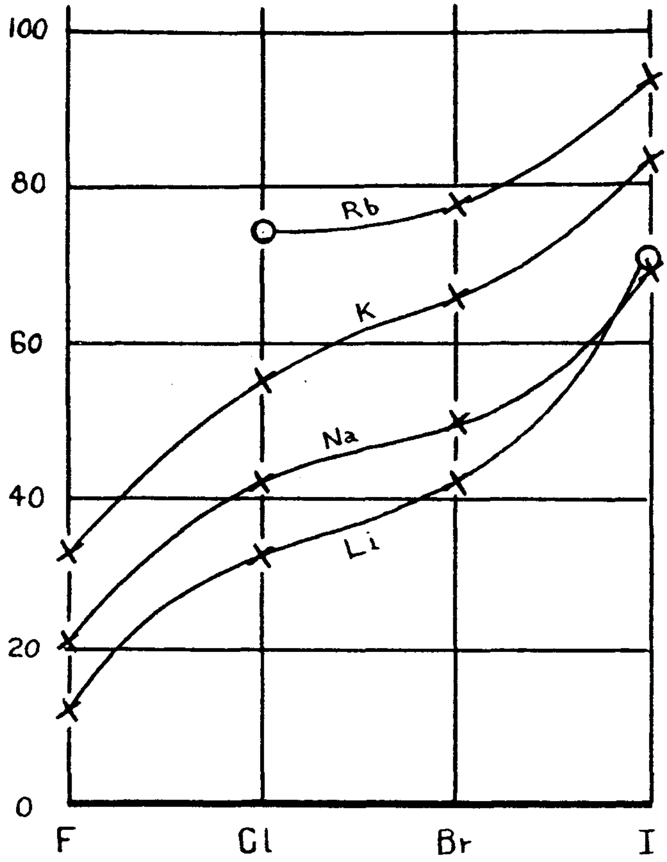

Figure 2. The compressibility at room temperature and atmospheric pressure of a number of the alkali halides. The curves connect the points for the compounds with the same alkali metal.

in Figure 2, in which the compressibilities at atmospheric pressure of various alkali halides are plotted with the successive halogens equispaced along the axis of abscissas. The two points for RbCl and LiI, which lie distinctly off the curve, are taken from measurements of Richards and Saerens,[11] which were made by a different method. It would probably be worth while to repeat these measurements.

One might perhaps expect the same sequence of conpressibilities on passing from one halogen compound to another in the series of the alkali earths as in the series of the alkali metals. That this is not the case is shown by Figure 3, in which are plotted the compressibilities found above for BaF_2 and SrF_2 together with the values found by Madelung and Fuchs[12] for $BaCl_2$, $BaBr_2$, $SrCl_2$, and $SrBr_2$. The sequence is not maintained, but there is a reversal of order at fluorine as compared with chlorine and bromine. The data for CdF_2 and $CdCl_2$ are also plotted in the same diagram; here there is an enormous variation on passing from fluorine to chlorine, which almost might suggest that the crystal structures of these two compounds is not the same.

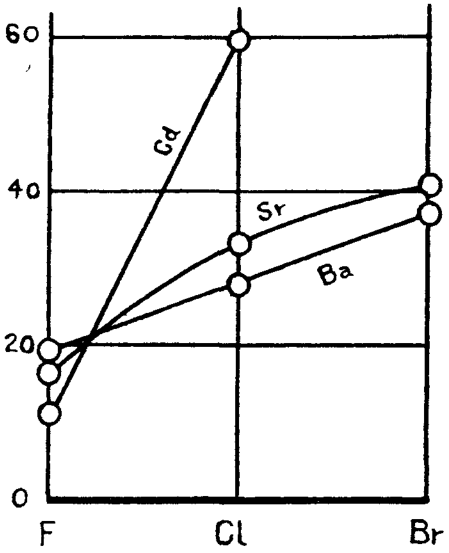

FIGURE 3. The compressibilities at atmospheric pressure and room temperature of the fluorides, chlorides and bromides of Sr and Ba, and the fluoride and chloride of Cd.

The compressibility of AlSb might perhaps be expected to be some sort of a mean between the compressibilities of its components. Perhaps the most natural sort of a mean is one in which the compressibilities of the pure metals are weighted according to their atomic volumes. Taking the initial compressibility of Sb as 27.0×10^{-7} and its atomic volume as 18.2, with 13.4×10^{-7} and 10.0 as the corresponding figures for Al, the compressibility to be expected is

$$\left(\frac{18.2}{28.2} \times 27.0 + \frac{10.0}{28.2} \times 13.4\right) \times 10^{-7} = 22.1 \times 10^{-7}.$$

The actual compressibility is 18.0×10^{-7}, 19% less. It is significant that the actual compressibility is less than that calculated; it is highly probable that at least part of this difference is to be explained by the effect of the internal cohesive pressure that the late Professor T. W. Richards was so fond of discussing. This cannot be the whole story, however, as is obvious from the non-cubical character of pure Sb and its greatly differing compressibility in different directions.

The compressibility of the two tellurides, CdTe and HgTe, differs even more from what may be calculated by the rule of mixtures. Taking the compressibilities of Te, Cd, and Hg (solid) as 50.8, 22.8, and 33.1×10^{-7}, and the corresponding atomic volumes as 21.2, 13.0, and 14.3, the calculated compressibility of CdTe is 40.1 and that of HgTe 43.7×10^{-7}, against the experimental values 23.3 and 19.8×10^{-7} respectively, which are thus about one-half the calculated values.

The low compressibilities of TiN and TiC is what would be expected from the high melting points and great mechanical hardness. Again the compressibility is very much less than that of the metallic constituent, the compressibility of pure Ti being about 8×10^{-7} against 3.3 for TiN and 4.7×10^{-7} for TiC. It is interesting that the compound with C is more compressible than that with N, although in the pure state solid N is much more compressible than C either as diamond or as graphite.

The electrical resistance of these two compounds puts them in the metallic class, the specific resistance of TiN being about 0.23, and that of TiC about 1.32 fold that of metallic Bi. The temperature coefficient of resistance of TiN, 0.00221, puts it not far from the pure metals; that of TiC, 0.00024, is in a different class. As far as the effect of pressure on resistance goes, the behavior is again in the class of the metals; the coefficient of TiN, -9.4×10^{-7}, is not far from that of metallic Co, and the coefficient of TiC, -1.37×10^{-6}, is not far from that of Mo. It would appear in general that the temperature coefficient of resistance is a more reliable criterion of the metallic character of a substance than either the specific resistance or the pressure coefficient of resistance.

Magnesium is of interest because it is the first hexagonal metal measured in which the axial ratio corresponds to a piling of *spheres* in close packed hexagonal array. The piling in Zn and Cd, for example, is that of close packed ellipsoids. It is, of course, well known that the close packed hexagonal arrangement of spheres differs little from the close packed face centered cubic arrangement. In fact, if the two arrangements are built up by piling over each other layers perpendicular to the hexagonal axis, the only difference between the two arrangements is a slight relative displacement of the third layers, the first and second layers in the two arrangements being the same. Hence the forces on the atoms in the two arrangements can differ only by the contributions of the more distant atoms, which are unimportant because of the rapid falling off of atomic forces with increasing distance. It follows that a close packed hexagonal arrangement of spheres would not be expected to differ greatly in physical properties from a close packed cubical arrangement, and in particular, the compressibility of the hexagonal arrangement would be expected to be nearly the same parallel and perpendicular to the axis, because the compressibility of a cubic crystal as the same in all directions.

This expectation is strikingly borne out by the behavior of the compressibilities of magnesium; at 30° the initial compressibility in the two directions is practically the same. There are, however, small differences; the second degree terms differ, so that at high pressures magnesium is more compressible parallel to the axis than perpendicular. The temperature behavior in the two directions is also different; perpendicular to the axis the temperature coefficient of compressibility is negative, a rather unusual effect. Furthermore, the negative coefficient perpendicular to the axis more than compensates for the positive coefficient parallel to the axis, so that the initial temperature coefficient of volume compressibility is also negative. The second degree terms act differently, however, so that at high pressures the volume compressibility is greater at the higher temperature, as is normal.

The specific resistance of magnesium differs more in different directions than would be expected from the considerations above and from the fact that the compressibility is so nearly the same in all directions. Furthermore, the resistance is abnormal in that it is least parallel to the hexagonal axis, that is, the resistance is least for flow across the basal plane. The only other example of this sort of behavior is antimony, and here the abnormality disappears at high pressures. The usually greater resistance across the basal plane is associated with the functioning of this plane as the plane of easiest slip or cleavage. The mechanical properties of single magnesium crystals have not yet been sufficiently studied to show whether in it the basal plane is also a slip plane or not.

After the manuscript of this article went to press, a paper has appeared by O. Sckell, Ann. d. Phys. 6, 932, 1930, in which it is shown that mercury (tetragonal) also has a smaller resistance parallel to the crystallographic axis.

The pressure coefficient of resistance does not vary as much with direction as does the specific resistance, but there is nevertheless a noticeable variation, and the variation is the same in character as that of the specific resistance, the pressure coefficient being least numerically in the direction in which the resistance is also least. It will be found on making the numerical calculation that the relative pressure coefficients are such in different directions that the ratio of resistance in the two principal directions is independent of pressure.

The temperature coefficient also varies with direction in the same way, the temperature coefficient being greatest in the direction in which the specific resistance is the greatest. The numerical values are such that the abnormal ratio of resistance in the two principal directions becomes accentuated at high temperatures.

I am much indebted for financial assistance to the Rumford Fund of the American Academy of Arts and Sciences and to the Milton Fund of Harvard University. I am also much indebted to my assistants: Mr. W. A. Zisman for making most of the readings, Mr. George Langreth for preparing the samples of NaF, BaF_2, SrF_2, CdF_2, AlSb, CdTe, and HgTe, and Mr. Sanford Goldman for preparing the single crystals of magnesium.

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RESEARCHES ON THE ROTATION OF PERMALLOY AND SOFT IRON BY MAGNETIZATION AND THE NATURE OF THE ELEMENTARY MAGNET.

By S. J. Barnett.

RESEARCHES ON THE ROTATION OF PERMALLOY AND SOFT IRON BY MAGNETIZATION AND THE NATURE OF THE ELEMENTARY MAGNET.[1]

By S. J. Barnett

§ 1. *The gyromagnetic ratio.* The most important quantities associated with the elementary magnet in ferromagnetic substances are its magnetic moment μ_0, its angular momentum M_0, and its gyromagnetic or magneto-mechanical ratio $\rho = M_0/\mu_0$. The gyromagnetic ratio of an elementary magnet consisting of a Weber-Rutherford-Bohr magneton, or electron moving in an orbit about a nucleus or atomic core, is $\rho_B = 2m/e$; that of a Lorentz electron spinning on a diameter is $\rho_L = m/e = \frac{1}{2}\rho_B$; that of an Abraham electron, uniformly charged throughout its volume and spinning about a diameter, is $\rho_A = 6/7 . m/e$; the quantity m/e being the ratio of electron mass to electron charge. Although the magnetic moments of many free atoms have now been determined by the method of Gerlach and Stern, no method of determining M_0 or μ_0 separately and with certainty in magnetic aggregates has yet been devised; but in several investigations $\rho = M_0/\mu_0$ has now been determined with considerable precision for many ferromagnetic substances.

§ 2. *Magnetization by rotation. The gyromagnetic anomaly.* The first successful work in the domain of gyromagnetic phenomena was an investigation on the magnetization of iron by rotation[2] first published by me in 1914 by presentation to the Ohio Academy of Sciences and the American Physical Society. This work established for the first time the actual existence of Ampère's molecular currents, proved that the electricity in motion in them is negative, and furnished data from which it is possible to calculate $\rho = M_0/\mu_0$ for the magnetic element in iron. The value of ρ calculated from these data and similar data obtained a few months later by the same general method is equal, within less than the experimental error of some 10 or 15 per cent, to m/e ($2m/e = 1.13 \times 10^{-7}$ e.m.u), and thus to one-half the gyromagnetic ratio for an electron orbit—a result which has come to be known as the *gyromagnetic* or *magneto-mechanical anomaly.*

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[1] Reports of progress on this work presented to the American Physical Society have been published in abstracts as follows: Physical Review 30, p. 964, 1927; *ibid.*, 31, p. 1116, 1928; *ibid.*, 36, p. 789, 1930.

[2] S. J. Barnett, Phys. Rev. 6, p. 239, 1915.

This work on magnetization by rotation was later (1917–1924) extended to other ferromagnetic substances, the last paper[3] dealing with iron, nickel, cobalt, Heusler alloy, and iron-nickel, iron-cobalt, and cobalt-nickel alloys. The mean gyromagnetic ratio obtained for all these substances was 1.06 × m/e, with an error estimated as not greater than 2 per cent, and with no certain evidence of any difference between the gyromagnetic ratios of the elementary magnets in different substances. The theory of the experiments requires that the change in the magnetic moment of the body, and the deflection

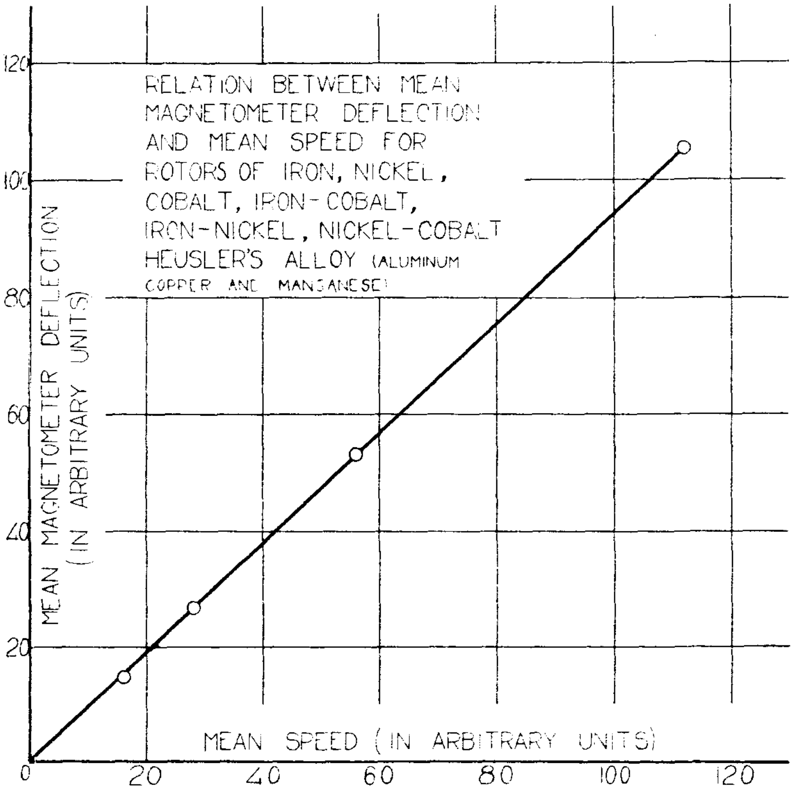

FIG. 2–1.

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[3] S. J. Barnett and L. J. H. Barnett, Proc. Amer. Acad. 60, p. 127, 1925.

of the measuring instrument produced thereby, be proportional to the speed of rotation. The experimental relation between speed and reduced deflection for the last and most extensive series of observations on all the above mentioned substances is given in Fig. 2–1, from which an idea of the degree of precision of the work can be obtained. The reduced deflection for each rotor is the actual deflection divided by the change of moment produced by a standard weak axial magnetic field intensity.

§ 3. *Rotation by magnetization.* The converse phenomenon, viz., the rotation of a ferromagnetic substance by magnetizing it, was discovered in 1915 by Einstein and de Haas,[4] who were not then acquainted with the (earlier) work on magnetization by rotation. Instead of m/e, their work, upon iron, gave for ρ the value $2m/e = \rho_B$, which they estimated correct within about 10 per cent; but additional investigations,[5] made much later in the same year and in the next year, were necessary to give them any certain evidence as to the sign of the electricity involved. This later work showed that the electricity was negative, as had been already definitely proved in the work on magnetization by rotation.

The work of Einstein and de Haas was limited to a few observations on iron. A more extensive and thorough investigation, on both iron and nickel, was made by John Q. Stewart[6] in this country in 1918. He obtained for both substances a value of ρ half that obtained by Einstein and de Haas, and thus equal to that obtained in the converse experiments several years earlier.

This work has been followed and confirmed by several other excellent investigations, in Switzerland, Sweden, and England. Of this work probably the most precise has been done in England, where Chattock, Sucksmith, and Bates[7] have made studies of iron, nickel, cobalt, magnetite and Heusler's alloy. In every case they believe they have, like all others except Einstein and de Haas, found for ρ the value m/e within their experimental error. In their best work, which was done on iron and nickel, they believe the experimental error not greater than one per cent. There is thus a discrepancy of about 6%

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[4] A. Einstein and W. J. de Haas, Verh. d. D. Phys. Ges. 17, p. 152, 1915.

[5] W. J. de Haas, Verh. d. D. Phys. Ges. 18, p. 423, 1916; A. Einstein, ibid. 18, p. 173, 1916.

[6] J. Q. Stewart, Phys. Rev. 11, p. 100, 1918.

[7] Phil. Trans. Roy. Soc. A, 223, p. 257, 1922; Proc. Roy. Soc. A, 104, p. 499, 1923; Proc. Roy. Soc. A, 108, p. 638, 1925.

between their mean result and the value obtained from experiments on magnetization by rotation.[8]

§ 4. *The present investigation.* On account of all the discrepancies, and the great theoretical importance of the phenomena, I have for several years been making a new and elaborate investigation of the rotation of certain substances by magnetization. An account of the investigation is given in this paper.

§ 5. *General experimental methods and theory.* In all experiments on the Einstein and de Haas phenomenon a cylinder of the substance under investigation is suspended with its axis vertical from a fixed support by means of a vertical wire or fiber along the axis extended. The cylinder is generally mounted coaxially within a magnetizing coil of insulated wire, and the motion of the cylinder produced by changes of axial magnetization is studied.

Let μ_0, as above, denote the magnetic moment of one elementary magnet, M_0 its angular momentum, and ρ its gyromagnetic ratio, so that

$$M_0 = \rho\mu_0 \tag{5–1}$$

Let j' denote the resultant angular momentum of all the elementary magnets in the cylinder, or *rotor*, as it will be called, and μ the rotor's axial magnetic moment. Then

$$j' = \Sigma M_0 \cos\theta = \rho\Sigma\mu_0 \cos\theta = \rho\mu$$

where θ is the angle made by the axis of an elementary magnet with the geometrical axis of the rotor. When the magnetization changes, the total angular momentum of all the elementary magnets will change at the rate dj'/dt; so that, in accordance with the principle of the conservation of momentum, the angular momentum of the *rotor and solenoid together* will change at the rate

$$g = dj/dt = -\,dj'/dt = -\,\rho d\mu/dt \tag{5–2}$$

which is the *gyromagnetic torque* on the rotor and solenoid.

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[8] The discrepancy is very probably due mostly, if not entirely, to the failure of the British physicists to eliminate the important errors discussed below in § 9 (2), § 11 (c), § 12 (b), § 17, and § 21, which were not considered by them or by any earlier investigators, but which are usually of considerable magnitude, and sometimes very large.

In ballistic methods of investigation equation (5–2) is used in the modified form

$$\int g\,dt = -\rho\Delta\mu \qquad (5\text{–}3)$$

i. e., the angular impulse is determined for a given change $\Delta\mu$ in the magnetic moment. In alternating current methods equation (5–2) is used directly, always with more or less complete resonance in order to obtain sensibility and to eliminate certain errors.

In nearly all the experimental work done hitherto the coil magnetizing the rotor has been fixed to the earth, and only the momentum communicated to the *rotor* has been measured, whereas the theory gives only the combined torque upon *rotor and solenoid.*[9] In the investigations by de Haas, however, the magnetizing coil was rigidly wound on the rotor, so that the total torque was measured; but this was done for practical, not theoretical, reasons; and the precision of the work was so low that no conclusion can be drawn from it with regard to the effect of the rigid connection between coil and rotor.

In the investigation described here many precise experiments have been made with the magnetizing coil wound rigidly upon the rotor, and also many with the coil fixed to the earth; and it has been definitely proved that the reaction to the electron change of momentum is at least almost entirely upon the rotor.

§ 6. *Alternating current null method.* All the work described here has been done by alternating current methods. The method to be described first, but not the principal method used, is a null method, and resembles, but with great differences, a method used by Sucksmith and Bates, and suggested by Chattock. The torque given to the rotor by changing its magnetization has its effect in producing rotation annulled by another torque, as suggested by Chattock. This annulling torque is produced by the action of an electric current traversing a fixed coil of wire, which exerts a torque on a small permanent magnet rigidly attached to the rotor. A diagram of some of the chief parts of the apparatus is given in Fig. 6–1.

The rotor, *F*, is axially suspended by a thin German silver wire or strip, *D*, attached above to a brass rod, *C*, passing through a torsion head, *B*, of amber. Hanging from the lower end of the rotor by an essentially rigid joint is a small brass rod, *J*, carrying on opposite sides a pair of parallel mirrors, *I*, and below, a group, *L*, of small permanent magnets, all turned in the same direction. Hanging from

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[9] Attention was directed to the remote possibility of error from this source by O. W. Richardson in Phys. Rev. 26, p. 248, 1908.

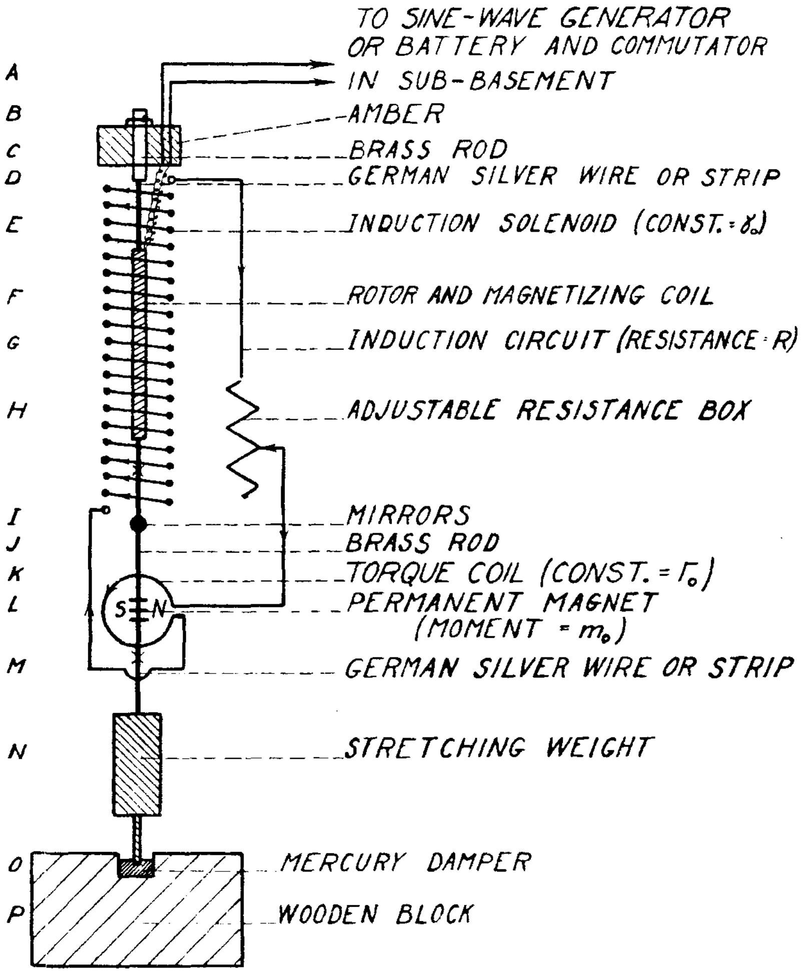
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
TO SINE-WAVE GENERATOR OR BATTERY AND COMMUTATOR IN SUB-BASEMENT
AMBER
BRASS ROD
GERMAN SILVER WIRE OR STRIP
INDUCTION SOLENOID (CONST.=γ_0)
ROTOR AND MAGNETIZING COIL
INDUCTION CIRCUIT (RESISTANCE=R)
ADJUSTABLE RESISTANCE BOX
MIRRORS
BRASS ROD
TORQUE COIL (CONST.=Γ_0)
S
N
PERMANENT MAGNET (MOMENT = m_0)
GERMAN SILVER WIRE OR STRIP
STRETCHING WEIGHT
MERCURY DAMPER
WOODEN BLOCK

FIG. 6–1. Diagram illustrating chief experimental methods.

the lower end of the brass rod, *J*, by another German silver suspension, *M*, is a stretching weight, *N*, which keeps the axes of the rods in line and the suspensions tight. A small rod projecting downward from the weight dips into some mercury, O, in a wooden cup, P, and helps to dampen out any lateral motion of the hanging bodies.

In the diagram the magnetizing coil is shown wound directly upon the rotor, F. Current is led to and from the coil by two fine wires soldered to small brass rods passing through the torsion head and connected by the wires A to an alternating current generator, or to a storage battery through a rotating commutator, driven by a motor in the subbasement. In the diagram, for the sake of clearness, the two fine wires forming the ends of the magnetizing coil are shown coiled separately. In the actual apparatus they were twisted together, and then the double wire twisted into a loose spiral around the suspension D. In another form of the apparatus the rotor is magnetized by a fixed solenoid located inside the induction solenoid referred to below. Most of the work was done with commutators and storage battery, not with the sine-wave generator, chiefly in order to make the phase angles α, γ, δ, § § 10–13, small.

When the commutator or generator is in operation the rotor is acted on by an alternating torque. In order to make the rotary vibration of the rod thus produced as great as possible, the speed of the commutator (or generator), controlled by devices described below, is adjusted until approximate resonance occurs, when the amplitude of vibration is very many times as great as it would be from a single impulse. The range, or double amplitude, of the vibration is examined by lamp, mirror, lens and scale.

The rotor is surrounded with a much longer and uniformly wound cylindrical coil of wire, E, which will be called the *induction solenoid;* and the group, L, of small permanent magnets is at the center of a small double coil, K, the *torque coil*, the planes of whose turns are parallel to the magnets. Currents sent in opposite directions through this coil will produce opposite torques upon the magnets and thus upon the vibrating system; so that, if such currents are properly timed, and of the proper magnitude, they will just annul the gyromagnetic torque.

This condition is brought about as follows: The induction solenoid E is connected through an adjustable resistance box H in circuit with the torque coil K. When the magnetization of the rotor is changing it induces a current in this circuit, whose magnitude can be increased or decreased at will by changing the resistance in the box. Thus, at the same time as a gyromagnetic torque is given to the rotor system, a torque of electromagnetic origin, which may be made exactly equal to it, is also given to the system. If the terminals of the torque coil are connected in one way to the rest of the circuit, the two torques will be in the same direction and the resultant torque twice one of them; if the connections are reversed the two torques will annul one another, and the vibration will disappear. From the directions in which the coils are wound, and the permanent magnet turned, it is easy to predict in advance how the terminals should be connected to produce the compensation, provided we know whether the electricity in Ampère's whirls is negative or positive, the connections being opposite for the two signs. The connections found necessary always show that the *electricity is negative*, and are correctly indicated in the diagram for the given azimuth of the magnet L.

If R_0 is the resistance of the circuit when the vibration is annulled, m_0 the moment of the group of permanent magnets, and Γ_0 and γ_0 the *constants* of the torque coil and induction solenoid, it is easy to show that, with due attention to signs (see § 7),

$$-\rho = \frac{\Gamma_0 \gamma_0 m_0}{R_0} \tag{6-1}$$

provided that the two effects we have been considering are the only ones involved.

§ 7. *Conventions with respect to signs of torques etc.* In what follows all directed quantities associated with the diagram of Fig. 6-1 which can be represented by vertical lines in a plane parallel to that figure will be considered positive when directed upward; and the right handed screw convention will be adopted. Thus an axial torque which would move the ends N and S of the magnet L from and to the reader, respectively, would be *positive*. The current in the torque coil K will be considered positive when it is clockwise. Horizontal quantities will be considered positive when directed to the right or when directed from the reader.

§ 8. *Formula for null method in ideal case.* To develop the formula (6-1) it will be necessary to take from § 5 the relation

$$g = -\rho d\mu/dt \tag{8-1}$$

which is true whatever the form of the wave of electromotive force or magnetic moment.

Let ϕ denote the magnetic flux through the induction circuit produced by μ, and suppose the flux produced by the magnetizing coil negligible. Then, if the constant γ_0 of the induction solenoid is uniform throughout the volume occupied by the rotor,

$$\phi = \mu\gamma_0 \tag{8-2}$$

If, as will be assumed in accordance with the facts, the reactances of the induction circuit for all appreciable harmonics are negligible in comparison with its resistance R, the current q produced in this circuit by the changing flux will be

$$q = -\frac{1}{R}\frac{d\phi}{dt} = -\frac{\gamma_0}{R}\frac{d\mu}{dt} \tag{8-3}$$

and will thus have the same form as g.

Thus the torque on the vibrator due to the action of the field of the torque coil upon its permanent magnet, with moment m_0, will be

$$c = \Gamma_0 m_0 q = -\frac{\Gamma_0\gamma_0 m_0}{R}\frac{d\mu}{dt} \tag{8-4}$$

with the same form as that of g.

If R is given such a value R_0, and the connections between the torque coil and the induction solenoid are so made, that $c = -g$ the resultant torque will vanish at every instant, and we shall thus have

$$-\rho = \frac{\Gamma_0\gamma_0 m_0}{R_0} \tag{8-5}$$

which is identical with (6–1)

§ 9. *Sources of disturbance.* Actual experimental conditions are of course never so nearly ideal as they are assumed to be in § § 6 and 8. There are in general many effects which tend to mask the gyromagnetic effect being studied, and whose elimination, by processes described later, is the source of most of the difficulties involved in the work. Some of these effects are as follows:

(1) *Mechanical vibrations* due to the continual shaking of the building. These can be largely eliminated by hanging the apparatus from the ceiling by springs and damping the motion with cotton waste,[10] and by working as far as possible when the earth's tremors are

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[10] *Note added in proof.*—Since this was written the behavior of the apparatus with respect to vibrations has been greatly improved by discarding the cotton waste (sometimes removed before) and applying adhesive tape along the springs and from spring to spring and by adopting also the method of internal damping suggested by R. Müller (Ann. des Phys. 1, p. 613, 1929). This con-

slight. The motor, gears, and generator, in the subbasement, never produced any appreciable vibration, either rotary or other, of the rotor.

(2) *Magnetostriction*, or changes in the rotor's dimensions due to changes in magnetization. The effects are relatively small in permalloy and soft iron, and much worse in other substances. Even in permalloy they may produce errors of many per cent. Some of them appear to be reduced by sealing the parts of the suspended system together with shellac or duco cement (which is better), and by changing the suspended weight. A serious systematic error due jointly to magnetostriction, inequality of the two half-cycles of magnetizing current, and either asymmetry of the suspended system or the variation with tension of the twist of the suspension, or both, can be eliminated by periodically reversing the connections between the magnetizing coil and the source of power. If an alternating current generator is used the same result may be accomplished by reversing its field. The effect can be reduced in some cases by using magnetizing currents strong enough to produce approximate saturation of the magnetic material. See also § 45.

Even when the half-cycles of current are equal, a similar magnetostrictive torque may exist if the rotor has a sufficient permanent vertical moment, due to earlier vertical magnetization or to failure to compensate the vertical part of the intensity of the earth's field. In the latter case the effects can be eliminated by neutralizing or reversing this part of the field. With certain rotors this effect is very pronounced when the current is weak and the residual vertical field not entirely compensated. In the former case the behavior of the rotor can be improved by putting it through a demagnetizing process.

(3) *Leakage* between the magnetizing circuit and the induction circuit. The effect of this can be made negligible by taking proper precautions with respect to insulation.

(4) *The presence of the earth's magnetic field*, which acts on the rotor and rotor coil. This can be largely compensated by means of three coils carrying steady electric currents, one to compensate the vertical part of the earth's intensity Z, one to compensate the northerly

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sists simply in placing pans of oil on the suspended apparatus. For referring me to Müller's work I am indebted to Dr. John Strong. As a further improvement, made possible by the changes already mentioned, small vanes were placed on the German silver rod of Fig. 6–1, and the mercury was replaced by a reservoir of light oil into which the weight N dipped.

component X, and one to compensate the easterly component Y. These compensations are more troublesome than they would otherwise be because the earth's intensity is continually varying in both magnitude and direction, and because the work has to be done in a building which contains iron, interfering with the uniformity of the field.

(5) *Failure,* in the rotor and other parts of the vibrating system, *of complete symmetry* about a central vertical line. To make the system sufficiently symmetrical mechanically requires the greatest care and skill on the part of the instrument maker. But even a mechanically perfect system is of course in general not magnetically so. The magnetic asymmetry may be made small with less difficulty in the case of easily magnetizable substances like soft iron and permalloy, than in the cases of at least most others.

(6) *Magnetic actions,* in addition to those given above, *of the different electric coils* on the rotor and the small group of permanent magnets. These can be made negligible or eliminated as indicated below.

(7) *Electron-inertia,* on account of which the change of current in the magnetizing coil fixed on the rotor, or the change of magnetic flux through the rotor, inducing currents therein, produces an axial torque. This is very minute, but not negligible in all cases. When not negligible, it can be computed and allowed for.

(8) *Inductive action* of the current in the magnetizing coil on the secondary or induction circuit. This can be computed and allowed for, or can be eliminated by the use of extra compensating coils.

(9) *The Elihu Thomson electrodynamic repulsion effect.* This is not in general appreciable. If it were appreciable in the case of equal half-cycles of current, it would have a negligible effect in the resonance experiments, as its frequency would be twice that of the vibration. In the case of unequal half-cycles there would be a component of the same frequency as that of the vibration, which could be eliminated by the process of reversal mentioned in discussing (2), above.

(10) *Magnetic impurities* in critical parts of the testing magnetometers, torque coil, induction solenoids, and other apparatus assumed to be free from iron. In this work careful tests of all material of importance have shown that no appreciable error could result from such impurities as were present.

In what follows all the torques which can produce appreciable systematic effects and have not already been adequately treated, as well as some others, are discussed in sufficient detail.

§ 10. *The normal torques.* The frequency of the generator or commutator being $\omega/2\pi$, the first harmonics of the vertical and horizontal magnetic moments of the rotor will be assumed to be

$$\mu = M \sin \omega t \quad \text{and} \quad \nu = N \sin (\omega t - \alpha) \tag{10-1}$$

respectively; and the angular momentum of the complete vibrating system will, on account of resonance, be assumed to be

$$p = P \sin (\omega t + \beta) = KA \sin (\omega t + \beta) \tag{10-2}$$

where A is the amplitude of the strip of light on the scale and K is a constant. Then the torques acting upon the vibrating system, aside from the disturbing torques, will be as follows:

(1) *The total torque* τ. This will be equal to the rate change of angular momentum. Thus

$$\tau = \frac{dp}{dt} = \omega P \cos (\omega t + \beta) = \omega KA \cos (\omega t + \beta)$$
$$= T \cos (\omega t + \beta) \tag{10-3}$$

(2) *The gyromagnetic torque* g. From (8-1) and (10-1) this torque is evidently

$$g = -\rho\omega M \cos \omega t = G \cos \omega t \tag{10-4}$$

(3) *The torque* c due to a current $q = Q \cos \omega t$ in the torque coil. As above,

$$c = \Gamma_0 m_0 q = \Gamma_0 m_0 Q \cos \omega t = C \cos \omega t \tag{10-5}$$

(4) *The frictional torque* f. This will be given by the equation

$$f = -\sigma p = -\sigma KA \sin (\omega t + \beta) = \sigma KA \sin (\omega t + \beta + \pi)$$
$$= F \sin (\omega t + \beta + \pi) \tag{10-6}$$

where σ is a positive constant.

(5) *The suspension torque.* This torque, in phase with τ, will be

$$s = S \cos (\omega t + \beta) = \omega KA \frac{\omega_0^2}{\omega^2} \cos (\omega t + \beta) \tag{10-7}$$

where $\omega_0/2\pi$ is the natural frequency of the vibration.

§ 11. *The disturbing torques.* (1) *Magnetic torques.* Undoubtedly the most important of the disturbing torques to be eliminated are the *magnetic torques.* These are due either to the residual part of the earth's field (A), or to the action of the electric coils on the movable system (B).

(A) *Torques due to the earth's field.* Let ΔH denote the horizontal intensity of the residual magnetic field in which the rotor is placed, and ΔX and ΔY its northerly and easterly components; also let ΔZ denote the vertical intensity of the residual field.

(a) Then there will be an axial torque on the rotor due to ΔH, which may be written

$$e_h = h\Delta H\nu = E_h \sin (\omega t - \alpha) \qquad (11\text{-}1)$$

If the torsion head is turned through 180° this torque will be reversed and will become

$$e'_h = - e_h = - E_h \sin (\omega t - \alpha) \qquad (11\text{-}2)$$

E_h of course vanishes with ΔH, ν, and the angle between them.

(b) ΔH, acting on the vertical moment, will produce a torque

$$l = L \sin \omega t \qquad (11\text{-}3)$$

about a horizontal axis. The natural frequency n of vibration about such an axis is clearly very much greater than the natural axial frequency $\omega_0/2\pi$, with which at least approximate resonance is always maintained, and the damping is great. Thus the resulting amplitude in altitude is very minute. If the system is not axially symmetrical, however, the torque will, in general, give rise to an axial torque

$$b = B \sin (\omega t - \chi) \qquad (11\text{-}4)$$

and thus to axial oscillations. This torque changes sign when the torsion head is turned through 180°. In addition to the axial oscillations due to this torque, a group of minute free axial vibrations with the frequency n will be set up during each half cycle of the magnetization.

All these effects may be made very minute, and vanish with ΔH.

(c) Although no earlier investigator has considered it necessary to annul Z, the residual intensity ΔZ, acting on the horizontal moment ν, may produce effects quite similar to those just considered, and for the same reason. The effective axial torque may be written

$$e_z = E_z \sin (\omega t - \alpha) \qquad (11\text{–}5)$$

It does not change sign or magnitude when the torsion head is turned in azimuth, but does change sign with ΔZ and E_z. In the cases of the permalloy and soft iron rotors this torque, as would be expected from the small value of ν, is very small even when ΔZ is a considerable part of the vertical intensity of the earth's field. As examples, all of them complicated however with effects of magnetostriction, two series of experiments on the iron rotor $I^2{}_{2\cdot4-2}$ may be mentioned, together with one on a permalloy rotor.

With 50 milliamperes traversing the magnetizing coil on the rotor, and only approximate resonance, the mean amplitude with the earth's vertical intensity compensated was 13.22 cm; with the earth's intensity alone, 13.20 cm; with the earth's intensity doubled, 13.16 cm. The same night, with 100 milliamperes, the three arrangements gave, respectively, 11.30 cm, 11.05 cm, and 11.29 cm.

With the same rotor a number of complete sets of observations have been taken by the latest method with the earth's vertical intensity under and over compensated by equal amounts and have shown a variation in the calculated value of ρ of only about 4.4 % for a change of more than 20% in the vertical intensity.

In the case of the permalloy rotor ${}_8P_{2\cdot4-2}$ changes in the vertical intensity equal to more than 12% of the earth's vertical intensity made no appreciable effect on the value of ρ.

(d) If the magnetizing coil is wound upon the rotor, there will be a torque upon the system due to the action of the residual field upon the wire, which may be written

$$i = I \sin (\omega t + \gamma) \qquad (11\text{–}6)$$

where γ is very small for rectangular waves. This torque retains its magnitude but changes its sign when the torsion head is turned through 180°.

§ 12. (B) *Torques due to the action of the electric coils on the moveable system.* (a) If the magnetizing coil is fixed to the earth, and if the rotor has a permanent horizontal magnetic moment ξ, there will, in general, be an axial torque d upon the rotor due to the alternating field of the fixed coil, whose direction is not, in general, exactly normal to ξ. This torque, with the commutator producing the alternating current, will be nearly in phase with μ. Thus we may write

$$d = D \sin (\omega t + \delta) \qquad (12\text{–}1)$$

where δ is small. This torque also will evidently be reversed in sign by turning the torsion head through 180°. It is often of very considerable magnitude, and may be difficult to eliminate completely. When rectangular waves are used D can probably be reduced by inserting a suitable condenser in the magnetizing circuit—a device used with effect by Einstein (Footnote 5 § 2) in the ballistic method.

(b) In this same case there is an axial torque due to the action of the alternating field on the alternating horizontal moment ν; but if the half-cycles of current and magnetization are equal, its fundamental frequency is twice that of the current, and so its effect is negligible. If the half cycles are not equal, there may be also a residual torque, with the frequency of the alternating field, which may have any phase relation to that of g. This torque may be written

$$j = J \sin (\omega t + \zeta) \qquad (12\text{–}8)$$

So far as the half-cycles of current are concerned, it can be eliminated by making the reversal referred to in § 9, (2).

(c) The torque coil produces a magnetic field extending upward into the region occupied by the rotor, where its lines of intensity are horizontal. Thus, if the rotor's magnetization is not symmetrical about the axis of suspension, an axial torque may be produced. If the magnetization is strictly alternating (and half-cycles equal), this torque will have twice the frequency of the alternation, and may be left out of account in resonance experiments. If there is a residual permanent moment, however, produced by an uncompensated part of the earth's field or otherwise, there will be an axial torque of the frequency of the current and in phase with (or in opposition to) the gyromagnetic torque. In the work described here this torque was entirely negligible.

Thus at a point 20 cm distant from the center of the compensating coil (less than the distance between the lower end of rotor and center of coils) and on a line normal to the axis and passing through the center, the magnetic intensity was found to be only one thousandth that at the center.

If now the iron rotor $F^{2}_{2\cdot3-2}$ (a rotor with one of the largest moments), were permanently magnetized to the maximum magnetization attained in the experiments, and were hung so unsymmetrically as to have the center of its lower end displaced one-half mm. from the vertical line through the center of the upper end, it is easy to show that the axial torque upon it, due to this field intensity, could not be greater than about 3×10^{-3} that upon the small permanent magnet at the center—equal to the gyromagnetic torque in the null experiments. The assumptions made are of course extreme, and in the actual experiments the torque could have been only a minute fraction of that calculated above, and was thus entirely inappreciable.

Moreover, in a special series of experiments, this rotor was driven by an alternating current of its own frequency in the torque coil, while a direct current of 100 milliamperes traversed its own solenoid, first in one direction, then in the other, the mean amplitudes for the two directions differed by less than 0.1% of either.

If the torque were appreciable, the effect on the determination of ρ could be eliminated by making two sets of experiments, with rotor azimuths differing by 180° with respect to the axis of the small magnet. This was done in some of the work, but was quite unnecessary so far as the effect under discussion is concerned.

If the half-cycles are unequal, there may be a residual torque with the frequency of the current, whose effect, if appreciable, is eliminated by making the reversals mentioned in § 9, (2).

(d) If the induction solenoid is not exactly vertical, but displaced from the vertical by a small angle α, the intensity of its magnetic field will have a horizontal component $h = \alpha\gamma_0$ per unit current, and may exert torques on the rotor due to its horizontal permanent and alternating moments ξ and ν. The intensity at the bottom of the longest rotors due to the compensating coils is $c = \gamma_0/1000$, approximately, per unit current. Since $\gamma_0/\Gamma_0 = 0.12$, approximately, h/c is approximately 120α. Thus even if α were as great as 1/12 radian (and it was always very much less indeed), the maximum possible axial torque on any permanent horizontal moment of the rotor due to h would be no greater than the maximum possible torque due to c (when the permanent pole off the axis is at the bottom of the rotor).

Even if such a torque were not negligible, its effect on the determination of ρ would be eliminated in the process, almost always adopted, of observing with the rotor in two different azimuths differing by 180°, the connections of the induction solenoid not being reversed with the rotor, like those of the torque coil.

If either the fixed magnetizing coil or the induction solenoid does not extend in both directions considerably beyond the rotor, so that even with the coil axis vertical there is a radial intensity near the ends, the torques due to asymmetry will be much greater than otherwise and much more difficult to eliminate by changing azimuths.

§ 13. C. *Torque due to leak between magnetizing and induction circuits.* If there is such a leak there will be a torque

$$\lambda = \Lambda \sin (\omega t + \gamma) \qquad (13\text{–}1)$$

on the vibrating system. The sign of the torque will be unchanged when the suspension is turned through 180° and at the same time the connections between compensating coil and induction solenoid are reversed. The coefficient Λ is made very minute by using high insulation,[10] and γ is negligible, at least when the commutator is used. So that the effect is both very minute, and also in quadrature with the gyromagnetic torque.

§ 14. D. *Torques due to mutual induction between magnetizing coil and induction circuit* were always made negligible, or else calculated and the corrections applied, as indicated below.

§ 15. E. *Torques due to electron inertia in rotor coil and rotor.*[11]

(1) *Electron inertia in rotor coil.* When the current in the coil, and therefore the angular momentum of the free electrons, changes, an equal and opposite change occurs in the angular momentum of the rotor. Thus there is an electron inertia torque upon the rotor. If the first harmonic of the current in the coil is $i = I \sin (\omega t + \gamma)$, the the first harmonic of this torque, which is the only one effective on account of resonance, consists of two terms t_1 and t_2 as follows:

$$t_1 = -2\frac{m}{e} \cdot M'\omega \cos (\omega t + \gamma) = T_1 \cos (\omega t + \gamma) \qquad (15\text{–}1)$$

and

$$t_2 = -2\frac{m}{e}\,\frac{M'\omega}{I} \cdot near\,\omega\Theta \cos (\omega t + \beta) = -\,T_2 \cos (\omega t + \beta) \qquad (15\text{–}2)$$

where M' is the magnetic moment of the coil, a the cross-section of the wire, r the mean radius of the coil, n the number of free electrons per unit volume and Θ the amplitude of the rotor's vibration.

When the commutator is used the angle γ is negligible, so that t_1 is in phase with g. In the null method t_2 vanishes with Θ; in the deflection methods β is nearly $\pi/2$, so that t_2 is in quadrature with g and its effect on the determination of ρ (see § 18) vanishes even when its magnitude is appreciable.

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[10] In most of the work the insulation was entirely of amber, hard rubber, paraffin, soft rubber, and sulfur.

[11] See S. J. Barnett, Phys. Rev. 36, p. 786, 1930, and a forthcoming article in the *Philosophical Magazine* Ser. 7, Vol. 11.

For the quantity T_1/G we obtain

$$\frac{T_1}{G} = \frac{2\frac{m}{e}\cdot M'\omega}{\rho\omega M} = \text{approx. } 1.9\,\frac{M'}{M} \qquad (15\text{–}3)$$

a fraction which must be subtracted from the value of ρ obtained without applying the correction. The magnitude of the correction in the experiments described here ranges from about 0.1% to about 2%. For the rotor $I^2_{2\cdot 4-2}$ when the current is 100 milliamperes the fractional correction is 0.004. For this rotor we may take, as approximately correct, $r = 0.14$ cm, $a = 0.00005$ cm^2; and we may assume $n = 10^{23}$ per cm^3, and $e = 1.6 \times 10^{-20}$ e.m.u. In standard experiments we may take $\omega = 40$ radians per second, $I = 0.01 \times 4/\pi$ e.m.u., $m = 1270 \times 4/\pi$ e.m.u., $M' = 2.5 \times 4/\pi$ e.m.u. and $\Theta = 0.01$ radian. From these data T_2/G = approximately 0.004.

(2) *Electron inertia in the rotor.* There are also electron-inertia torques upon the rotor due to the induction of currents therein by the changing flux, but it is easy to show that they are very much less even than those just considered. One of the two terms vanishes in the null method; both are in quadrature with g in the deflection method.

§ 16. *Experiments with rotors of brass and glass.* In order to study disturbing torques not due to the rotor's magnetization, rotors of brass and glass were prepared. The electron inertia torque on the rod itself is too minute to be observed for brass, and of course for glass; while the electron-inertia torque on the coil does not depend much on the exactness of its winding. The torques due to the action of the residual magnetic field on the coil, however, depend greatly on the winding, on the arrangement of the leads, or both. In the case of a glass rotor these torques were originally very small, but were greatly increased when the rotor was rewound, although with great care.

In 1927 the following experiments, with others, were made on a brass rotor, which was 0.24 cm in diameter and of the same length as the standard magnetic rotors. It was wound with two layers of DCC copper wire No. 36, like some of the earlier magnetic rotors, and was suspended in the usual way. Its natural frequency was about 6.65 cycles/sec.

(1) With a standard current of 135 milliamperes in the solenoid, and with Y and Z fields very nearly balanced, the resonance amplitude

was measured as a function of X. The results of one of the two sets of concordant observations are given in Fig. 16–1.

(2) With the same solenoid current, and with the X and Z fields very nearly balanced, the amplitude was measured as a function of Y. The results of one of the two concordant sets of observations are given in Fig. 16–2.

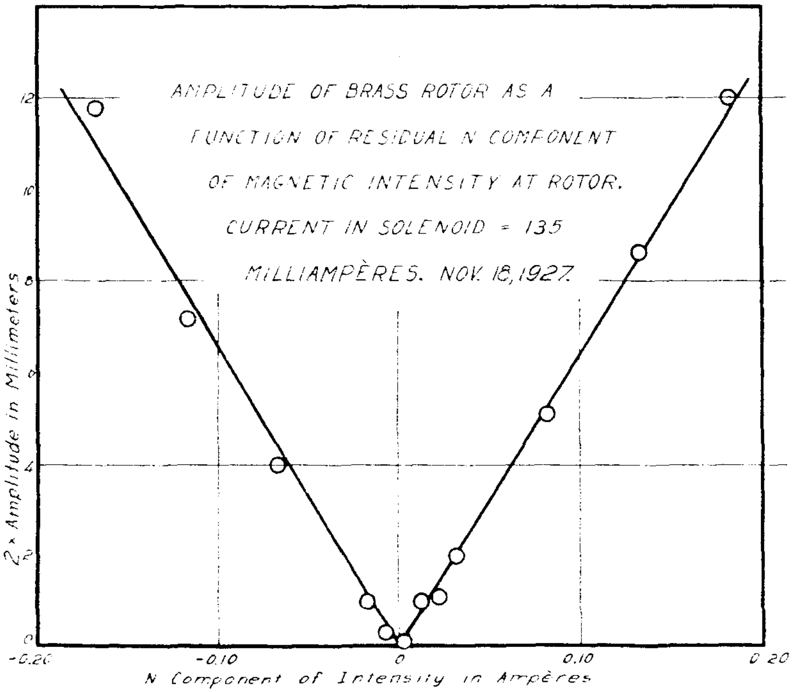


Fig. 16–1.

(3) With X and Y balanced, it was found that Z produced no effect.

(4) With Y and Z balanced and X about $1/3$ the northerly component of the earth's intensity, the amplitude was measured as a function of the solenoid current. The results are given in Fig. 16–3.

(5) A current of 10^{-5} ampere (rectangular wave) at the frequency of resonance was sent through the small compensating coils, and produced a deflection amplitude of 6.4 cm. The amplitude of the first harmonic of the torque was then about $1.4 \times 1.4 \times 10^{-3}$ dyne cm.

From this result and Fig. 16–1 it follows that the X component of the earth's horizontal intensity uncompensated would produce on the rotor winding when its current is 135 milliamperes, a torque whose amplitude is about $1.4 \times 2 \times 10^{-3}$ dyne cm.

In the same way, from Fig. 16–2, it follows that the whole uncompensated Y component of the earth's horizontal intensity would produce a torque with amplitude about $1.4 \times 7 \times 10^{-5}$ dyne cm.

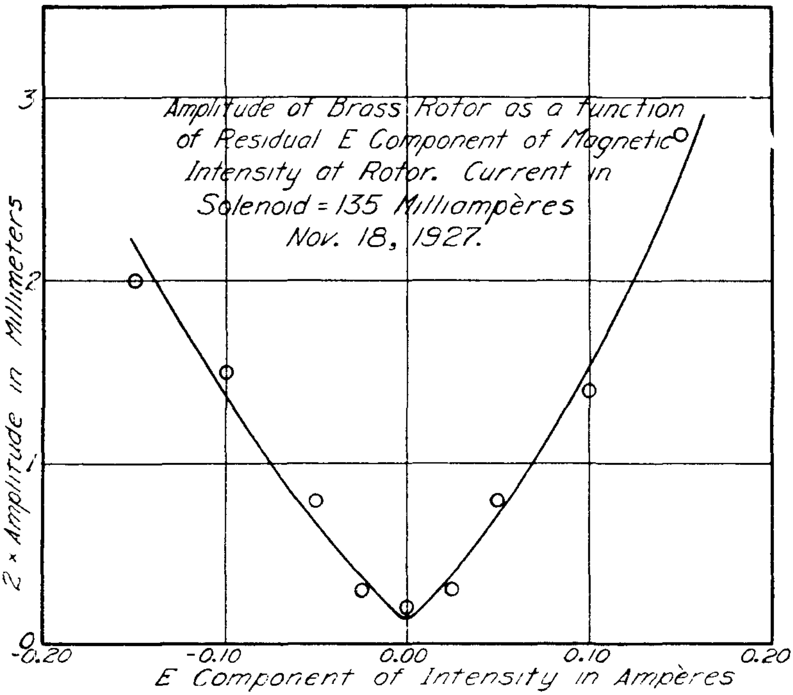


FIG. 16–2.

The amplitude of the first harmonic of the gyromagnetic torque on one of the small permalloy rotors when nearly saturated in the experiments, is about $1.4 \times 1.7 \times 10^{-3}$ dyne cm, and the gyromagnetic torques on most of the other rotors are greater, as they have greater moments.

Thus an error of one per cent in setting X would produce on the brass rotor a torque only about 1 per cent of the torque on the permalloy rotor; and an error of one per cent in setting V would produce on the brass rotor a torque of only about one twentieth of one per cent of the gyromagnetic torque on the permalloy rotor.

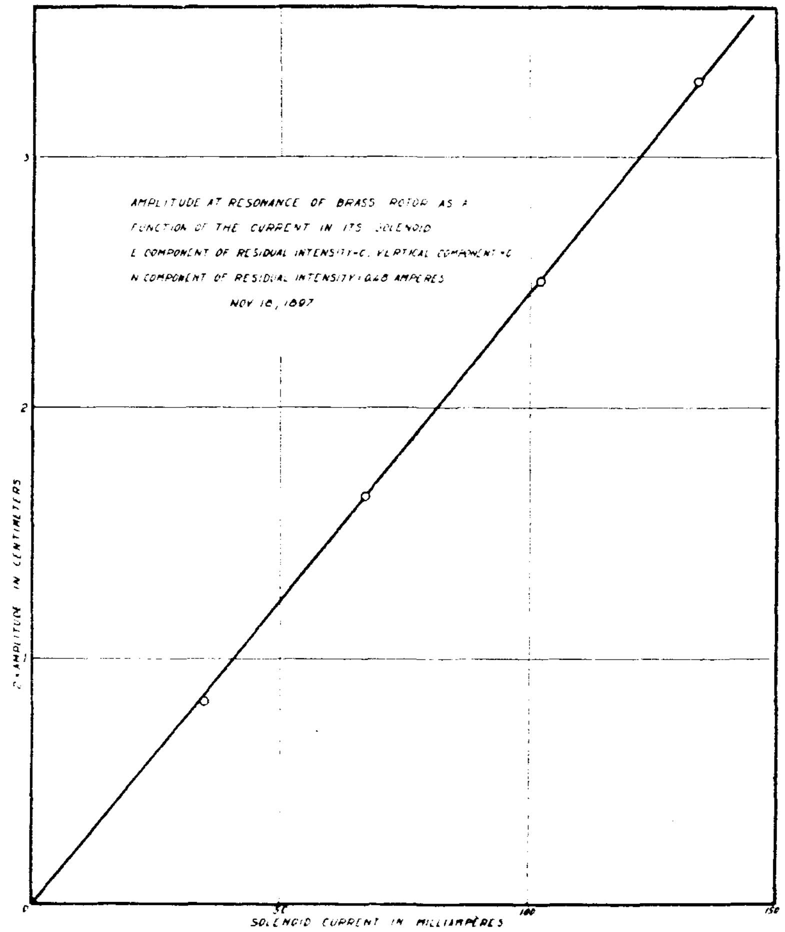


FIG. 16-3.

Similar torques act on the coils of the magnetic rotors; but even when appreciable they have no effect on the determination of ρ, as already explained.

The small residual amplitudes in Figs. 16–1 and 16–2 are doubtless due in part to electron inertia.

In addition to these experiments many additional experiments have been made on this rotor (original and rewound), and a rotor of glass, already referred to. There are minute disturbances which often occur and are not yet wholly explained; nevertheless, the observations have been sufficient to measure, with an error of only a few per cent, the electron-inertia torque of § 15, which is itself very small in comparison with the gyromagnetic torques investigated in this paper. The disturbances referred to cannot therefore affect seriously the value of ρ obtained in this work.

§ 17. F. *Torque due to magnetostriction.* Since the change in rotor length due to magnetization is independent of the direction of the magnetization, no magnetostrictive torque of the same period as that of the first harmonic of the magnetization can arise if the two half-cycles of magnetization are similar. If they are dissimilar, however, either because of a residual vertical magnetic moment, or because the two half-cycles of magnetizing current are dissimilar, there will be an elongation with a first harmonic of the period of the magnetization. For effects of upper harmonics see § 45.

Hence if the rotor is unsymmetrical, or if the twist of the suspensions depends on their tensions, as it is found to do with German silver, there will be a torque of magnetostrictive origin upon the rotor, whose first harmonic may be written

$$h = H \sin (\omega t + \theta) \qquad (17\text{–}1)$$

If when the two half-cycles of current are similar the residual magnetization is reversed (as in most cases by reversing the earth's residual vertical intensity; or, when the half-cycles are dissimilar but the residual moment vanishes, if the connections between battery and commutator are reversed, or the generator field current is reversed, or the connections between the terminals of the rotor coil and the rest of the circuit are reversed, so that the half-cycles producing positive and negative magnetization of the toror are reversed—the torque changes sign without change of magnitude. Its effect can thus be eliminated by taking observations in which such reversals are periodically made.

By driving the generator or commutator at such speed as to produce a current with half the natural frequency of the rotor system, enormous amplitudes due to magneto-striction can sometimes be obtained.

§ 18. *Equation of motion and fundamental formula* when the magnetizing coil is wound on the rotor, and effects D and E of §§ 14 and 15 are neglected. If the frequency of the first harmonic of the current in cycles per second is denoted by $\omega/2\pi$ it is clear from §§ 10–13 that the equation of motion is

$$(G + C)\cos\omega t - \sigma KA\sin(\omega t + \beta) + \omega KA\frac{\omega_0^2}{\omega^2}\cos(\omega t + \beta)$$
$$+ (E_h + E_z)\sin(\omega t - \alpha) + B\sin\omega t$$
$$+ (I + \Lambda)\sin(\omega t + \gamma) + H\sin(\omega t + \theta)$$
$$= \omega KA\cos(\omega t + \beta) \qquad (18\text{–}1)$$

From this equation we obtain for the components of the amplitude of the resultant impressed electromagnetic torque in phase with g and leading g by a quarter period, respectively, the quantities

$$T_{\parallel} = G + C - (E_h + E_z)\sin\alpha + (I + \Lambda)\sin\gamma + H\sin\theta$$
$$= KA\left\{\frac{\omega^2 - \omega_0^2}{\omega}\cos\beta + \sigma\sin\beta\right\} = UKA = \psi A_{\parallel} \qquad (18\text{–}2)$$

and

$$T_{\perp} = -B - (E_h + E_z)\cos\alpha - (I + \Lambda)\cos\gamma - H\cos\theta$$
$$= KA\left\{\frac{\omega^2 - \omega_0^2}{\omega}\sin\beta - \sigma\cos\beta\right\} = VKA = \psi A_{\perp} \qquad (18\text{–}3)$$

where U and V are abbreviations for the expressions in curled brackets, $A_{\parallel}$ and $A_{\perp}$ the amplitudes of the deflection which would be produced by the two components separately, and ψ is a constant. The amplitudes $A_{\parallel}$ and $A_{\perp}$ are connected with A by the relation

$$A_{\parallel}^2 + A_{\perp}^2 = A^2 \qquad (18\text{–}4)$$

In the work described here (except special tests) $E_z\sin\alpha$ and $\Lambda\sin\gamma$ are negligible. Cancelling them, we obtain from equation (18–2)

$$G + C + H\sin\theta - E_h\sin\alpha + I\sin\gamma = \psi A_{\parallel} \qquad (18\text{–}5)$$

If G, θ and ω are kept constant, while the torsion-head is turned through the angle π (from azimuth A to azimuth B) and the connections between the torque coil and the rest of the induction circuit are reversed, and if the amplitude of the current in the coil is given the value C', we obtain in place (18–5) the equation

$$G + C' + H \sin \theta + E_h \sin \alpha - I \sin \gamma = \psi A'_{11} \qquad (18\text{–}6)$$

By addition of (~~16–1~~ 18-6) and (~~16–6~~ 18-5) we obtain

$$2G + C + C' + 2H \sin \theta = \psi(A_{11} + A'_{11}) \qquad (18\text{–}7)$$

If now, with G and ω still unaltered, one of the reversals mentioned in § 17, F is made, changing θ by π, we have for current amplitudes C and C' corresponding to azimuths A and B, as above,

$$G + C_1 - H \sin \theta - E_h \sin \alpha + I \sin \gamma = \psi A_{1\,11} \qquad (18\text{–}8)$$

and

$$G + C_1' - H \sin \theta + E_h \sin \alpha - I \sin \gamma = \psi A_1'_{11} \qquad (18\text{–}9)$$

which together give

$$2G + C_1 + C_1' - 2H \sin \theta = \psi(A_{1\,11} + A_1'_{11}) \qquad (18\text{–}10)$$

Combining (18–7) and (18–10), and denoting the mean values of the current and deflection amplitudes by $\bar{C}$ and $\bar{A}_{11}$, we obtain finally

$$G + \bar{C} = \psi \bar{A}_{11} \qquad (18\text{–}11)$$

from which the effects of all disturbing torques (except D and E) are eliminated and which is the fundamental formula sought.

§ 19. *The null-graphical method.* This has already been treated for the ideal case in which all disturbing torques vanish. In a single experiment the amplitude A of the deflection may then be reduced to zero by making c oppose g and giving R a suitable magnitude R_0.

When disturbing torques are present the amplitude $A_\perp$ is in general appreciable, so that only a minimum amplitude ($A_\perp$ itself) can be obtained by varying R. A_{11} is then zero. Four experiments are necessary, two with values of θ differing by π for each of the two azimuths A and B. The effects of all the disturbing torques (except D and E) are then eliminated by using equation (18–11), which becomes simply

$$G = -\bar{C} \quad \text{or} \quad -\rho = V_0 \gamma_0 m_0 \overline{\left(\frac{1}{R_0}\right)} \qquad (19\text{–}1)$$

when $\overline{\left(\frac{1}{R_0}\right)}$ is the mean value of the reciprocals of the four resistances necessary to produce the four minimum amplitudes of deflection.

§ 20. *Large deflection method I.* In the determination of ρ by this method a comparison is made between the amplitude A_G of the deflection produced by the gyromagnetic torque when there is no current in the torque coil ($C = 0$), and the amplitude A_C of the deflection produced by a current $q = Q \cos \omega t$ in the torque coil when there is no current in the magnetizing coil ($G = 0$). As in the method just described, four experimental arrangements are necessary to eliminate all the disturbing torques (except D and E); and in each case it is also necessary to measure $A_\perp$ and to calculate $A_\parallel$ from A and $A_\perp$ by equation (18–4). When $C = 0$, equation (18–11) and (18–4) give from the four arrangements

$$G = \psi \bar{A}_\parallel = \psi \sqrt{\overline{A^2} - \overline{A_\perp{}^2}} = \psi A_G \tag{20–1}$$

and when $G = 0$,

$$C = \psi A_C \tag{20–2}$$

Since $G = -\rho\omega M$, and $C = \Gamma_0\, m_0 Q$, the last two equations give

$$-\rho = \frac{\Gamma_0 m_0 Q}{\omega M} \cdot \frac{A_G}{A_C} \tag{20–3}$$

When rectangular waves are used, Q and M are the amplitudes of the first harmonics, or $4/\pi$ times the steady values to which the current and magnetic moment quickly rise.

The quantities $A_\perp$ are usually so small in comparison with the corresponding quantities A that the latter, which are directly observed, can be used for A_G or A_C without appreciable error. For equal amptitudes the error vanishes completely.

The method is open to the objection that it requires the precise measurement of a number of quantities; nevertheless it is capable of considerable precision. The precision is, of course, greatly enhanced by using resonance. The method is open to the additional objection that the harmonics in q and c, which are not now, as in the null method (and other deflection methods), produced by mutual induction between the primary and secondary circuits, are not identical with those in g; so that formula (20–3) is from this cause not quite exact. On account of resonance, however, and the great preponderance of the first harmonic in the angular momentum, the error from this source is negligible. No such harmonics are appreciable.

§ 21. *Methods with the magnetizing coil fixed to the earth.* The equation of motion and the other equations applicable to this case are identical with those already developed except (1) that the torque i and its phase angle γ are replaced by the torque d and its phase angle

δ; and (2) that there is added another torque $j = J \sin(\omega t + \zeta)$ (§ 12, B, (b)) which may be treated precisely like the torque h, being eliminated in the same way. The effect of the torque d, which changes sign when the torsion-head is turned through 180°, is eliminated in the same way as the effects of the torques i and e_k. While these latter torques are very minute, however, the torques d and j are often large, so that the apparent values of ρ for the two azimuths and the two positions of the reversing switch are often considerably different.

§ 22. *Corrections due to mutual induction.* In what precedes the variable part of the magnetic flux through the induction circuit has been taken as $\varphi = \mu\gamma_0$, that is the flux through the induction solenoid due solely to the magnetization of the rotor.

The magnetizing coil, however, also produces a flux φ' through the solenoid, and the magnetizing coil and rotor together produce a flux φ'' through the torque coil, which is a part of the induction circuit.

The apparatus was always so symmetrical that the flux φ'' either vanished or was entirely negligible in comparison with φ, as careful tests showed.

In the fixed solenoid method, and sometimes in the moving solenoid method, the flux φ' was compensated by means of a pair of coils, one similar to the induction solenoid, but longer, the other within the first, having nearly the same magnetic moment per unit current as the magnetizing coil. When φ' was appreciable, compensation could always be effected with much greater precision than necessary by moving one of the coils relatively to the other, or adding or subtracting a few extra turns, until the mutual induction vanished.

This was quite important in the fixed solenoid method as there φ' and φ were always of the same order, of magnitude, though φ' was always less than φ.

In the case of the moving solenoid, however φ'/φ was always small, ranging from about 0.2% to about 1%.

Since with the rectangular wave commutator the current was practically in phase with the vertical magnetic moment, the existance of φ', when small, and uncompensated by mutual induction, could always be allowed for by applying to the value of ρ determined on the assumption that $\varphi'/\varphi = o$ a correction

$$\Delta\rho = \rho \times \frac{\varphi'}{\varphi} = \rho\,\frac{M}{M'}\,.$$

The connection is clearly always positive. On account of the smallness of the correction it could be, and has been, applied in the same way without appreciable error in the case of experiments made with sine waves.

There is also a harmonic flux φ''' though the induction circuit due to the motion of the permanent magnet, with moment m, with respect to the compensating coil. The quantity φ'''/φ however, is entirely negligible. For the iron rotor $I^2_{2\cdot 4-2}$ its magnitude was about 5×10^{-5}.

When the rotor does not hang symmetrically through the induction solenoid and the torque coil, some of the flux produced by the rotor threads the torque coil. Even with asymmetry far greater than ever existed in the experiments this flux was entirely negligible. Thus, for example, with the center of the torque coil only 10 cm below the end of the rotor $I^2_{2\cdot 4-2}$ (less than $1/2$ the distance in nearly all the work) and axially displaced 4 mm., the flux through the torque coil was not measurable when the flux through the induction solenoid produced a deflection of about 83 cm.

§ 23. *Experimental curves in the null method.* In making a series of observations by this method, G (and the remainder of T_{11}) is kept constant, while C, otherwise constant, is varied by changing R, which occurs in its denominator. The amplitude of the impressed torque, when no disturbing torques in quadrature are present, is thus $T_{11} - \psi/R$, where ψ is a constant. The deflection amplitude A is proportional to the inpressed torque, so that we may write

$$A = \alpha(T_{11} - C) = \alpha\left(T_{11} - \frac{\psi}{R}\right) = \beta - \frac{\gamma}{R} \qquad (23\text{–}1)$$

where α, β, and γ are constants.

The equation (23–1) between A and R gives the two rectangular hyperbolae drawn in Fig. (23–1). The experimental parts of the curves are drawn in heavy continuous and chain dotted lines.

The curves obtained by experiment agree with such hyperbolae closely, except that, when there are disturbing torques in quadrature with the gyromagnetic torque, the curves are rounded near the junction, as indicated in the figure. The abcissa R_0 of the junction at which the minimum amplitude occurs is not affected, only the ordinate. When there are disturbing torques in phase with or in opposition to the gyromagnetic torque, the junction is shifted to the left or right.

§ 24. *Transformation from hyperbolae to straight lines.* If, instead of plotting the relation between the vibration amplitude and the resistance R, we plot that between the amplitude and the conductance $X = 1/R$, we obtain the straight lines of Fig. (24–1), which are so drawn as to make clear the relation of their parts to the corresponding parts of the hyperbolae of Fig. (23–1).

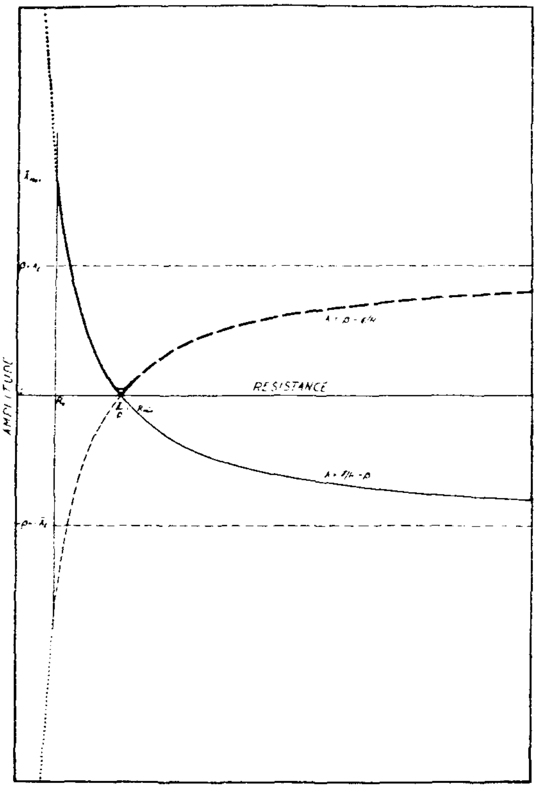
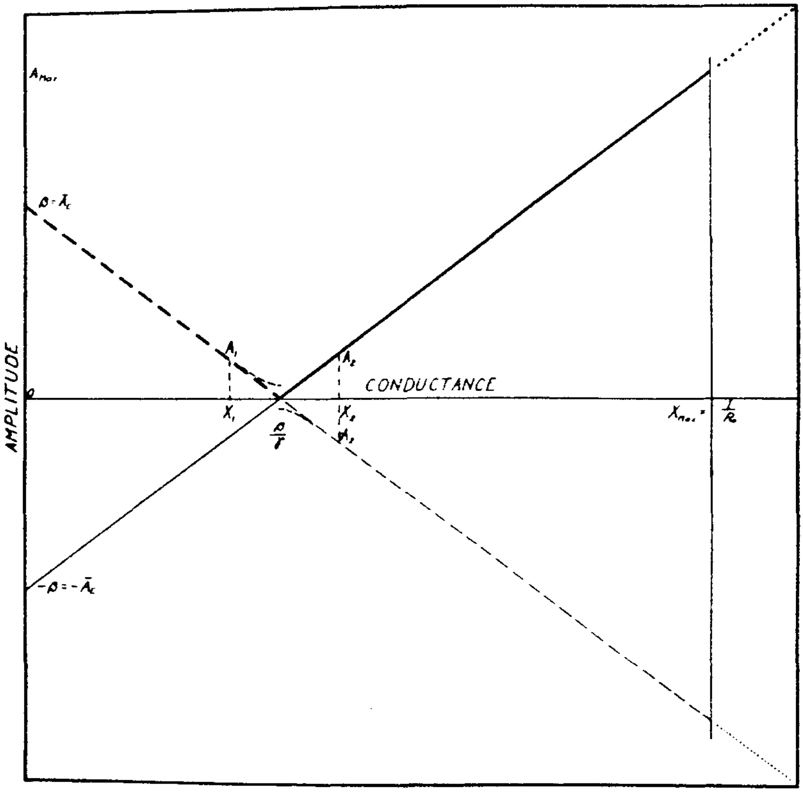


Fig. 23-1.

Each of the straight lines cuts or touches the axis of conductance at the point $X_0 = 1/R_0 = \beta/\gamma$. Reproductions of some of the straight lines obtained from actual experiments are given in Fig. (46–1). From these the hyperbolae were drawn, and are given in the same figure. The observed points are given by the small circles.

In finding X_0 graphically by means of such straight lines, it is better to plot the amplitudes obtained for conductances greater than X_0 as negative quantities, and draw a single straight line, as in the curves of Fig. (46–1).

§ 25. *Method of interpolation from small deflections.* While in much of the earlier work experimental curves, either hyperbolae or straight lines, were drawn for a number of points, particularly in the neighborhood of the minimum amplitude, and the resistance or conductance for the minimum taken from the curves, a different procedure was adopted in some of the work, interpolation being made from a few small amplitudes taken for conductances on opposite sides of that corresponding to the minimum amplitude.



AMPLITUDE
A_{Max}
$\rho = A_c$
A_1
A_2
CONDUCTANCE
0
X_1
X_2
$\frac{\rho}{\gamma}$
A_2
$X_{Max} = \frac{1}{R_0}$
$-\rho = -A_c$

FIG. 24–1.

Thus if A_1 and A_2, Fig. (24–1), denote the vibration amplitudes corresponding to conductances X_1 and X_2, less and greater, respectively, than X_0, the conductance for vanishing amplitude, we obtain

$$X_0 = X_1 + (X_2 - X_1) \cdot A_1/(A_1 + A_2) \qquad (25\text{–}1)$$

When X_1 and X_2 differ by about the same amount from X_0, the same relation holds even when the lines are curved, as in the dotted part of Fig. (24–1), since symmetry still exists.

§ 26. *Large deflection method II.* The same figure leads to a large deflection method far superior to that of § 20 and used in most of the later work. Thus, if we make $X = X_1 = 0 (R_1 = \infty)$, we obtain the amplitude A produced by the gyromagnetic torque and the (usually small) disturbing torques; and if we make $R = 1/X = 1/X_2$ about one-half the value for the minimum amplitude A_0, we obtain a nearly equal amplitude A_2. We then have simply

$$X_0 = X_2 \frac{A}{A + A_2} \qquad (26\text{–}1)$$

As the torques in quadrature with g produce very little effect on the amplitudes when large, results obtained in this way are almost independent of such torques. The effect of the quadrature torques when not negligible and yet not too great can be eliminated by substituting in the formula for the amplitudes A and A_2 the quantities $\sqrt{A^2 - A_0^2}$ and $\sqrt{A_2^2 - A_0^2}$.

As will be seen below, results obtained by this method agree very closely with those obtained by the other methods.

Although formula (26–1) is based on the assumption that only the first harmonic is present in the angular momentum, it holds also when higher harmonics are appreciable, since these harmonics enter in exactly the same way into both A and A_2.

In all methods it is, of course, necessary, in general, to find the X_0 from each of the four different experimental arrangements mentioned above, and to calculate ρ from the mean.

§ 27. *Method of remote induction solenoid.* If we have two rotors A and B exactly alike and wound with magnetizing coils exactly alike, so that equal currents produce equal moments, A may be mounted in the usual way to obtain the gyromagnetic torque, while B, inside the induction solenoid (or a similar solenoid) and at rest, may be mounted at a distance. With the rectangular wave commutator, however, it is unnecessary that the two rotors, or the rotor A and the rod B, have the same moments or be of the same material, or be wound alike. It is only necessary to know the ratio M_B/M_A of the maximum moments when the two magnetizing coils, in series, are traversed by the same maximum current. In this case we have, in place of (19–1), the equation

$$-\rho = \frac{M_B}{M_A}\,\Gamma_0\gamma_0 m_0\overline{\left(\frac{1}{R}\right)} \qquad (27\text{–}1)$$

A number of measurements made in this way gave results agreeing closely with those made in the usual manner, which avoids any error in determining M_B/M_A.

§ 28. *The apparatus adjacent to the rotor.* The way in which the principal apparatus near the rotor was mounted is illustrated in Fig. 28–1. The induction solenoid A was held in a special wooden clamp B between the boards CC attached (with ebonite insulation in between) to a heavy wooden frame E resting on a heavy wooden table F. A third board D carried the compensating coil, also insulated with ebonite. The frame and table also supported the coils for neutralizing the horizontal intensity of the earth's magnetic field. It was provided at the top with adjustable diagonal brass rods for correcting distortion; and two boxes HH, for holding sand, were attached to the bottom of the table. The table, together with all it supported, was suspended from the ceiling by twenty-four heavy brass springs attached in equal groups of six each to the wooden beams GG, below the table top, and two similar beams at the ceiling. The lower ends of some of the springs are shown disconnected in the figure. Spirit levels, J and K, in brass cases, and sensitive to 10″ or thereabouts, made it possible to set the apparatus with precision, where once the induction tube, neutralizing coils, etc. had been placed in correct relative adjustment by means of plumb lines, etc. The table was leveled, and stability secured, by means of sand in the boxes HH. The legs of the table nearly touched the floor, and damping of its motion was produced when desired by placing small amounts of cotton waste between the floor and the legs.[12]

The large rectangular frame carrying the coil for neutralizing the vertical intensity of the earth's magnetic field surrounded the other apparatus and was supported from wooden beams at the ceiling by brass wires attached at the bottom to adjusting slides of brass provided with slow motion by screws.

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[12] For later improvements in the method of damping see Footnote 10, § 9.

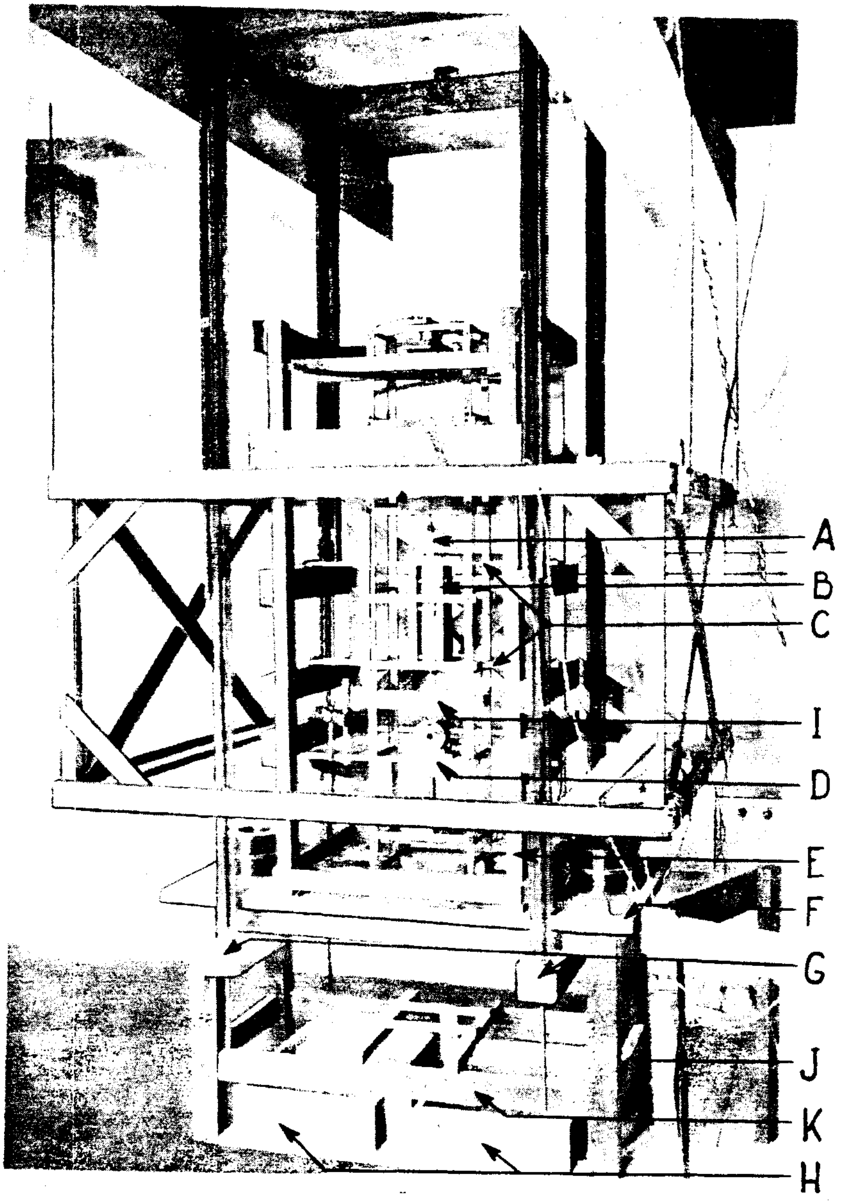
A
B
C
I
D
E
F
G
J
K
H

FIG. 28-1. The central apparatus.

At *I* is a light wooden support carrying a spectacle lens by which, and the mirror on the vibrating apparatus, an image of the vertical filament of an electric lamp is formed on a translucent scale between four and five meters distant.

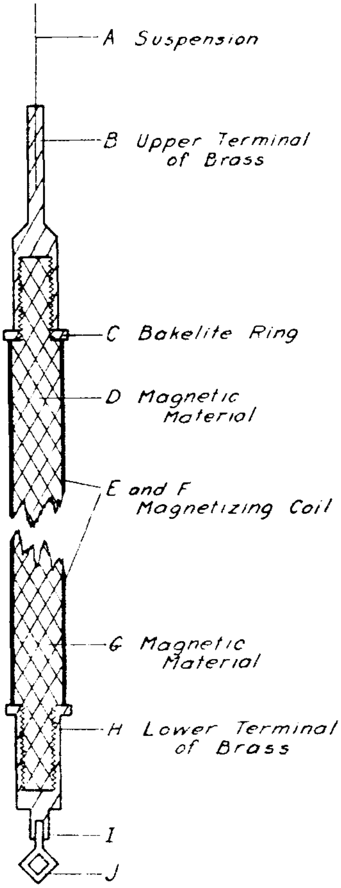
A Suspension
B Upper Terminal of Brass
C Bakelite Ring
D Magnetic Material
E and F Magnetizing Coil
G Magnetic Material
H Lower Terminal of Brass
I
J

FIG. 29-1. Construction of standard wound rotor.

§ 29. *The rotors.* The magnetic rotors, of annealed Armco electrode iron and Bell Telephone permalloy, were of two types, those with a magnetizing coil wound upon them, and those without a winding.

Most of the rotors with the coil wound upon them were constructed as indicated in Fig. 29–1. The whole length of the rotor was about 31 cm, and that of the magnetic material about 27 or 28 cm. An approximately circular and cylindrical rod of the magnetic material was made as nearly straight as practicable. The ends were threaded, and on them were screwed upper and lower terminals of brass of the forms indicated. The upper terminal was drilled and drawn for the suspension of German silver wire, which was very carefully soldered in. If the hole did not fit the wire closely, instead of resorting to drawing, the terminal was sometimes symmetrically squeezed onto the wire with a three-jawed chuck. The lower terminal ended in a rectangular ring, with rectangular cross-section, in which the magnet-mirror holder hung. Between the upper terminal and the main cylinder of the rotor was a small bakelite washer which was screwed on to the rotor and pressed tightly against its shoulder by the upper terminal as it was screwed into place. The rotor was closely wound, between the bakelite ring and the lower terminal, with from two to eight layers of silk covered wire. No. 36 was at first used, No. 40 later; and in almost all cases the terminals were twisted together, drawn through a small hole in one side of the washer, and wound in a spiral about the upper terminal and suspension, being finally soldered to two small brass rods in the torsion head, as indicated in Fig. 6–1. Everything was done to make the whole body as nearly symmetrical about the axis as practicable. As the best rotors are exceedingly soft, it was usually necessary to restraighten them after winding.

In some cases the bakelite washer was replaced by a ring of brass, which was then made integral with the upper terminal. This is, of course, a better construction mechanically, but endangers the insulation.

In one rotor the upper and lower ends of the magnetic material, above *E* and below *F* in the figure, were replaced by brass, so that the whole magnetic part of the rotor was in an approximately uniform field.

One rotor was similar to those first described except that the magnetic part was about 8 cm shorter, and the lower terminal, of brass, about 8 cm longer. This made it possible to mount the rotor in the fixed magnetizing solenoid in such a way that the magnetic material was wholly inside the uniform part of its field.

The rotors without windings were similar to the last, but slightly shorter; they could, of course, be used only with the fixed-coil methods.

Other rotors had their upper terminals shaped for suspension from a flat strip instead of a round wire. One rotor was similar to the first except that the magnetic material was wholly replaced by brass. Another was of glass, with brass caps at the ends.

The rotors without windings were about 1.5 mm. in diameter. The diameters of the cores of the magnetic rotors with windings ranged from about 2 mm. to 3.2 mm. The core of the brass rotor was 2.4 mm. in diameter, that of the glass rotor 4.8 mm.

In the designations of the rotors the symbol I refers to iron, and P to permalloy; the subscript s prefixed indicates that the rotor was of the shorter type, all without this subscript being of the longer type; the subscript 36 prefixed, that it was wound with No. 36 wire, all others being wound with No. 40 wire, or being unwound; the exponents 1 and 2, that the rotor core was used in two states, before and after rewinding. The first subscript suffixed indicates the diameter in mm., the second the number of layers of wire. Thus, for example, ${}_{36}I^{1}_{2.4-2}$, means an iron rotor 2.4 mm. in diameter of the standard length wound with 2 layers of No. 36 wire, and indicates that the rotor was later rewound.

Photographs of two of the rotors, with their suspensions and torsion heads, are reproduced in Figs. 29-2 and 29-3. Resonance curves of two rotors, one wound, the other unwound, are given in Fig. 29-4.

§ 30. *The magnets, mirrors, and magnet-mirror holders.* The small magnets and mirrors indicated in Fig. 6-1 were carried on long brass holders as shown in Fig. 30-1. Two holders Nos. 1 and 2 were used in the earlier work. They were 23 cm long and otherwise nearly alike except that the straight part of one was of rod about 2.4 mm. in diameter, while the straight part of the other was about 1.5 mm. in diameter. At the upper end is a rectangular hook, at the lower a rectangular ring or hook, each with rectangular cross-section. The upper hook fits into the ring in the lower end of the rotor, the lower ring or hook into the upper hook of the lower suspension. At the distance of 8 cm from the top of one holder and 11.5 cm from the top of the other, two small thin brass plates were soldered parallel on opposite sides of the rod. By means of small projections and a small amount of universal wax, the plates carried two small parallel silver-on-glass mirrors. The area of each mirror was about 6 × 3 mm.[2] For a centimeter or so, at the mean distance of 20 cm from the top, the rod was flattened and two similar groups of small hardened steel rods normal to the brass rod, and nearly parallel to the mirrors,

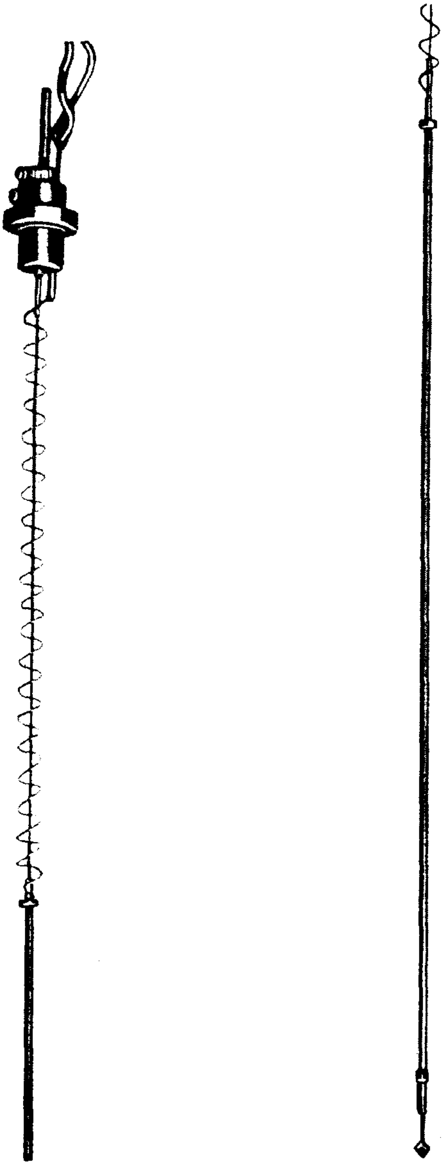

FIG. 29–2. The rotor $I^2_{2\cdot4-2}$ with suspension and torsion head.

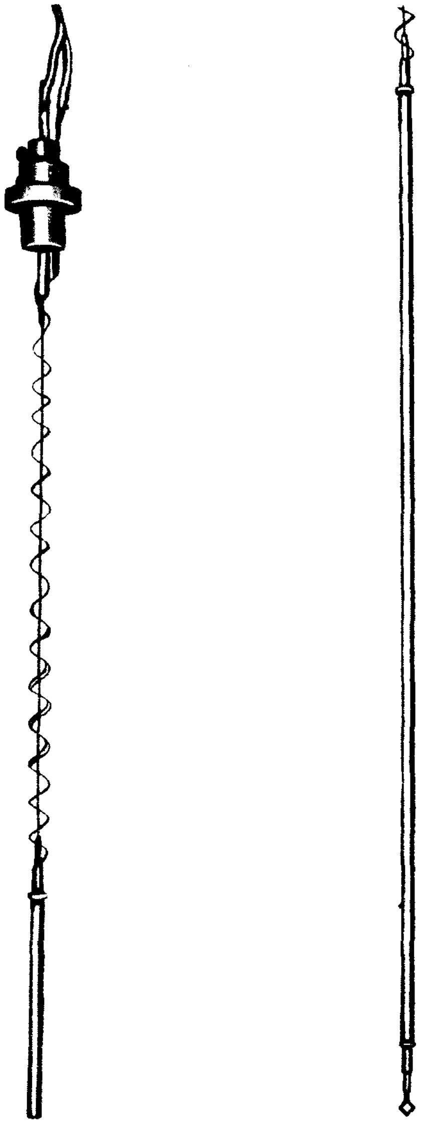

FIG. 29-3. The rotor $P^{2}_{3\text{-}2\text{-}4}$ with suspension and torsion head.

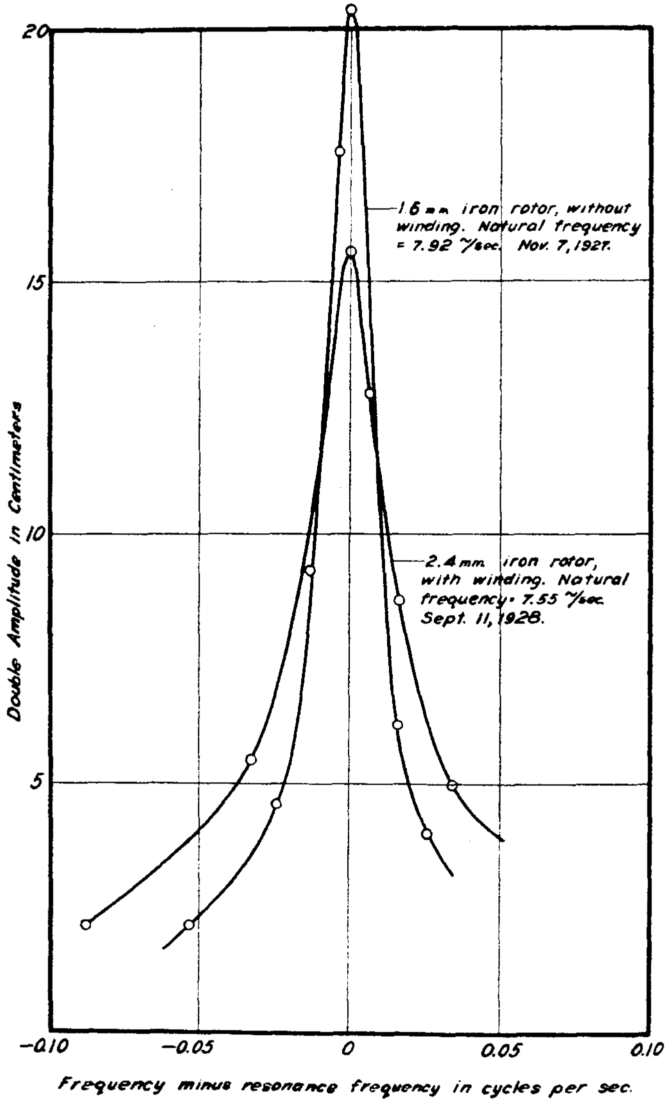

FIG. 29–4. Resonance curves for two types of rotor.

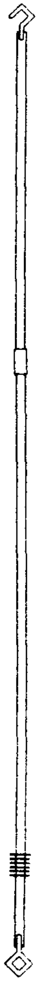

FIG. 30-1. Magnet-mirror holder No. 2.

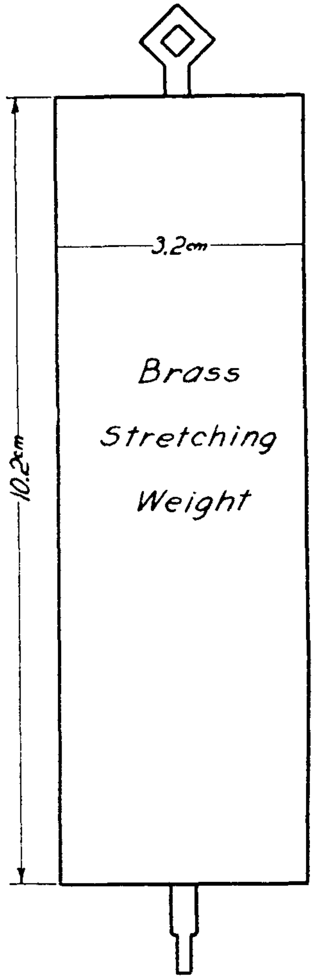

FIG. 31-1.

were securely cemented on with shellac. The steel rods on the thicker holder were similar to those used in the magnetometer during the latest work on magnetization by rotation. They were 5 mm. long, 0.17 mm. thick and 0.35 mm. wide. They were mounted with their axes 1 mm. apart in two groups, one on each side of the rod. In all the later work there were 6 magnets in each group, though as many as 8 were sometimes used earlier. The magnets of the completed group were magnetized together in the intense field of a suitable electromagnet. In the case of one of the holders the angle between the planes of the magnets and the planes of the mirrors was negligible; in the case of the other, this angle was about 4 degrees. The mirrors gave very good images with lamp and scale at distances greater than 4 m.

A third holder, used in all the latest work, was similar to the second, except that it extended 10 cm farther below the magnets and that the mirrors were thicker, had greater areas (nearly 6 × 7 mm.) and were mounted strictly parallel to the magnets, each resting against and being cemented to three points in its brass support. The surfaces were at first silvered, later gilded.[13]

§ 31. *The upper suspensions, lower suspensions and stretching weights.* All the upper suspensions were of German silver wire No. 28, 30, 32, or 34, or of strip rolled from No. 28 German silver wire. Most of them were about 15 cm. long. Several lower suspensions of German silver wire and strip, were used. They were all of about the same length, viz. 7.5 cm, but had different torsional constants in order to vary the frequency of the vibrating system conveniently. The wire of each suspension was soldered to rectangular hooks of the sort already described, the upper hook fitting into the ring or hook at the bottom of the magnet-mirror holder, and the lower into a rectangular ring at the top of the stretching weight.

Two weights, one of copper, the other of brass, were used. Each had a central rectangular ring at the top and a small axial rod of German silver at the bottom. The masses of the copper and brass weights were 361 and 685 grams, respectively. The latter is illustrated in Fig. 31–1. Weights were interchanged to move spurious resonance peaks farther from that at the vibrator's fundamental frequency.

§ 32. *Induction solenoids.* Several induction solenoids were used in the course of the work. The solenoid used in a great deal of the earlier work with magnetizing coil attached to the rotor was wound on a circular cylindrical bakelite tube 56 cm in length and with internal and external diameters of ½ and ¾ inch, respectively. The coil, of No. 30 cotton enamel copper wire, was wound with precision on the lathe in a single layer with a pitch of 44 turns per inch, successive turns not touching one another, and was shellaced in place. The coil was divided into two parts, one (the upper) 48 cm long, the other (the lower) 8 cm long; and the bakelite tube was likewise divided into two parts, one screwing into the other and bringing the two coils very near together. The two parts of the coil were provided with thin flexible twisted leads, and all were properly bound, first with friction tape, then with linen tape.

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[13] For the gilding, which was beautifully done, I am indebted to the kindness of Dr. C. Hawley Cartwright.

The torsion-head carrying the rotor fitted exactly in the top of the tube, and the ring at the lower end of the rotor then protruded a few millimeters from the lower end of the upper and longer part of the tube. After the rotor was in position, the magnet-mirror holder was slipped through the shorter tube and hung on the rotor's ring; then the shorter tube was screwed into the longer, and the leads properly connected together through a reversing switch.

Another tube, designed for work with both moving coil and fixed coil methods, and used in nearly all the later work, was similar to that just described except that its internal and external diameters were 1 inch and 1¼ inch, respectively, and that the short part slipped over the lower end of the longer part, instead of being screwed into it. The tube carrying the magnetizing coil generally used in the fixed coil method was held coaxially in this tube by means of amber bushings; and the torsion head carrying the rotor was then mounted in the upper end of this latter tube. It could also be mounted in a special amber bushing mounted directly in the top of the induction solenoid. In this case the fixed magnetizing coil was removed. In either case the ring at the lower end of the rotor was a few mm. below the lower end of the upper part of the induction solenoid, just as in the case described above. The arrangement is illustrated in Fig. 32–1. The inductance of this solenoid was about 2.4×10^{-3} henry.

Two of these coils were measured after long use, and each pitch found to be uniform to a degree much beyond the requirements of this work, and its mean value in exact agreement with that expected from the lathe, screw, and gears.

In the earlier part of the work other solenoids, less precisely wound, with approximately 70 turns per inch, 46 turns per inch, and 136 turns per inch, were used.

§ 33. *Magnetizing coils.* In the early part of the work, after preliminary experiments, the coils wound on the rotors consisted of two layers of No. 36 (SSC) copper wire, closely wound. The rotors wound later had either two, four or eight layers of closely wound SSC No. 40 copper wire. We were unsuccessful in getting proper insulation with enamelled wire.

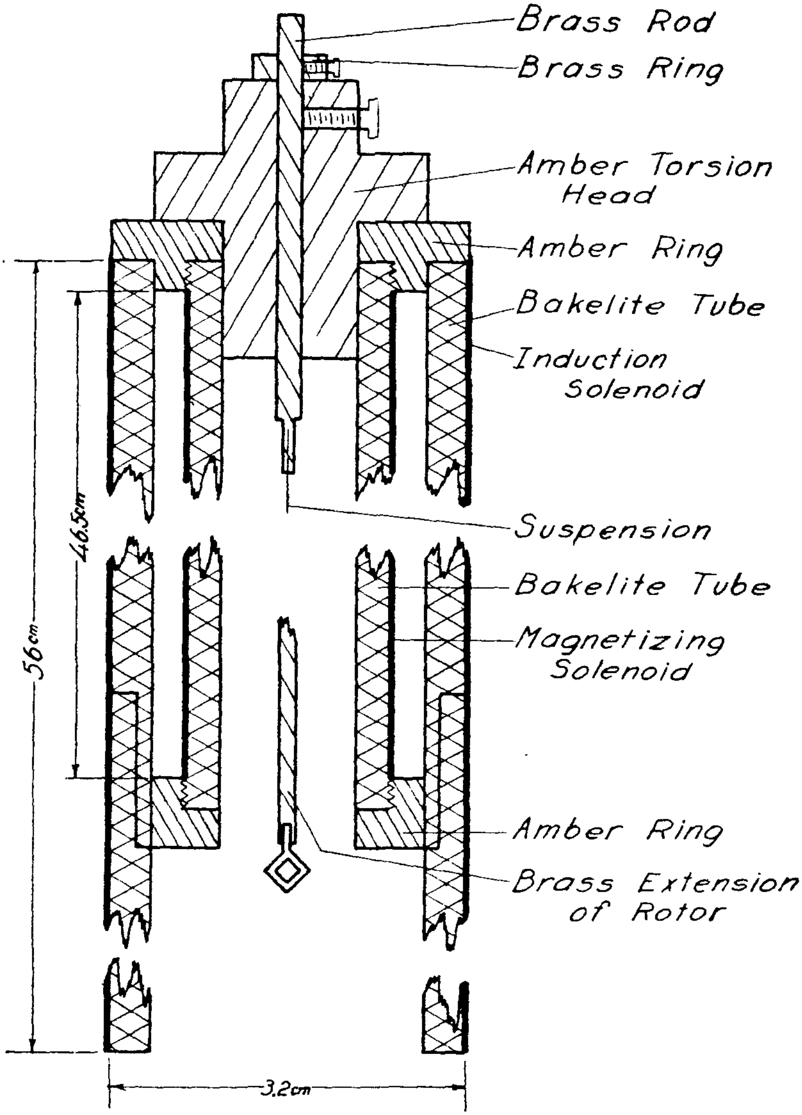
Brass Rod
Brass Ring
Amber Torsion Head
Amber Ring
Bakelite Tube
Induction Solenoid
Suspension
Bakelite Tube
Magnetizing Solenoid
Amber Ring
Brass Extension of Rotor
46.5cm
56cm
3.2cm

FIG. 32-1. Torsion head, rotor terminal, and induction and magnetizing solenoids.

The magnetizing coil used in nearly all of the work done by the fixed coil method was wound of No. 28 DDC copper wire on a circular cylindrical bakelite tube with amber bushings at the ends. It was 46.5 cm long (between amber bushings). The coil was in two layers, each wound to pitch very nearly 46 turns per inch. The inner and outer diameters of the tube were ½ inch and ¾ inch, respectively. The coil was provided with flexible leads, and all was properly taped.

For the production of stronger fields in the fixed coil method, another magnetizing solenoid thirty-eight cm long and with eight layers of the same kind of wire was wound on a thin glass tube of small diameter provided at the ends with amber bushings, by which it was held in place in the larger induction coil symmetrically about the rotor. The inner and outer diameters of this coil were 7 mm. and 14 mm., respectively.

§ 34. *The torque coil.* The small torque coil in the induction circuit acting on the suspended magnet was wound in two halves on bakelite bobbins of the dimensions indicated in Fig. 34–1. Each half coil was approximately 1.7 cm wide, and had internal and external diameters of 3 cm and 5 cm, respectively. Each coil contained 20 × 28 turns of No. 28 DCC copper wire. The bobbins, *A*, *B*, were held by two circular bakelite discs *C*, *D*, 10 cm in diameter and wound with subsidiary coils, the use of which, however, was abandoned early in the course of the work. The larger discs were carried by a bakelite base *E*, by which they were attached either to the framework carrying the vibrating apparatus, or to a special magnetometer for determining the constants of the coils. The resistance of the compensating-coil was about 33 ohms and its inductance about 0.004 henry.

§ 35. *The compensating coils* used to balance the induction effect of the magnetizing coil on the induction circuit were in the form of long solenoids. The secondary coil, 83 cm long, was wound with precision to the pitch 44 turns per inch of No. 30 cotton enamel wire on a circular cylindrical bakelite tube with internal and external diameters of 1 inch and 1¼ inch, respectively. As it turned out to be necessary, a few extra turns were wound around the secondary solenoid and connected in series with it.

The primary coils were duplicates of the 46.5 cm and 38 cm magnetizing coils described above, and similarly mounted, except that in the case of the first the amber bushings at both ends were of the proper diameters to hold the coil coaxially anywhere within the secondary.

§ 36. *Equipment for the production of the magnetizing current.* The current was obtained at will either from a 2 KW sine wave alternating current generator or from a storage battery and one of several commutators giving approximately rectangular waves. The rotating part of the commutator was mounted on the shaft of the generator; and the brush-holder, of bakelite, wasr igidly secured to its frame.

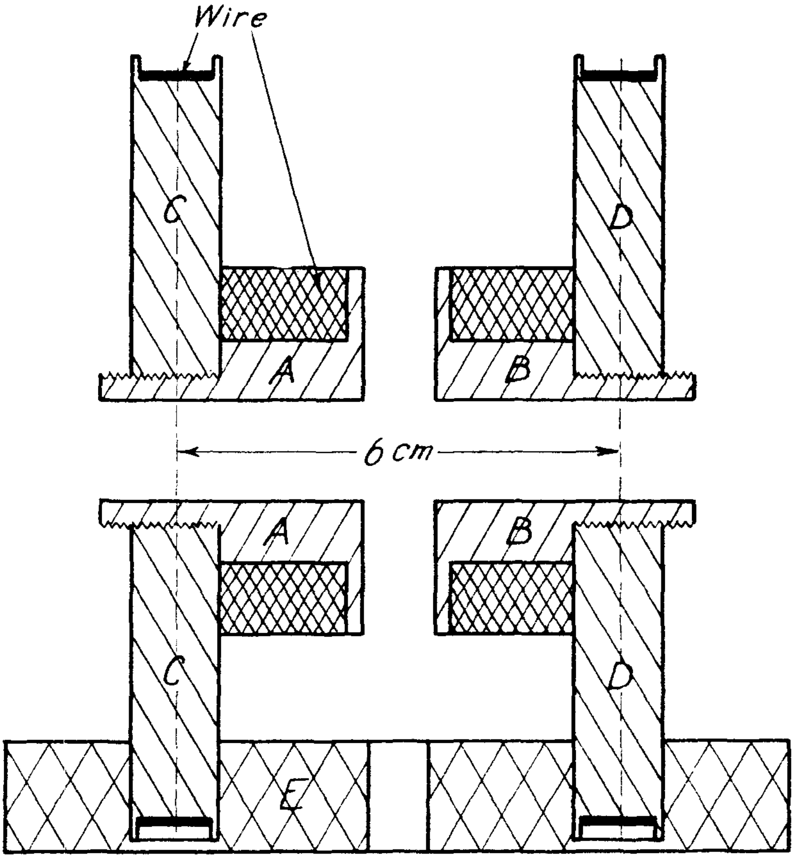

Fig. 34–1. The torque coils.

The generator was driven, through two spiral gear-boxes in series, by a 3 H.P. shunt-wound motor supplied from a storage battery of large capacity. The first gear-box reduced the speed of its driving shaft to 1, 1/2, 2/7 that of the motor; the second box reduced the speed in the same ratios. Thus the shaft of the generator, for any speed of the motor, could be driven at various speeds from that of the motor to 4/49 that of the motor, whose normal speed was about 30 revolutions per second.

The gear-boxes are the same as were used in the last investigation on magnetization by rotation previously referred to.

Several commutators making two reversals per revolution were used in the course of the work. The method of construction finally adopted is illustrated in Fig. 36–1. The rotating parts were made of bakelite and brass. The parts marked C and D were of bakelite; those marked A and B, of brass. All the conductors A_1 were connected together, likewise all the conductors A_2. The two brushes E_1 and E_2 were connected together, and similarly the brushes E_3 and E_4. The cylinders on which the brushes pressed were about 4 inches in diameter. In one commutator the insulating gaps were about 1/8 inch across; in two others about 1/64″ and 3/4″ respectively.

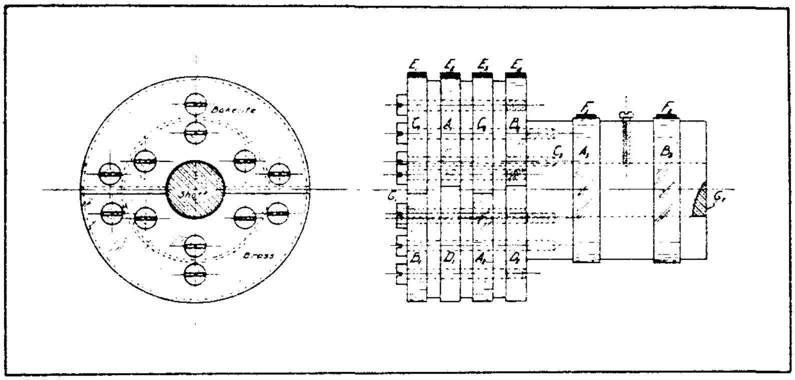

Fig. 36–1. One of the Commutators.

A fourth commutator was constructed in much the same way except that each circle contained three bakelite and three brass sectors, so as to produce 6 reversals per revolution. The insulating gaps of this commutator were about 1/32 inch across.

The brushes first used were of rolled German silver strip so bent and filed at the ends as to make contact over a very narrow strip of the cylindrical surface. Although these brushes worked well for a time when properly lubricated with vaseline and when the proper pressure was applied, they later gave a great deal of trouble. Laminated brushes of phosphor bronze, bound together with rubber bands or tubing to assist the laminations in the prevention of vibration, and applied tangentially, and also heavy graphite brushes with blunt wedge-shaped ends, pressed against the commutator surface by spiral springs are far superior. Such graphite brushes were used in most of the later work. As the figure shows, all the brushes were applied along the same element—and an upper element—of the cylindrical commutator surface.

All the apparatus described in this section was mounted in the sub-basement of the laboratory, two stories below the room in which the vibration experiments were made.

§ 37. *Frequency control.* Inasmuch as all the methods of experimentation used were resonance methods, it was necessary that the current frequencies, and therefore the motor speeds, should be maintained very nearly constant for long intervals. For this purpose, the method of motor control devised by F. Wenner some years ago, but never published by him[14] so far as I know, was adopted, with comparatively slight modifications, and has proven very satisfactory.

In the simplest form of Wenner's method, a resistance coil in series with the motor field coils is automatically short-circuited one quarter of the time during normal operation when the motor shaft makes exactly one revolution in the period of an electrically driven tuning-fork. This tuning-fork and a commutator on the motor shaft together increase the time of short-circuit (thus strengthening the mean motor field) if the motor gains on the fork, and decrease the time of short-circuit (thus weakening the mean motor field) if the motor lags behind the fork. The motor and the commutator are thus automatically made to rotate in the period of the fork with fluctuations of the phase difference between fork and commutator which may be made very slight.

A diagram of the arrangement used in this work is given in Fig. 37-1, where d is the resistance coil automatically short-circuited, α the tuning fork and (2)-(5) the commutator.

The tuning fork contact $\alpha\beta$ is closed one-half the time, as indicated by the imaginary equivalent brush and rotating circle (1). One semi-circle (black) is supposed to be of metal and permanently connected with the fork α, the other (white) to be of insulating material, while the wire β is connected to the brush. The whole circle rotates uniformly with the period of the fork in the direction of the arrow.

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[14] A brief description is given in F. A. Laws' *Electrical Measurements*, p. 432 (N. Y., 1917).

In the type of commutator (2)–(5) usually used, there are four coaxial circles of brass and bakelite firmly screwed together and mounted on the motor shaft, but all insulated therefrom. Circle (2) is of brass, while the remaining circles, or rings, are half brass and half bakelite, the bakelite halves being shown white. The metal parts of all four circles are in permanent electrical connection. The four circles are pressed upon by four laminated phosphor-bronze brushes along a line parallel to the shaft. The phase relations between the conducting semi-circles are permanently as indicated in the figure, (4) being a quarter period ahead of (3), and (5) being a quarter period behind (3). In the figure, (3), the semi-circle which short-circuits the resistance coil, is a quarter period behind the tuning fork (1)—the normal phase difference for steady running. In this position, as the figure shows, the resistance d is short circuited just one quarter of the time when the switch γ is open or thrown to the right.

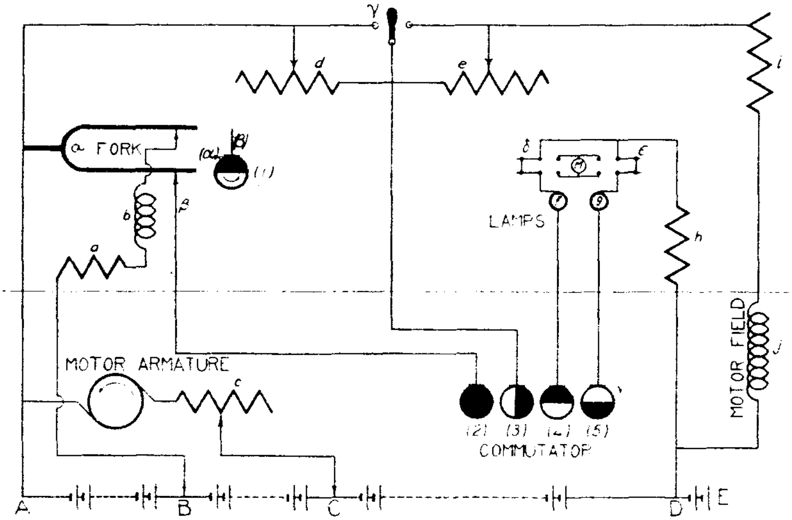

Fig. 37-1. The frequency control arrangement.

In order to use the same tuning fork in controlling the motor when driven at very low speeds, a second type of commutator was built on the same principles as the first, but with the semi-circles (3), (4), (5) replaced by quadrants in the same phase relations. With this commutator the fork of course controlled the motor when driven at half its own frequency.

In Wenner's apparatus two similar incandescent lamps *f* and *g* connected to the brushes of circles (4) and (5) and, through switches δ and ε and (if desired) a rheostat *h*, to the battery serve to indicate the phase relation between (3) and (1).

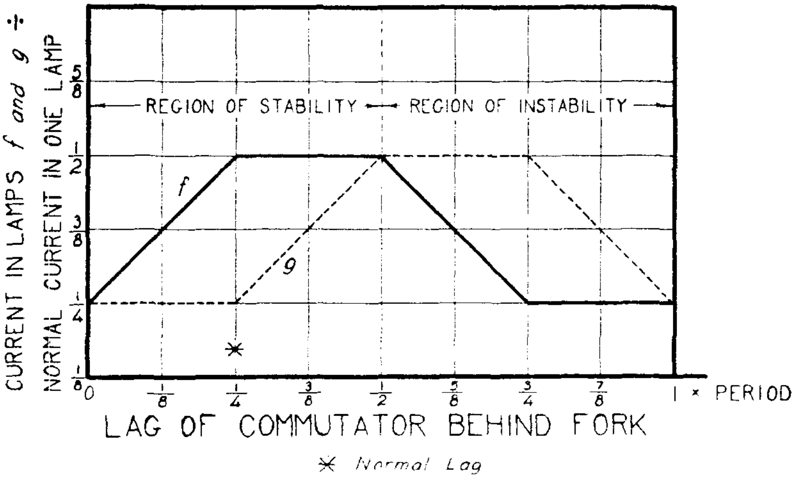

FIG. 37-2.

The arrangement shown in the figure permits an ammeter to be used, and more effectively, for the same purpose. If the current in *f* is to be read, both switches are thrown to the right; if that in *g* is to be read, both are thrown to the left. If the sum of the two currents is to be read, δ is thrown to the right and ε to the left. If the lamps are to be used alone, δ is thrown to the left and ε to the right. The relations between the currents in *f* and *g* and the lag of the commutator (semi-circle (3)) behind the tuning fork (1) are easily worked out from the diagram and are given in Fig. 37–2. In the region of stable running *f* is greater than *g*, the difference being a maximum for the normal lag.

When the switches are thrown so that the ammeter reads the sum of the currents in f and g, the relation between current and lag is that given in Fig. 37-3. With this arrangement the current for normal running is $\frac{3}{4}$ the normal current in one branch. If the current is greater, the motor is too slow; if it is less, the motor is fast.

To get the motor into synchronism with the tuning-fork, the switch δ is first thrown to the left, thus cutting out the resistance d and inserting in the motor field a resistance $e = \frac{3}{4} d$. The effect of e on the speed is the same as that of d during normal running, since then d is short circuited one quarter of the time.

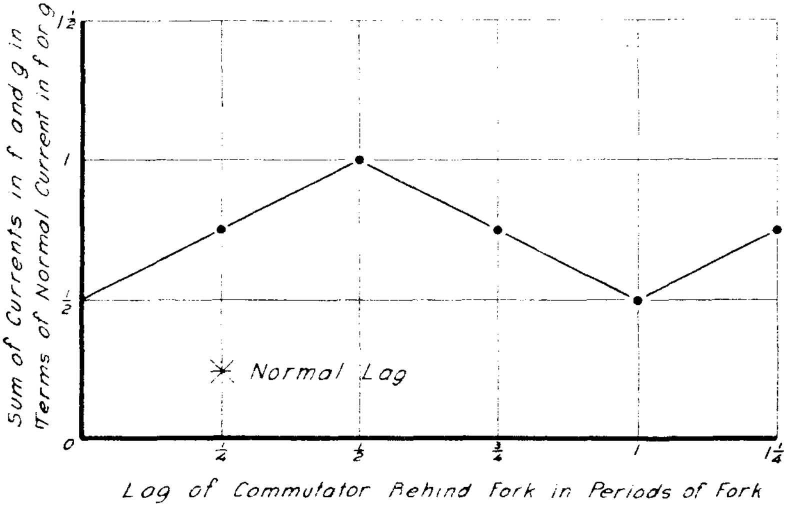

FIG. 37-3.

The motor is made to run at very nearly synchronous speed by adjusting the armature and field voltages AC and AD, and the armature and field resistances c and i. As synchronism is nearly approached the fluctuations of the lamps or ammeter become very slow. When this state has been attained, and when the current in f is the maximum and that in g the minimum (or their sum equal to the mean of the extreme values of the current when the motor is out of synchronism), the switch γ is thrown to the right. Automatic control then sets in. If the motor has been in operation at nearly the correct speed for some time, so that steady temperatures have been reached, the synchronous speed may be maintained for hours, with only slight fluctuations in the phase difference between commutator and fork. With the arrangement corresponding to Fig. 37–3 the fluctuation of phase difference is clearly equal to the fluctuation of the current divided by twice the difference between the minimum and maximum currents when the motor is thrown slightly out of synchronism.

The tuning-fork was one of the Western Electric Company's standard electrically driven forks having a normal frequency of about 48 periods per second. It was provided with several pairs of brass and bronze weights which, one or more pairs at a time, were symmetrically clamped by steel screws to the prongs in suitable positions to give (through the gear-boxes) frequencies equal to the normal frequencies of the vibrating systems (and other frequencies for certain tests). Rough adjustments could be made by changing or moving the heavier weights near the free ends of the prongs; finer adjustments by moving smaller weights at greater distances from these ends. In Wenner's apparatus provision was made for frequency adjustment by continuous alteration of effective prong-lengths. A modified form of Wenner's device was used in the earliest part of this work, but was soon abandoned in favor of the devices just mentioned. The contact $\alpha\beta$ (Fig. 37–1) was so adjusted as to be closed one-half the time when the fork was in operation. The fork was ordinarily operated by a 30 volt battery through 500 ohms external resistance, the mean current through the electromagnet $a\ b$ being then about 0.02 ampere.

§ 38. *The constants of the induction solenoids.* The mean constant of either solenoid for the full length of any one of the rotors used is very nearly equal to $4\pi \times$ the number of turns per cm, i. e. $4\pi \times 44/2.54 = 217.68$ e.m.u. $= G$. For any of the arrangements the mean constant γ_0 is given by the relation $\gamma_0 = G(1 - \alpha)$, where α is very small because the ratio of solenoid diameter to length is small, because the solenoid always extended far beyond the rotor ends,[15] and because the gaps between the upper and lower parts of each solenoid were small. For the different arrangements the quantitites α were carefully determined, and ranged from 0.0015 for a short or long rotor in the smaller solenoid to 0.0043 and 0.0078 for a short and long rotor in the larger solenoid.

The inductances of the two solenoids were approximately 6×10^{-5} henry and 17×10^{-5} henry.

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[15] Except in a few early experiments.

§ 39. *The large Helmholtz coil.* The constant Γ_0 of the small compensating coil was determined electrically by comparison with the constant Γ_0' of a specially constructed Helmholtz coil. The same Helmholtz coil, together with a standard current, served to determine the intensity of the magnetic field by which the moment of the vibrator's magnet was found.

The diameter D of the Helmholtz coil was approximately 26 cm, and the axial distance A between centers of corresponding wires in the two halves was approximately 13 cm. Each half-coil contained 10 turns, placed close together, and was nearly 6 mm. wide.

The two half-coils were wound in accurate cylindrical grooves about 11 mm. wide turned in a round tube, 3.2 mm. thick, of bakelite micarta; see Fig. (39–1). The wires were of DCC copper No. 25 and were pressed tightly against one another and the outer shoulders of the grooves. They were held in position by tightly wound fish-cord filling the rest of the grooves and bound to them with shellac along the inner edges.

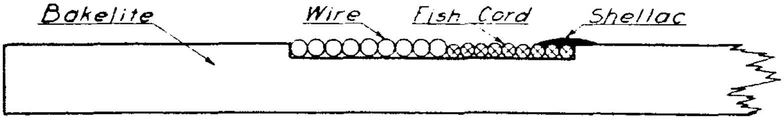

FIG. 39–1.

The end corrections and lead corrections were negligible. The axial distances A were measured with a dividing engine kindly loaned by Mr. Babcock of the Mt. Wilson Observatory. The mean value from several hundred properly spaced measurements is 13.170 cm and is correct to a degree of precision much beyond the requirements of this work.

The overall diameters were measured with calipers and a steel scale by Brown and Sharpe; also with a steel tape compared with the same scale. Both sets of measurements (taken over two years apart) agreed closely, but the latter were more precise. The tape measurements, properly corrected for tape thickness and wire diameter, gave for the mean diameter $D = 25.93_2$ cm. The diameters of the two half-coils differed by only about 1 part in 4000.

The constant of the coil is thus

$$\Gamma_0' = \frac{2\pi \times 20 \times (12.966)^2}{\{(12.966)^2 + (6.585)^2\}^{\frac{3}{2}}} = 6.869 \text{ e.m.u.} \qquad (39\text{–}2)$$

While in use, this coil was mounted with its axis horizontal in a brass frame as indicated in Figure 39–2, the frame being mounted on a brass tripod.

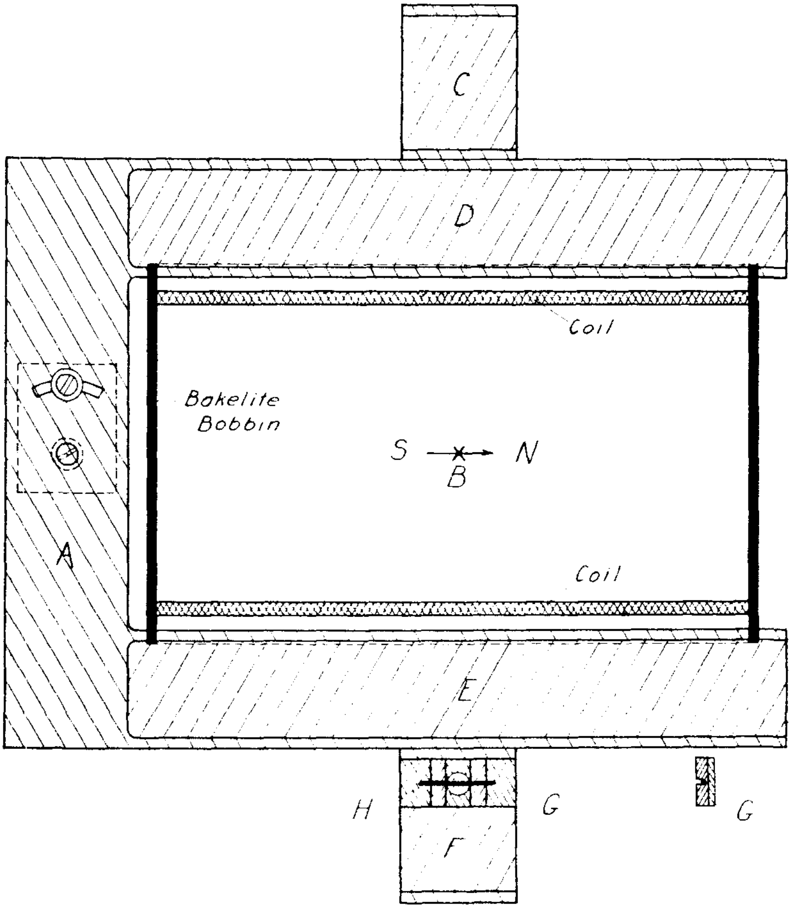

FIG. 39–2.

§ 40. *The constant of the torque coil.* For making the comparisons between the constant Γ_0 of this small coil and that of the larger Helmholtz coil, a special magnetometer and stand were constructed. When the small coil was removed from the vibration apparatus and set properly on this stand, and the stand leveled and adjusted in height, the magnetometer magnet, almost exactly similar in construction to that on the vibrator, was at the center of all the coil systems, and their turns were parallel to one another.

In making the comparisons each coil, as is usual, was connected in series with a suitable rheostat; the two systems were then placed in parallel and connected through a reversing switch with the terminals of a storage battery. One of the rheostats was adjusted until the magnetometer needle gave a very much smaller deflection on reversal of the currents than that due to either coil separately, and then interpolation was made for zero deflection. The constants of the two coils were then proportional to the resistances of the respective branches.

In this way, in February, 1929, long after earlier less precise measurements, Γ_0/Γ_0' was found to be 266.52, corresponding to $\Gamma_0 = 1830.7$ e.m.u. A later determination, made in November and December, 1930, gave $\Gamma_0/\Gamma_0' = 266.72$ and $\Gamma_0 = 1832.0$. The difference between the results of the two measurements is 0.07 per cent, which is much beyond the possible experimental error. It cannot be attributed to a change in Γ_0', as control observations on the latter several years apart have shown.

The smaller value of Γ_0 has been used in the calculations for all observations before 1930; the larger for those of 1930.

It was originally planned to use the intermediate Helmholtz coil in every series of observations in order to redetermine the constant Γ_0 for the actual position of the vibrator magnet, and also in order to produce deflections of the vibrator magnet from which its moment, once determined absolutely, might be readily found as a function of the time. Experience, however, proved that the small errors made in setting the vibration system produced negligible errors[16] in the constant of the smaller coil; and it turned out to be more convenient to use the magnetometer for the necessary moment determinations than to use the Helmholtz coil.

§ 41. *The constants of the principal magnetizing coils.* The constants of the solenoids wound on the rotors were approximately 940 and 710 e.m.u. per layer for the No. 40 and No. 36 wire, respectively, and for points at the centers. The constants at the centers of the fixed magnetizing coils with 46 turns per inch per layer were approximately 270 e.m.u. per layer.

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[16] Thus an axial displacement of the magnet from the center by about 1.25 mm. changed the constant by less than 1 part in 3000 and a lateral or vertical displacement of 1 mm. changed the constant by about 1 part in 3000; while in the vibration work the adjustment was in general correct to a fraction of a millimeter.

§ 42. *The moments of the vibrator magnets.* These were determined at suitable intervals with the aid of a simple magnetometer and the large Helmholtz coil.

The magnet of the magnetometer was one of the circular gages of polished steel, 1 mm. thick and 20 mm. in diameter, made at the Bureau of Standards, magnetized along a horizontal diameter, and suspended by a long strip of phosphor-bronze. One of the polished surfaces served as a mirror, for reading deflections by lamp and scale. The magnetometer box was of copper, and provided adequate damping. The scale distance was nearly 2 m.

The Helmholtz coil, carried by its bakelite bobbin, was mounted on its brass frame *A*, Fig. 39–2, with its center coincident with that of the steel magnet *B* and its axis horizontal and normal to the mirror, in the magnetic prime vertical.

The horizontal frame *A* which carried the coil had parallel grooves *C*, *D*, *E*, *F* milled in it symmetrically on both sides the coil and perpendicular to its axis. Small carriers *G*, two of brass and one of translucent bakelite, each with rectangular base and sides normal thereto, were accurately constructed to hold the vibrator magnets *H* symmetrically with the magnetic axes horizontal and normal to the greater lengths. The width of each magnet-carrier, and the distances between corresponding groove-edges on opposite sides of the magnetometer, were accurately measured. The groove-edge distance principally used was measured with the engine mentioned above. The carrier could be so set, by pressing it to the edge of a groove and moving it along this edge, that the axis of the vibrator magnet pointed to the center of the magnetometer magnet.

Since in the course of a determination of moment the carrier was placed the same number of times on each (east and west) side of the magnetometer; and since the magnet, fixed in the carrier, was turned east and west the same number of times; twice the mean distance between the centers of the two magnets was equal to the distance between the groove edges plus or minus the width of the magnet carrier, according as inner or outer groove edges were used.

In nearly all of the work the brass carrier No. 2, 1.93_4 cm wide, was placed against the inner edges of two grooves distant 25.96_8 cms apart. Thus the mean magnet distance was 13.95 cm.

Let D denote the mean double deflection produced by reversing a standard current C in the Helmholtz coil, d the mean double deflection produced by reversing the axis of the vibrator magnet, m_0 the moment of this magnet, and r the distance between the centers of the two magnets. Then, if we can neglect the linear dimensions of the magnets, and assume proportionality of deflections and intensities,

$$\frac{d}{D} = 2m_0/r^3 \div \Gamma_0'C \quad \text{or} \quad m_0 = \frac{\Gamma_0'Cr^3}{2} \cdot \frac{d}{D}$$

The finite length of the vibrator magnets, viz. 4.2 mm. (= 5 mm. × 5/6), necessitates a negative correction − 0.18 per cent to m_0 as given by the above formula. The finite diameter of the magnetized mirror, viz. 20 mm., on the assumption that it is uniformly magnetized parallel to the horizontal diameter with poles distributed around the circumference, necessitates two corrections to m_0: one, equal to + 0.76 per cent, on account of the fact that the lines of induction from the vibrator magnet make (at the edge of the disc) an angle with the disc's axis equal approximately to $3 \times \frac{1}{2} \times \frac{1}{14}$ radian; and the other, equal to + 0.77 per cent, because the distance from the center of the vibrator magnet to the edge of the disc is greater by about 0.036 cm than the distance between the centers of the two magnets.[17]

The deflections are not, as above assumed, strictly proportional to intensities, and d/D must be multiplied by the correction factor 1.0040, or increased by 0.4 per cent, as careful experiments have shown.

The total correction is thus (+ 0.77 + 0.76 − 0.18 + 0.40) per cent = + 1.75 per cent. The true[18] value of m_0 is thus given, with negli-

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[17] The correction whose theoretical value on the approximate assumption made is 0.76% + 0.77% = 1.53% was also determined experimentally with great care. The experimental value is 1.56%. Thus the difference between the correct value, which must be somewhat *less* than the theoretical value, and this theoretical value is negligible.

[18] When the observations are being made, the magnetometer magnet, whose moment is about 17 e.m.u., is always turned in such a way as to produce a magnetic intensity at the position of the vibrator magnet whose component h parallel to the axis of this magnet is in the direction of the axis and thus tends to increase the moment. The magnitude of h, however, for the maximum deflections was only 1.4×10^{-4} e.m.u., so that its effect on m_0 was entirely negligible.

Also, if θ denotes the angle between the axis of the magnet system and the gible error, by the equation

$$m_0 = \frac{\Gamma_0' C r^3}{2} \frac{d}{D} (1 + 0.017_5) \text{ e.m.u.} \tag{42-1}$$

The current C was determined by means of a Wolff 15,000 ohm potentiometer, a Weston cell, and a standard coil with resistance 9.994 ohms, all properly checked by comparison with other standards and known to introduce only negligible errors.

A representative example of one determination of the moment of the vibrator magnet-system mostly used (No. 3) follows:

A. Time of beginning: $1^h\ 30^m$ a. m. June 27, 1928.

B. Current $C_1 = 0.004209_5$ ampere.

Deflection $D_{12} = 43.51 \pm 0.03$ cm. (from 18 scale readings at equal intervals).

Current $C_2 = 0.004209_5$ ampere.

C. Magnetometer readings and deflections 1 (from 32 readings at equal intervals):

N pole E		N pole W		N pole W		N pole E	
(1)	− 4.51 cm	(2)	+ 4.23 cm	(3)	+ 4.08 cm	(4)	− 4.40 cm
(8)	.47	(7)	.21	(6)	.07	(5)	.40
(9)	.48	(10)	.27	(11)	.10	(12)	.37
(16)	.43	(15)	.30	(14)	.11	(13)	.37
(17)	.46	(18)	.30	(19)	.12	(20)	.32
(24)	.40	(23)	.33	(22)	.15	(21)	.35
(25)	.41	(26)	.32	(27)	.18	(28)	.28
(32)	.40	(31)	.35	(30)	.17	(29)	.30

From (1) − (8) and (25) − (32), d = 8.60 cm
From (9) − (24), d = 8.60 cm
d_1 = 8.60 cm

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horizontal when the moment is measured, the quantity obtained is $m_0 \cos \theta$ instead of m_0. *An error as great as 2° is setting the magnet, however,* would affect the measurement by not much more than 1/20%, so that the error made by assuming $\theta = 0$ is negligible. Such error as there is makes the estimated values of m_0 and ρ too small.

When the magnet is in position at the center of the torque coil, the angle θ' between its axis and the planes of the turns of the coil was, in all the later work, made so small that the difference between $m_0 \cos \theta'$, to which the torque is proportional, and m_0 was negligible. Slight errors from this source, making the estimated value of the quantity ρ too large, occurred in some of the earlier and less precise work, and may account for a part of the small difference between the earlier and later results.

D. Current $C_3 = 0.004209_5$ ampere.
Deflection $D_{34} = 43.47 \pm 0.01$ cm (from 26 scale readings at equal intervals).
Current $C_4 = 0.004209_0$ ampere.

E. Magnetometer readings and deflections II:

Thirty-two observations similar to those of C. From (1)-(8) and (25)-(32), $d = 8.61$ cm. From (9)-(24), $d = 8.62$ cm, Mean $d_{II} = 8.615$ cm.

F. Current $C_5 = 0.004209_0$ ampere.
Deflection $d_{56} = 43.47 \pm 0.02$ cm (from 20 scale readings at equal intervals).
Current $C_6 = 0.004209_2$ ampere.

G. Time of ending: $2^h\ 50^m$, a. m.

From B, C, and D, $m_{0I} = 0.777_0 \times 1.0175$ e.m.u.
From D, E, and F, $m_{0II} = 0.778_2 \times 1.0175$ e.m.u.
Mean $m_0 = 0.777_6 \times 1.0175$ e.m.u.

The moment of each magnet-system was plotted as a function of the time, and the value needed for any determination of the gyromagnetic ratio was taken from the appropriate curve. The average departure of the observed values from the curves was not greater than about 1 part in 1000.

§ 43. *Neutralization of the earth's magnetic field.* The total intensity of the earth's field in the region occupied by the rotor was very nearly annulled by passing steady electric currents through three compensating coils.

One coil, A, compensated the northerly component X; another, B, the easterly component Y; the third, C, the vertical component Z. A and B are what might be called oblong Helmholtz coils, designed to produce very nearly uniform fields in a central cylindrical region. In each system are two similar elongated approximately rectangular coils so mounted in parallel planes that the wires form approximately edges of a rectangular parallelepiped. Each system is about 2 meters long.

Each coil of A has five turns of heavily insulated wire, the longer sides being about 36 cm apart, and the planes of the two coils being about 21½ cm apart. Each coil of B has only one turn of bare wire, the longer sides being about 23 cm apart and the planes of the two being about 13 cm apart. Each coil pair produces in the central region an intensity parallel to the shortest sides of its parallelepiped.

The coils were mounted in a bakelite frame, with brass distance pieces at the ends to keep the horizontal dimensions of *A* constant, and wood distance pieces for *B*. The coils were always kept stretched tight, the wires of *B* being mounted with permanent tension, those of *A* being stretched by a spring device at the top of the frame.

On account of the large amount of iron used in the construction of the laboratory, the magnetic field about the rotor was not strictly uniform. Careful measurements of the component *X*, however, showed that it increased upward linearly by only about ½ per cent in the length of a rotor. On this account the wires in each coil of *A* were not left strictly parallel, but were set closer together at the top by nearly ½ cm for each meter of length. The field produced by *A* itself over the length of a rotor, when its wires were parallel, was uniform to 1 part in 1800.

The uniformity of *Y* was not investigated; but *Y* was only a small part of *X*, viz. about one twelfth.

System *C* is a pair of horizontal square coils 1.75 m. on a side, mounted with their wires parallel and coaxial one below the other at a mean distance of about 0.9 m., and traversed by currents in series in the same direction. Each coil has 45 turns of DCC No. 20 copper wire. These square coils are easier to build than true (circular) Helmholtz coils, and produced an axial field nearly enough uniform. Indeed, the variation of their axial intensity from the center of one of the larger rotors to either end is only about 1 part in 600. The degree of uniformity of *Z* was not investigated, as only comparatively rough vertical compensation was necessary. The coil system *C* was hung from the ceiling by heavy brass wires attached below to brass slides permitting vertical adjustment, with set screws, and making it easy to level the coils with exactness.

The currents traversing *A*, *B*, and *C* were in general furnished by storage batteries used at the same time for no other purpose, and were measured at first with ammeters; later the potentiometer and adjunct ammeters were used for *A* and *B*.

§ 44. *Standardization of the compensating coils.* The proper currents to compensate *X* and *Y* at various times were determined by means of a special flip coil about an inch in diameter and as long as a rotor, the currents being adjusted until the flips produced a small deflection with a sufficiently sensitive galvanometer, and the currents for zero deflection being found by interpolation. The *X* and *Y* compensating currents were determined to about 1 part in 1000 and 1 part in 100 respectively. The currents to compensate the vertical component *Z* was determined in much the same way, an earth inductor being used as a flip coil. The Z compensating current was set to about 1 part in 900, but the calibration was not so precise as this.

Determination of compensating currents at time of principal experiments. It was, of course, impossible to make the standardizing observations except at infrequent intervals. Hence, as the earth's magnetic field is continually fluctuating, it was necessary to make in connection with them other observations which would render it possible to determine the proper compensating currents at any time at which the principal experiments were in progress.

The following procedure was adopted: Before and after the flip coil determinations for X and Y, magnetometer scale readings were taken for a known current reversed through the standard Helmholtz coil; and similar magnetometer and current readings were taken at the times of the principal experiments.

Let X_0 and Y_0 denote the correct values of the compensating currents, D_0 the mean deflection of the magnetometer (without control magnets) produced by the current C_0, θ_0 the magnetic declination, and R_0 the zero reading on the magnetometer scale, all at the mean time T_0 of the standardization; and let X, Y, D, C, θ, and R denote the corresponding quantities at any other time T at which principal experiments are to be made. Then, very approximately,

$$X = X_0 \frac{C}{C_0} \cdot \frac{D_0}{D} \tag{44-1}$$

and

$$Y = Y_0 \left(1 + \frac{X - X_0}{X_0} + \frac{\theta - \theta_0}{\theta_0} \right) = Y_0 \left\{ \frac{C}{C_0} \frac{D_0}{D} + 6 \frac{(R - R_0)}{d} \right\} \tag{44-2}$$

since $\theta_0 = 1/12$ radian, and $\theta - \theta_0 = (R - R_0)/2d$, where d is the scale distance.

Similarly, an earth inductor is set up near the magnetometer in a permanent position, and at the time of calibration is reversed through a galvanometer, first in such a way as to cut only the horizontal lines of the earth's field, giving the deflection D_{H0}; second, in such a way as to cut only the vertical lines, giving the deflection D_{Z0}. Similar observations at the time T give deflection D_H and D_Z. Then, very approximately, if Z_0 denotes the vertical intensity at the center of the coils at the time T_0, and Z the intensity at the time T,

$$Z = \frac{Z_0}{X_0} \frac{D_{H_0}}{D_{Z_0}} \frac{D_Z}{D_H} \cdot X = Z_0 \frac{D_{H_0}}{D_{Z_0}} \cdot \frac{D_Z}{D_H} \cdot \frac{C}{C_0} \frac{D_0}{D} \tag{44-3}$$

From (44-1), (44-2), (44-3), X, Y, and Z can be found at any time T.

45. *Observations and results.* By the methods now described a large amount of work has been done on rotors of permalloy and soft iron. They are illustrated by examples below and summarized in six groups, designated as Groups *O*, I, II, III, IV, and V. Group *O* contains the observations made by the null-graphical method, the others those made by deflections. The latter are also referred to as Deflections I, II, III, IV, V.

Groups *O*–III were made without any attempt to eliminate by reversals the effect of any inequality of the half-cycles of current. In the course of these observations, however, the magnetizing coils were removed from and connected into the circuit a great number of times with no attention whatever as to which wire from the generator or commutator was connected to a given terminal. The connections were thus probably reversed many times in the course of the work, and the error involved largely eliminated. That this is the case is indicated by the close agreement between the means of the results obtained. In much of this work the effects of the inequality were small, but they became prominent in a part of Group III, and led to the discovery of the source of the trouble.

Among the magnetostrictive effects alluded to in § 9, and in addition to that treated in § 17, is a small ripple, usually of (relatively) high frequency, superposed on the fundamental vibration. It appears to be a forced vibration due to axial asymmetry of the suspended system and the magnetostrictive elongations produced by one or more of the harmonics of the magnetization of the rotor. The ripple frequencies are thus twice the frequencies of these harmonics. When the ripple exists, the horizontal band of light on the scale is crossed by vertical shaded bands, or lines if the amplitude is small. When the rotor is unsuccessfully mounted, or the asymmetry otherwise too great, the appearance is not steady, and precise observations cannot be made. When the rotor is well mounted, but still without perfect symmetry, the appearance is very steady. The band of light in this case, when the amplitude is not too large, resembles somewhat a line spectrum with a continuous background.

This magnetostrictive ripple was present in many of the earlier observations; but it was judged to be absent in all or nearly all of those in Group V. Its presence, however, does not produce any error in the calculated value of the gyromagnetic ratio, either in the null-graphical method, or in the deflection methods of Groups II–V when the amplitudes A_1 (or A) and A_2 are equal. When they are only approximately equal (as is usually the case) the error is of the second order and negligible. The vanishing of the error is due to the fact that the ripple affects amplitudes on both sides the minimum (when equal) exactly alike. In the method of Group I, where only the amplitude A_G can contain the ripple, the calculated value of ρ is slightly too large if the ripple is present.

The earliest observations were made in Group O, when the details of the procedure necessary to secure the best operation of the apparatus were imperfectly understood. A number of sets of observations were rejected because of a high minimum (probably due largely to the unsuspected effect of half-cycle inequality) in one of the two halves of the set, or in both halves, or because the compensating currents necessary to produce low minima differed by more than very small amounts. In almost all cases throughout the investigation the currents were set and maintained practically constant throughout the complete set; while in successive sets the first half was made with the axis of the vibrator magnet pointing alternately east and west, in order to correct, on the whole, for any slow fluctuation in the earth's field.

Most of the work described in this article has had to be done at or near the time of the sun-spot maximum. The work has therefore been greatly interfered with by numerous, and sometimes violent, magnetic storms, which have made work impossible on a large number of nights.

§ 46. *Observations for* ρ *by the null-graphical method.* (Group O). In the practise of this method, the earth's field is neutralized (or approximately neutralized), and the vibrating system set into resonance (or approximate resonance) so that a large amplitude is obtained with the induction circuit open; then the circuit is closed and the resistance adjusted until the amplitude is cut down to a few cm at most. Then the amplitude is determined for a number of different resistances, first decreasing until after the minimum is well past, then increasing until the minimum is well past again. A curve (or the reduced straight line) plotted from all the observations near the minimum gives the resistance (or the conductance) for the minimum approximately independently of the changes in conditions during the process. The observations are then repeated with the torsion head turned through 180°, and the connection between induction solenoid and compensating coil reversed. In order better to eliminate errors due to changes in the earth's field, these pairs of observation groups on successive days are made in opposite orders.

Examples of curves obtained in this manner are given in Fig. 46–1. These curves were obtained in October 1927, with the rotor ${}_{s}P_{2\cdot4-2}$ mounted in the 1¼″ induction solenoid, a current of 100 milliamperes (rectangular wave) traversing the coil wound on the rotor. The moment of the vibrator magnet when these results were obtained was 0.7860×1.0175. From this, the resistances or conductances for minimum amplitude given by the charts, the coil constants given in the text, and the correction $-$ 0.002 for coil moment and electron inertia, the value of ρ/ρ_L for $N\ P\ E$ is 1.054, that for $N\ P\ W$ 1.065; mean 1.060. The observations from which Fig. 46–1 was drawn were obtained under poor conditions when the apparatus was constantly swinging. Curves of the same kind obtained now and under good conditions would be much better still.

All the observations between May 21, 1927, when the first good results were obtained, and February 23, 1928 were made by this general method, although details were not always the same. In this interval 16 sets[19] gave for iron, with the current traversing a coil wound on the rotor, $\rho/\rho_L = 1.056 + 0.011$; while 2 sets in which the magnetizing coil was fixed to the earth and the iron rotor had no winding gave 1.037 ± 0.002. The mean for the iron rotors is 1.054 ± 0.010.

In this same interval 21[19] sets on permalloy rotors with windings traversed by the current gave $\rho/\rho_L = 1.063 \pm 0.017$, while 9 sets in which the magnetizing coil was fixed gave $\rho/\rho_L = 1.069 \pm 0.005$. In 4 of these latter the rotor was the same as in the 21 sets just mentioned; in 5 a rotor without winding was used.

The rotors used were as follows: ${}_{36}I^{1}_{2\cdot4-2}$, $I^{2}_{2\cdot4-2}$, ${}_{s}I_{1\cdot5-0}$; ${}_{36}P^{1}_{2-2}$, ${}_{36}P^{2}_{2-2}$, ${}_{s}P_{1\cdot5-0}$, ${}_{s}P_{2\cdot4-2}$, $P_{3\cdot2-2}$, $P_{3\cdot2-4}$. Of the 21 sets first mentioned all but 6 were made on rotors from 2 to 2.4 mm. in diameter. The remaining six were made on rotors 3.2 mm. in diameter. This group of six gave $\rho/\rho_L = 1.063 \pm 0.020$.

The mean for the permalloy rotors is 1.064 ± 0.013.

The strength of the magnetizing field in all these experiments except one was in the neighborhood of 20 gausses (uncorrected for end effects). In the exceptional case (one of the group of 6) it was about 40 gausses, but the result differed from those with the weaker field by less than they differed among themselves.

In 10 of the sets on iron and 17 of the sets on permalloy the magnet-

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[19] In two of these sets the remote induction solenoid was used (§ 27).

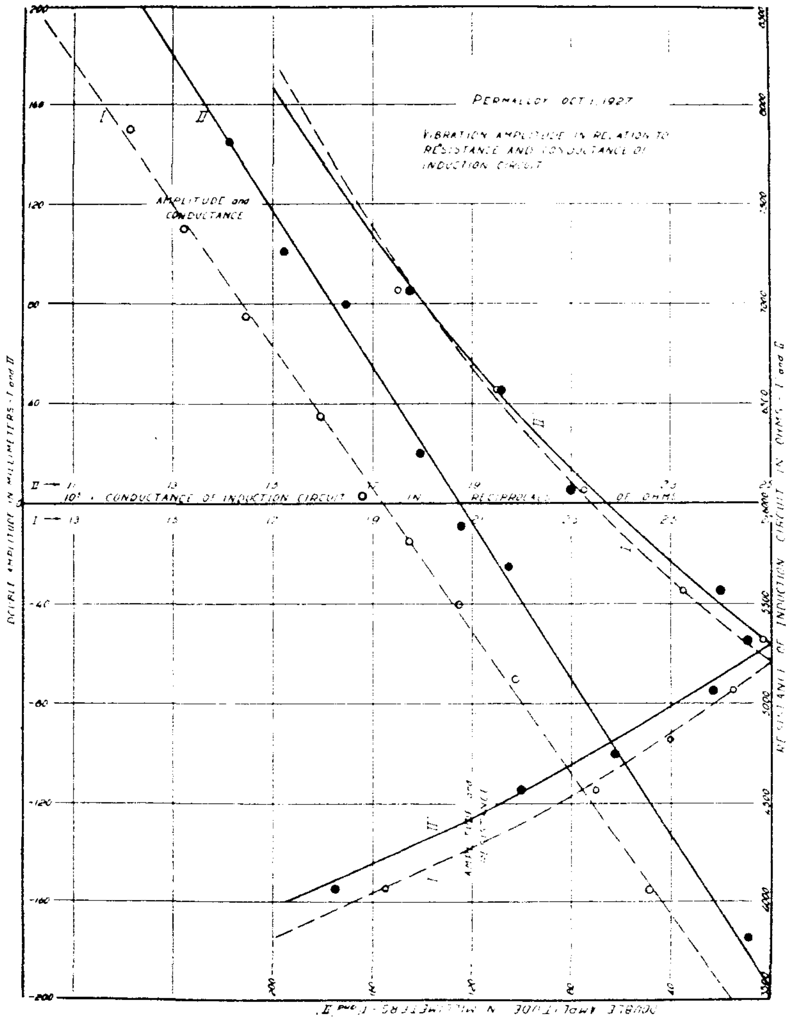

FIG. 46–1.

mirror system No. 1 was used. Long after the experiments were over the brass carriage in which the system was held when its moment was determined was found to be magnetized. If it was magnetized, and to the same amount, while in use, ρ/ρ_L for these 27 observations would have to be *increased* by 0.8%. The earlier and later observations, however, do not disclose any certain difference. When the carrier was magnetized is not known.

The frequencies ranged from about 4.1 to about 13.9 per second.

§ 47. *Example of observations for* ρ. *Large deflection method I.* (Group II). For reasons given in § 20 comparatively precise observations by this method were restricted to a single set, although the method is probably superior to any deflection method previously used by others. The following data, most of which, including the gyromagnetic deflections, were obtained in December, 1928, will suffice to illustrate the practice of the method.

The rotor was ${}_8P_{2\cdot4-2}$, and the experiments were made with the complete rotor in a magnetizing coil fixed to the earth, the coil wound on the rotor being on open circuit. The strength of the magnetizing field was about 20 gausses.

During each set of observations, made at approximate resonance, the fork control was not altered, and the amplitudes gradually decreased. The effect of this was practically eliminated by taking the observations in the order indicated. The natural frequency of the rotor as mounted at the time indicated was $\nu = 6.84$ cycles per second, giving $\omega = 2\pi\nu = 43.9_7$. The value of m_0 was $0.768_6 \times 1.0175$, and the value of $Q \times \pi/4$ was 1.248×10^{-6} e.m.u. The value of Γ_0 was 1830.7 gauss/e.m.u. current. The additional data necessary are given in Table 47-1.

These data, inserted in equation (20-3) give for $|\rho|$ the values $\rho_1 = 10^{-7} \times 0.5967$, and $\rho_2 = 10^{-7} \times 0.5947$. The mean, from which the effect of disturbing torques, except the possible effects due to inequality of the half-cycles and magnetostriction, apparently small at this time, are eliminated, is $|\rho| = 10^{-7} \times 0.5957$. This gives $\rho/\rho_L = 1.054$.

§ 48. *Observations by the second deflection method.* (*A*) *Large and small deflections.* Group II. The procedure and general character of the results are best illustrated by a table such as Table 48-1, which contains observations of March 22, 1928, made according to § § 25 and 26 with both small and large deflections. The observations by this method were in general made in the definite order indicated, and usually on a regular time schedule so as to eliminate errors arising from the inevitable changes in conditions during the progress of the work. In the example given the magnetizing current traversed the solenoid wound on the rotor. These observations also were obtained on a relatively poor night, as the building was in a state of continual

TABLE 47-1.

Group	Azimuth of vibrator magnet	Order of Obs	Magnetizing current	$=4\pi\times M$ e.m.u.	A^*_G	A^*_C
1	North pole west	(1)	337 milli-amperes	627	8.50 cm	cm
		(2)	—	—	—	9.25
		(3)	—	—	—	9.05
		(4)	337	627	8.35	—
	Mean amplitudes				8.425	9.15
2	North pole east	(1)	335	626	8.40	—
		(2)	—	—	—	9.20
		(3)	—	—	—	8.70
		(4)	335	626	8.00	—
	Mean amplitudes				8.20	8.95

* Minimum amplitudes with gyromagnetic torque balanced were not obtained on this occasion, but observations made shortly before and after make it evident that the quadrature torques had a negligible effect on A_G and A_C.

vibration; nevertheless those obtained with the different arrangements agree remarkably well. The residual amplitude was not measured, but was unnecessary, as the observations show.

Seventy sets of observations, complete except for systematic reversals to eliminate the effect of possible inequality of the half-cycles, were obtained in the interval Feb. 1928–Jan. 1929 by this method on four rotors of permalloy and soft iron, viz., $P^{2}_{2\cdot4-2}$, ${}_{s}I_{1\cdot5-6}$, ${}_{s}P_{2\cdot4-2}$, and ${}_{s}P_{1\cdot5-6}$; and the results are given in Table 48–2. The columns (1), (2), (3) correspond to columns 3, 4, and 2 of Table 48–1, respectively. On account of the possible error due to residual minimum amplitude, the results of columns (1) and (3) are more reliable than those of column (2).

§ 49. *Observations by the second large deflection method, con. (B). Large deflections only.* (Group III). The practice of observing with both small and large amplitude was later abandoned, and observations made with the large amplitudes only, the number of large amplitudes observed in one set being increased (ordinarily) to 12. Table 49–1 gives in tabular form most of the results obtained prior

TABLE 48–1.

OBSERVATIONS FOR GYROMAGNETIC RATIO, MARCH 22, 1928.

Rotor: $I^{2}_{2\cdot 4-2}$, Current: 100 milliamperes (field strength 20 gausses). Frequency: 6.7 per second. $m_0 = 0.779_6 \times 1.0175$ e.m.u.

I. Azimuth of vibrator magnet: N pole west. Times of beginning and ending: $2^h\ 25^m$ and $3^h\ 40^m$ a. m.

Resistance of induction circuit in ohms	∞	2755	4655	6055	5055	5555
Amplitudes × 2 in cms	*(1) 15.90	(2) 15.00	(15) 2.50	(16) 1.90	(3) 1.10	(4) 0.90
	(28) 15.70	(27) 14.60	(18) 2.45	(17) 1.90	(6) 1.45	(5) 0.84
	*The numbers in parentheses indicate the order of the observations		(19) 2.50	(20) 1.90	(7) 1.50	(8) 0.92
			(22) 2.45	(21) 1.90	(10) 1.18	(9) 0.90
			(23) 2.38	(24) 2.00	(11) 1.18	(12) 0.93
			(26) 2.30	(25) 1.95	(14) 1.10	(13) 0.91
Mean amplitudes × 2	15.80	14.80	2.43	1.92_5	1.25	0.90

II. Azimuth of vibrator magnet: N pole east. Times of beginning and ending, $4^h\ 15^m$ and $5^h\ 35^m$ a. m.

Resistance of induction circuit in ohms	∞	2755	4655	6055	5055	5555
Amplitudes × 2 in cms	(1) 15.46	(2) 14.50	(3) 2.40	(4) 2.05	(15) 1.50	(16) 1.26
	(28) 15.70	(27) 14.50	(6) 2.55	(5) 2.08	(18) 1.37	(17) 1.28
			(7) 2.55	(8) 2.10	(19) 1.36	(20) 1.11
			(10) 2.55	(9) 2.10	(22) 1.30	(21) 1.10
			(11) 2.50	(12) 2.00	(23) 1.30	(24) 1.40
			(14) 2.58	(13) 2.00	(26) 1.55	(25) 1.35
Mean amplitudes × 2	15.58	14.50	2.52	2.05_5	1.40	1.25

*ρ/ρ_L from I	1.043	1.041	1.044
*ρ/ρ_L from II	1.049	1.046	1.049
Mean*ρ/ρ_L	1.046	1.044	1.047
Mean†ρ/ρ_L	1.044	1.042	1.045

* Uncorrected for coil moment and electron inertia.

† Corrected for coil moment and electron inertia.

TABLE 48-2.

ρ'/ρ_L FROM SMALL (1), SMALLER (2), AND LARGE (3) AMPLITUDES (GROUP II).

A. IRON

Rotor	Approximate field strength in gausses	Frequency in cycles per sec.	No. of sets	(1)	(2)	(3)
$I^2_{2\cdot4-2}$	2	6.6	1	1.030	1.030	1.016
$I^2_{2\cdot4-2}$	5	6.6	6	1.038 ± 0.007	1.044 ± 0.005	1.032 ± 0.010
$I^2_{2\cdot4-2}$	10	6.6	14	1.038 ± 0.005	1.040 ± 0.005	1.036 ± 0.005
$I^2_{2\cdot4-2}$	20	6.6 & 8.6	18	1.046 ± 0.007	1.046 ± 0.004	1.046 ± 0.009
${}_sI_{1\cdot5-0}$	20	8.1	2*	1.046 ± 0.008	1.048 ± 0.003	1.026 ± 0.014
	Mean		41	1.042 ± 0.006	1.043 ± 0.004	1.039 ± 0.008

B. PERMALLOY

Rotor	Approximate field strength in gausses	Frequency in cycles per sec.	No. of sets	(1)	(2)	(3)
${}_sP_{2\cdot4-2}$	5	6.9	8	1.038 ± 0.008	1.042 ± 0.009	1.041 ± 0.007
${}_sP_{2\cdot4-2}$ & ${}_sP_{1\cdot5-0}$	5	6.9 & 7.6 ±	6*	1.042 ± 0.008	1.053 ± 0.006	1.023 ± 0.008
${}_sP_{2\cdot4-2}$	10	6.9	3	1.049 ± 0.002	1.048 ± 0.001	1.055 ± 0.004
${}_sP_{1\cdot50}$	10	7.4 & 7.7	4*	1.058 ± 0.005	1.061 ± 0.002	1.056 ± 0.003
${}_sP_{2\cdot4-2}$ & ${}_sP_{1\cdot5-0}$	20	6.9 & 7.6 ±	11†	1.047 ± 0.006	1.047 ± 0.005	1.043 ± 0.005
	Mean		32	1.045 ± 0.007	1.049 ± 0.005	1.041 ± 0.006

* With magnetizing coil fixed to earth.

† Includes 3 sets with coil fixed to earth.

to August 8, 1929, when further modifications were made in the method. Some of the observations were obtained under extreme conditions, and are quite rough. This is especially true of Groups 1 and 2, and in particular of Group 1, in which the heat dissipated in the winding of the rotor produced great irregularities in the vibrations. Several sets of observations made with a rotor in the 80 gauss field of a fixed magnetizing coil gave a considerably lower value of ρ and are not included in the table. The low value is probably due in part to the non-verticality of the axis of the magnetizing coil, and its different altitudes in the two half-sets (the error introduced being much greater in strong than in weak fields); and in part to the inequality of the half-cycles of current, the effects of which were now becoming manifest. Groups 3 and 7–8 are far more reliable than Groups 1–2 and 4–6 and alone of these observations are included in the final summary given in the next section.

§ 50. *Observations by the second large deflection method, con.* (*C*). *Observations in which the effect of inequality of half-cycles was approximately eliminated in each half set.* Group IV. The observations were in general similar to those recorded above, except that reversals were systematically made in each set to eliminate the effect of magnetostriction. The results of all but the final group of observations are given in Table 50–1.

§ 51. *Observations by the second large deflection method, con.* (*D*) *Final observations on permalloy and soft iron.* Group V. For these observations a more rigorous time schedule was adopted, and the observations were taken in such an order that the mean times for both large amplitudes in each half set (one for each azimuth of the vibrator magnet) were very nearly identical. An example is given in Table 51–1.

The results of all the observations made in this way, with the exception of one set on an iron rotor in a weak field, which was evidently vitiated by some unknown cause, and which gives a very large value of ρ/ρ_L, are summarized in Table 51–2.

[*Addition made in proof:* Since this paper went to press, and with the help of Dr. Otto Beeck, ten sets of observations have been made by this method on the large permalloy rotor $P_{3\cdot2-4}$ with a field strength of about 35 gausses and at a frequency of about 19 cycles per second. The observations are not so reliable as those of Group V, because the vibrator magnet system had to be reconstructed and remagnetized before the observations began, and because while the series was in

TABLE 49-1.

ρ/ρ_L FROM LARGE AMPLITUDES (GROUP III).

Material	Group	Rotor	Field strength in gausses	Frequency in cycles per sec.	No. of sets	Range of ρ/ρ_L	Mean ρ/ρ_L
Iron	1*	$I_{2\cdot4-8}$	80	17.4	6	1.027 – 1.158	1.111 ± 0.045
	2	$I_{2\cdot4-8}$	20	17.4	6	0.901 – 1.072	1.022 ± 0.024
	3	$I^2_{2\cdot4-2}$	20	8.1	3	1.026 – 1.051	1.042 ± 0.011
Permalloy	4	$P_{2\cdot0-8}$	40	19.7	5	1.003 – 1.052	1.023 ± 0.015
	5	$P_{2\cdot0-8}$	80	19.7 & 18.0	5	1.026 – 1.042	1.035 ± 0.007
	6†	${}_sP_{1\cdot5-6}$	20	19.7	7	1.040 – 1.076	1.064 ± 0.015
	7	$P^2_{3\cdot2-4}$	40	5.5	6	1.035 – 1.059	1.047 ± 0.007
	8	${}_sP_{2\cdot4-2}$	20	6.9 & 7.7	6	1.028 – 1.058	1.048 ± 0.008

* For two of these sets curves were obtained as in the earlier work, and the calculations were made from them, giving $\rho/\rho_L = 1.043 \pm 0.016$. These are the most reliable observations of the group.

† Coil fixed to the earth.

TABLE 50-1.

MAGNETOSTRICTION APPROXIMATELY ELIMINATED IN EACH HALF-SET (GROUP IV).

Material	Group	Rotor	Field strength in gausses	Frequency in cycles per sec.	No. of sets	Range of ρ/ρ_L	Mean ρ/ρ_L
Permalloy	1	${}_sP_{2\cdot4-2}$	20	6.9	16	1.022 – 1.090	1.055 ± 0.016
	2	${}_sP_{2\cdot4-2}$	5	8.0	11	1.047 – 1.075	1.058 ± 0.008
	1 & 2	${}_sP_{2\cdot4-2}$			27		1.056 ± 0.013
Iron	3	$P_{2\cdot4-2}$	20	10.0	14	1.020 – 1.052	1.040 ± 0.008

TABLE 51-1. MARCH 18, 1930.

EFFECTS OF HALF-CYCLE INEQUALITY COMPLETELY ELIMINATED IN EACH HALF-SET.

ROTOR $I^2_{2.4-2}$. FREQUENCY 9.9 PER SEC. FIELD STRENGTH 15 GAUSSES. MOMENT OF VIBRATOR MAGNET: 1.0175 × 0.7954 E.M.U. TIMES OF BEGINNING AND ENDING: $1^h\ 45^m$ AND $5^h\ 46^m$ A. M.

Azimuth	North Pole West				North Pole East			
Position of switch	I	II	II	I	I	II	II	I
Box resistance in ohms¶	∞	2600	∞	2600	∞	2600	∞	2600
Amplitudes × 2 in cm	(1)* 11.40 (8) 11.48 (9) 11.55 (16) 11.68 (17) 11.78 (24) 11.40	(2) 11.35 (7) 11.76 (10) 11.63 (15) 11.93 (18) 11.83 (23) 11.55	(3) 9.56 (6) 9.58 (11) 9.75 (14) 9.83 (19) 9.87 (22) 10.33	(4) 10.11 (5) 10.13 (12) 10.10 (13) 10.17 (20) 10.25 (21) 10.33	(1) 11.83 (8) 11.73 (9) 11.60 (16) 11.53 (17) 11.73 (24) 11.83	(2) 12.15 (7) 12.35 (10) 12.05 (15) 12.17 (18) 12.45 (23) 12.43	(3) 10.43 (6) 10.16 (11) 10.38 (14) 10.49 (19) 10.73 (22) 10.51	(4) 10.93 (5) 11.03 (12) 10.73 (13) 10.98 (20) 11.33 (21) 11.23
Mean amplitude × 2 in cm	11.55	11.68	9.82	10.18	11.71	12.27	10.45	11.04
Mean residual amplitude × 2 in cm	0.9		0.9		0.8		0.5	
†ρ/ρ_L	1.125 (from I)		0.966 (from II)		1.089 (from I)		0.973 (from II)	
Mean† ρ/ρ_L	1.046 (NPW, I & II)				1.031 (NPE, I & II)			
Mean ρ/ρ_L‡ for both azimuths	1.037 (NPW & NPE)							

* The numbers in parentheses indicate the order of the observations for each azimuth.

† Uncorrected for coil moment and electron inertia.

‡ Corrected for coil moment and electron inertia.

¶ Extra resistance in circuit, 69 ohms.

progress a piece of one of the twelve magnets broke off, making somewhat uncertain extrapolations necessary to determine some of the moments. These observations, corrected for nearly one percent electron inertia effect, give $\rho/\rho_L = 1.046 \pm 0.005$, with the range 1.038 $-$1.054. The result is entirely consistent with the results on permalloy in Group V. The vibrations were entirely free from magnetostrictive ripple.]

§ 52. *Summary of results.* The mean results obtained by the different methods are summarized in Table 52-1. There is but little disagreement between them. The results from the observations of Group V are far more trustworthy than the others, although these others are considered much more reliable than any obtained by earlier investigators. The new results are all entirely consistant with those published by the author and L. J. H. Barnett in 1925, which give as the mean value of ρ/ρ_L from experiments on magnetization by rotation the number 1.05_9. The differences are less than the sum of the probable experimental errors.

The work on magnetization by rotation, with a mean error between one and two per cent, was not sufficiently precise to reveal any probable differences between the gyromagnetic ratios for different substances. The results now obtained on rotation by magnetization, however, appear to show a distinct difference between permalloy and iron, the gyromagnetic ratio for the former being about one per cent greater than for the latter. This difference is considerably greater than the average departures from the means in the most reliable work.

The work has revealed no differences in the gyromagnetic ratio due to changes in dimensions of rotor, field strength, or frequency. It has also shown that the gyromagnetic torque acts upon the magnetic material of the rotor only, or at least that no appreciable part of it acts upon the magnetizing coil, which may be fixed either to the rotor or to the earth with no difference in result.

§ 53. *The error in ρ.* The error in ρ depends on the errors in Γ_0, γ_0, R_0, and m_0. The error in γ_0, and that in R_0 in so far as it depends on the correctness of the box resistances, may be considered entirely negligible. The error in Γ_0 is probably less than 1 part in 2000. The only error of any consequence arising from imperfect knowledge of the constants is that in m_0, which depends on Γ_0' C, r^3, and d/D. The error in each of the first three quantities is probably not greater than 1 part in 4 or 5 thousand, and that in d/D not as great as 1 part in 1000. Altogether, m_0 may be in error by about 0.2 per cent; but this is

TABLE 51·2.

RESULTS OF FINAL OBSERVATIONS ON PERMALLOY AND SOFT IRON (GROUP V).

Material	Group	Rotor	Field strength in gausses	Frequency in cycles per sec.	No. of sets	Range of ρ/ρ_L	Mean ρ/ρ_L
Permalloy	1	${}_{8}P_{2.4-2}$	20	10.0	17	1.046 – 1.055	1.049 ± 0.003
Iron	2	$I^{2}_{2.4-2}$	20	9.8	16	1.028 – 1.039	1.035 ± 0.002
	3	$I^{2}_{2.4-2}$	30	9.9	4	1.033 – 1.046	1.039 ± 0.004
	4	$I^{2}_{2.4-2}$	15	9.9	5	1.036 – 1.049	1.040 ± 0.004
Iron	2, 3 & 4	$I^{2}_{2.4-2}$			25		1.037 ± 0.003

TABLE 52-1.

SUMMARY OF RESULTS OBTAINED FOR THE GYROMAGNETIC RATIO.

	Material	Iron		Permalloy	
Group	Method	No. of sets	ρ/ρ_L	No. of sets	ρ/ρ_L
0 I	Graphical-Null Deflection I	18	1.054 + 0.010	30 1	1.064 ± 0.013 1.054
II* III** IV	Deflection II Deflection III Deflection IV	41 3 14	†1.040 + 0.007 1.042 + 0.011 1.040 + 0.008	29 12 27	†1.043 ± 0.006+ 1.048 ± 0.008 1.056 ± 0.013
V	**Deflection V**	**25**	**1.037 + 0.003**	**17**	**1.049 ± 0.003**

* From columns (1) and (3) of Table 48-2.
** Groups 1 and 2 and 4-6 of Table 49-1 are excluded.
† Mean from columns (1) and (3) of Table 48-2.

likely an over-estimate. So far as the constants are concerned, the error in ρ is not likely greater than 0.2%, even if all the componens errors are in the same direction, which is improbable. And this it less than the average departure from the mean in the final work. It therefore seems exceedingly probable that the final values of ρ are not in error by as much as one-half per cent.

§ 54. *The magnetic elements.* The differences of 4 or 5 per cent between the chief results of this investigation (as well as the investigation by L. J. H. Barnett and myself on magnetization by rotation) and those of the British physicists who have worked on rotation by magnetization are, as stated above, probably due, at least largely, to the failure of these investigators to eliminate the effects of magnetostriction and other effects depending on inequality of half-cycles of current, or on residual magnetization. There is no longer any evidence for the exact equality of ρ and m/e.

What interpretation must finally be given to the difference which is now confirmed is not yet certain. Inasmuch, however, as a gyromagnetic ratio equal almost exactly to m/e is apparently demanded in order to explain the Zeemann effect and a great mass of other spectroscopic phenomena, it seems probable that the chief magnetic element in ferromagnetic substances is the spinning electron with $\rho = m/e$, but that there is some contribution due to electron orbital motion. If this is true, we must expect to find for any substance from experiment (and calculations based on the hypothesis of only one kind of element) a mean value of ρ somewhere between m/e and $2m/e$; and in particular the small apparent difference between the values of ρ for permalloy and iron is not surprising.

§ 55. *Acknowledgements.* I am under obligations to many individuals and organizations for a great deal of help without which this work would have been impossible. Together with the institutions of learning mentioned below, I am especially indebted to the Carnegie Institution of Washington, which has furnished most of the standard equipment and much of the special equipment; to Mr. Wm. A. Arnold, of the Institute, who, as research assistant for several years, has given me indispensable help of many kinds; and to the instrument makers of the two institutions, especially Mr. G. H. Jung, of the University, who has done most of the more difficult machine work. For magnetic material I am indebted to Dr. H. D. Arnold and the Bell Telephone Laboratories, to Dr. T. D. Yensen and the Westinghouse Elec. and Mfg. Co., and to the Page Steel and Wire Co. For

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NOTE ON THE KELVIN SCALE TEMPERATURE OF FREEZING WATER.

By Frederick G. Keyes.

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NOTE ON THE KELVIN SCALE TEMPERATURE OF FREEZING WATER.*

BY FREDERICK G. KEYES.

THE values reported for the triple point temperature of water under atmospheric air pressure cover a range from 273.09 to 273.20 on the thermodynamic scale. The importance of the constant, which may be designated T_0, makes it desirable that its value be known to 0.01 degree or about 1 part in 27000; a degree of precision which is at present difficult to realize. To obtain such exactitude the coefficients of expansion of gases, preferably both at constant pressure and at constant volume, must be known to this degree of certainty. A consideration of the available and comparatively large number of coefficients reported for He, H_2, N_2 indicates that on average no single coefficient may be expected to be more exact than 1 part in 10000 corresponding to 0.03° in T_0. The mean deduced value of T_0 may of course be supposed to represent the true value of T_0 within narrower limits. The difficulties involved in measuring the coefficients are numerous but the more important are connected with the reproducibility of the temperature and pressure expansivity of the gas container, the adsorption of the gas on the container walls, the transfusion or solubility of the gas in the container material, besides all the difficulties of reading the pressures or volumes which require a precision of 1 part in 75000 to realize 0.01° in T_0.

The expansion coefficients of a gas may in themselves be sufficient, if measured at different pressures, to determine T_0 assuming, as at present appears legitimate, that as the pressure (p) approaches zero the gases may be assumed to be in a state where the pressure or volume, at constant volume or constant pressure, is proportional to the thermodynamic temperature. The additional assumption is usually involved that the coefficients vary linearly with the pressure: an assumption which evidently need not necessarily be adopted.

Recourse may also be had to the Joule-Thomson effect, or any other measure of the departure of actual gases from the ideal "perfect" state, to convert the actual coefficients into coefficients which the gases would have in the "perfect" state. Of the three measures of "imperfection" in gases the Joule-Thomson effect and the determination of the pressures at various volumes and temperatures are alone of immediate practical importance since the procedure for measuring the Joule, or free expansion, effect has not been sufficiently developed[1] to provide reliable values.

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* Contribution from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, No. 260.

The Joule-Thomson effect provides data immediately applicable for converting constant pressure gas thermometer readings to thermodynamic, or Kelvin temperatures. The effect can also be used to convert constant volume gas thermometer readings to Kelvin temperatures, but independent pressure, volume, temperature properties for the thermometric gases are then required, unless the use of certain forms of Equation of State are permissable.

By far the most painstaking and extensive measurements of the Joule-Thomson effect for any substance have been made by J. R. Roebuck for air.[2] The values of the effect $\left(\frac{dT}{dp}\right)_H$, have been tabulated in Roebuck's 1930 paper and values for the free expansion effect $\left(\frac{\partial u}{\partial v}\right)_T = \lambda$, computed. The latter quantity is immediately applicable for reducing constant volume gas thermometer readings to Kelvin temperatures.

On page 316, section 22, of Roebuck's 1930 paper, Chappius' constant volume expansive coefficient is used to obtain T_0 using the λ coefficients obtained from his μ values. The value of T_0 depends upon being able to compute the integral $\int \frac{\lambda}{T^2} dT$ between the limits T_0 and T_{100}. This Roebuck does in a manner which it appears to me may be improved upon. Incidentally attention is called to an error in Roebuck's final computation of T_0.

The expression for the free expansion effect, λ, may be written, where τ designates T^{-1}:

$$\lambda = -\left(\frac{\partial(p\tau)}{\partial\tau}\right)_v \tag{1}$$

Since by definition the thermometric constant-volume coefficient of a gas is $\frac{p_{100} - p_0}{p_0 100} = \alpha_v$ there results:

$$\alpha_v = \frac{1}{T_0} - \frac{T_{100}}{100 p_0} \int_{T_0}^{T_{100}} \lambda d\tau$$

$$T_0 = \frac{1}{\alpha_v + \frac{T_{100}}{100 p_0} \int_{T_0}^{T_{100}} \lambda d\tau} \tag{2}$$

On page 588 of Roebuck's 1925 paper use is implicitly made of the assumption that air may be represented to a close approximation in its pressure, volume, temperature properties by the equation $p(v + B) = RT$ where B is given by $\frac{A}{RT} - \beta$. The Joule-Thomson coefficient is given however by the thermodynamic equation:

$$\mu c_p = -\left(\frac{\partial(v\tau)}{\partial\tau}\right)_p \tag{3}$$

where c_p is the constant pressure specific heat. From the equation of state above there results accordingly:

$$\mu c_p = B + \tau\frac{\partial B}{\partial\tau} = \frac{2A}{RT} - \beta \tag{4}$$

The Joule-Thomson coefficients at one atmosphere are used by Roebuck to determine $\frac{2A}{R}$ and β, and the value of $\mu\, c_p$ from T_0 to T_{100} is evaluated. The available constant pressure coefficients at various zero degree pressures are then used to deduce values of T_0 which on averaging give $T_0 = 273.153$.

The computation of $-\int \lambda d\tau$ however, is made by selecting values of λ from a table of values at various pressures corresponding to constant volume, the pressure at zero degrees having the value one atmosphere. These λ values are then plotted in a diagram with τ and integrated between 0 and 100° in 25 degree stages assuming a linear relation in the variables. The value so obtained is 2.309×10^{-6} atmospheres per degree with the pressure at zero equal to one atmosphere. Chappius coefficient was obtained at 1.343 atmospheres, and Roebuck obtains for T_0 the number 272.79. The value is too low, however, because the value of $-\int \lambda d\tau$ used to obtain this value of T_0 was taken for 1 atm. instead of for 1.343 atm. corresponding to Chappius coefficient. The correct value of T_0, assuming Roebuck's vaue of $-\int \lambda d\tau$ to be correct at one atmosphere, is obtained by multiplying 2.309×10^{-6} by 1.343 atm. to bring the integral into correspondence with the Chappius coefficient relative to pressure. The value of T_0 then becomes 273.01 instead of 272.79, as stated by Roebuck.

The method of computing $-\int \lambda d\tau$ employed by Roebuck does not however appear to me to be sound. There is no reason, moreover, why the method employed by Roebuck (and others) for obtaining T_0 from the constant pressure coefficients should not be used. From the equation of state used for the $\mu\ c_p$ data at one atm. (Eq. 4) there results for λ:

$$\lambda = +\frac{R}{(V+B)^2}\frac{\partial B}{\partial \tau} = +\frac{A}{(V+B)^2} \tag{5}$$

Since B is small compared to v (1 in 1700 at 0°) λ may be assumed to vary inversely as v^2.

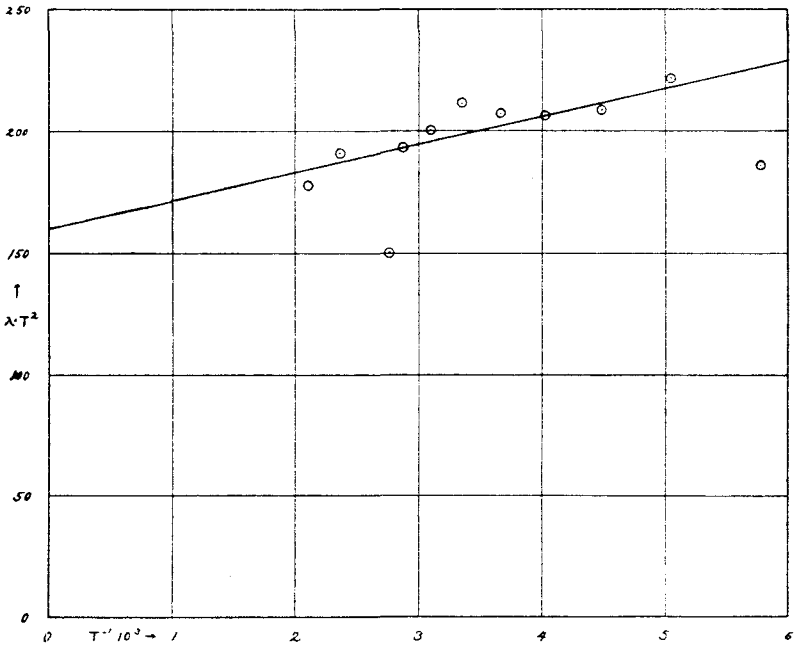

FIG. 1.

Roebuck lists values of λ from $-100°$ to $+200°$ at one atmosphere. To determine A it suffices therefore to assume the gas laws for v and plot the product of λ and T^2 with T^{-1}. A graph of this kind is shown in figure 1. It is to be observed that the data are erratic but the indication is that λT^2 is linear in T^{-1}. This result is to be expected since the theory of the equation of state for a van der Waals molecular model[3] leads to an expression for B of the form:

$$B = \frac{A}{RT}\left(1 + \frac{a_1}{T} + \frac{a_2}{T^2} \cdots\right) - \beta \tag{6}$$

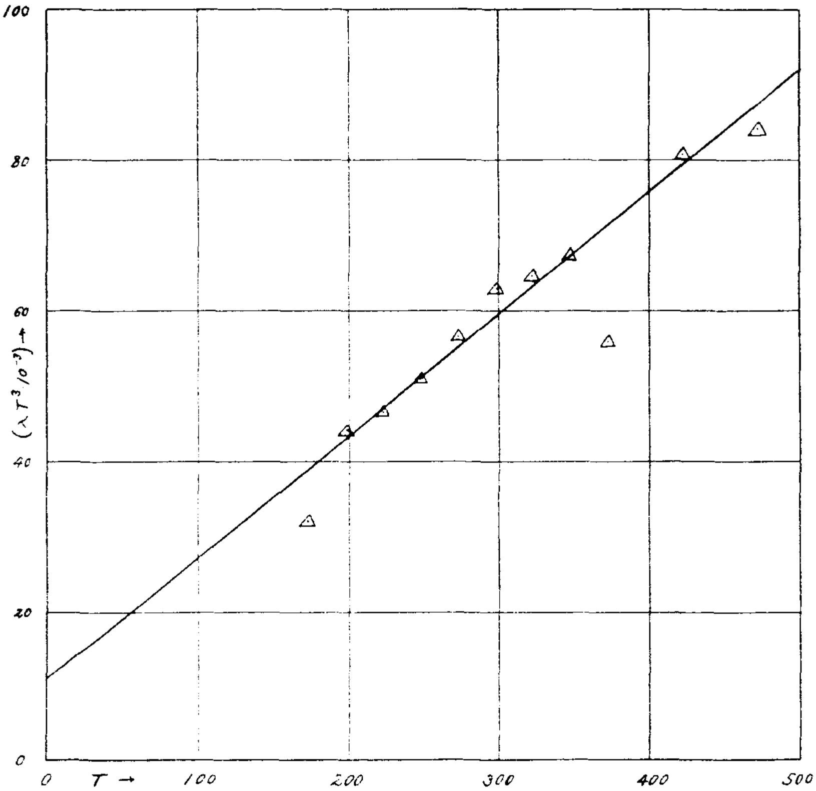

FIG. 2.

From fig. 1 the value of A/R^2 is given by the intercept. In figure 2 λT^3 is plotted with T and here A/R^2 is given by the tangent. The equation finally selected to represent λ for low pressures is:

$$\lambda = \frac{1293}{v^2}\left(1 + \frac{69.8}{T}\right)\frac{\text{atm.}}{\text{cc. per g.}} \tag{7}$$

Table I gives the comparison of the computed values of λ and those given by Roebuck.

TABLE I.

t	*−100*	*−75*	*−50*	*−25*	*0*	*25*	*50*	*75*	*100*	*150*	*200*
Roebuck	0.006182	0.005681	0.004194	0.003351	0.002779	0.002381	0.001919	0.001597	0.001078	0.001066	0.000795
Cal.	0.007536	0.005546	0.004245	0.003350	0.002709	0.002235	0.001875	0.001595	0.001373	0.001048	0.000825
Obs.-Cal.	−.001354	+.000135	−.000051	+.000001	+.000070	+.000146	+.000044	+.000002	−.000295	+.000018	−.000030

From equation (2) making use of (7) there results:

$$T_0 = \frac{1}{\alpha_v - 9.65 \times 10^{-6} p_0}$$

where p_0 is the pressure of the gas at 0° in atm.

Using the Chappius coefficient 0.00367442 measured at 1.343 atm. there is obtained for T_0 273.116. The mean value of T_0 obtained by Roebuck from the constant pressure coefficients is 273.153. The final mean of T_0 from air is therefore 273.135 which is very close to Buckingham's value (273.13) and mine[4] (273.14) computed some years ago.

It is of interest to note that the μc_p values at one atmosphere are given quite satisfactorily from 0 to 200° by the formula:

$$\mu c_p = \frac{2A}{RT}\left(1 + \frac{3}{2}\frac{a_1}{T}\right) - \beta$$

$$= \frac{912}{T}\left(1 + \frac{76.88}{T}\right) - 1.6105 \ \frac{\text{atm.}}{\text{cc/g.}} \qquad (8)$$

The value of A is here the same as that obtained from the λ values but a_1 is 51.26 instead of 69.8 showing that the λ and μ values are not quite consistent. The effect of this on the T_0 values is insignificant however.

In concluding this note it is worth emphasizing that the van der Waal's model of a molecule does not lead to an equation of state conforming to the indications of the available pressure, volume, temperature data.[5] Over short temperature ranges (100° to 150°) this molecular model may suffice, as illustrated in the computations above but errors will be introduced if representations of the type of (8) are assumed valid to very low or very high temperatures.

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[1] Sears and Keyes. Proc. Nat. Acad. Sci., *2*, 38 (1925).

[2] J. R. Roebuck, These Proceedings, *60*, 537 (1925); 64, 287 (1930).

[3] F. G. Keyes, Chem. Review, *6*, 175-(1929). Equation 7.

[4] F. G. Keyes, J. Am. Chem. Soc., *42*, 54 (1920).

[5] F. G. Keyes and R. S. Taylor, J. Am. Chem. Soc., *49*, 896 (1927); also F. G. Keyes, Chem. Review, *6*, 175 (1929).

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RADIOACTIVE DISINTEGRATION APPLIED TO THE MEASUREMENT OF GEOLOGIC TIME ILLUSTRATED BY APPLICATION TO THE WILBERFORCE URANINITE.

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Alfred C. Lane, Tufts College, Massachusetts.

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By Gerhard Kirsch, Radium Institute, Vienna, Austria, and Alfred C. Lane, Tufts College, Massachusetts.

This paper has three parts. The first general part is by Kirsch, revised with his permission by Lane. In particular the formulae are somewhat altered in expression both to make it easier for the printer and also to put them into more familiar form for computation. There has been also some revision with reference to recent papers by A. F. Kovarik[1] of Yale University, which had not reached Kirsch at the time.

In the second part Lane calculates the age of the Wilberforce uraninite not only to show that the uncertainties as to age are not so great after all (See Fig. 1), but also to show what data are needed to reduce these uncertainties. Even now important geological results may be obtained. The third part is by Kirsch with slight modifications by Lane, and includes certain general cautions as to age calculations.

I. General Calculation of Age.

Historical treatments of the subject of this paper have already been given by Holmes and Lawson[2] and quite recently by Kovarik.[3] Holmes and Lawson based their discussion on the usual approximate formula, introducing, whenever required, one or the other of certain correction factors, which they discussed. This formula is

$$(1) \qquad A = Pb \cdot C/(U + k\ Th)$$

where A is the age, C a constant, Pb, U and Th the percentage of lead respectively uranium and thorium in a mineral and k denotes the ratio between the rates of lead production from thorium and from uranium. The value and constancy of the fraction k is one of the things to be discussed. The ratio of the number of atoms disintegrating to the total number present, often expressed in terms of λ, which is n/N of Kovarik's nomenclature, we will here express in terms of T which is the time required that an element shall be half gone (T_u for uranium, T_{th} for thorium, etc.) and is naturally inversely proportional to the rapidity of disintegration λ. It is $0.693/\lambda$. In terms of it

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[1] Science (Aug. 1, 1930) 72: 122–5; Am. Jour. Science (Aug. 1930) 20: 81–100.

[2] Am. Jour. Sci. (1) 3 (1927) p. 327.

[3] Am. Jour. Sci. 20 (1930) pp. 81–100, and Science 72 (1930) pp. 123–125.

$$(2) \qquad k = \frac{\lambda_{Th}\dfrac{208}{232}}{\lambda_{U}\dfrac{206}{238}} = \frac{Tu \cdot 1.156}{T_{th} \cdot 1.115} = 1.038 \cdot T_u/T_{th}$$

and

$$C = 1.156 T_u/0.693 = 1.668 T_u$$

C obviously represents the inverse value of the rate of lead production from uranium. Formula (1) is symmetrical in relation to U and Th:

$$A = \frac{Pb}{U + k \cdot Th}(1.668 \cdot T_u) = \frac{Pb}{1/k\ U + Th} \cdot \frac{1.668 Tu}{k}$$

$$= \frac{Pb}{1/k\ U + Th} \cdot 1.607 T_{th} \qquad (1a)$$

and is thus applicable to minerals containing thorium or uranium or both elements together. When we consider, however, that the proportion of lead producing elements[1] has not remained constant through the whole period since a mineral was formed, we should bear in mind that the correction factors to this approximate formula are not identical for thorium and for uranium minerals. In fact they respectively differ very much as the difference between the age of a thorium mineral with definite "Pb ratio" ($Pb/k \cdot Th$!) and that of a uranium mineral with the same Pb ratio is larger than the difference between the true age of the former as computed from the exact formula:

$$A = \frac{T_{th}}{0.693} \cdot (2.3026 \log (1 + 1.115\, Pb/Th)$$

$$= 3.323 T_{th} \cdot \log (1 + 1.115 Pb/Th)$$

and that calculated from formula (1).

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[1] And that includes the proportion of isotopes of uranium, so that the atomic weight of uranium in Eq. 2 is not constant nor the value of k absolutely.

We should therefore like to suggest the following as one practical method of calculation for minerals in which neither of the two elements may be neglected: One calculates the terms:

$$3.323\,T_u \log\,(1 + 1.156\,Pb/(U + kTh))$$

and

$$3.323\,T_{th} \log\,(1 + 1.115\,Pb/(U/k + Th))$$

and finds a very nearly correct value by linear interpolation in the ratio $U : k \cdot Th$ between them, *in case there is no way* of telling how much lead should be assigned to the different isotopes of *Pb*. Moreover, owing to the different values of the time average corrections, one has to fall back on interpolation formulas in dealing with any mineral rich in thorium.

Although the disintegration constants of uranium and thorium are not known with the same degree of accuracy, this is of minor importance when comparing the relative ages either of different uranium minerals or of different thorium minerals. On the other hand, the exact value of the $T_u : T_{th}$ ratio is essential even for relative determinations of age as soon as a comparison of the ages of thorium and uranium minerals inter se is involved. This leads to a discussion of the so-called constants C and k, which appear in the formulas for the age and of the disintegration constants respectively of T_u and T_{th} by which they can be replaced according to the formulas above.

II. The Constants.

We fully agree with the views of Holmes and Lawson regarding the number of alpha particles emitted per second from one gram of radium, given as $3.72\ (10)^{10}$, which has lately been repeatedly confirmed but we find it necessary to subtract a correction from T_u as calculated indirectly from T_{Ra} and the $Ra : U$ ratio $(3.40\ (10)^{-7})$ in order to allow for the branching off of the actinium series. If we assume a branching ratio of three percent[1] T_u should be decreased by three percent since only 97 percent of the U contributes to the Ra and we therefore get:

$$T_U = 4.3\ (10)^9 \text{ years} = 4.3 \times \text{antilog } 9 \text{ years} \qquad (5)$$

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[1] Hevesy in Science, Nov. 21, 1930, says this is fixed, and this has been the general result of experiments. But confer Kovarik (Am. J. S. LXII, p. 125) and James E. Wildish (Jour. Am. Chem. Soc., Jan. 1930, p. 175). Also Grosse and Kurbatow report the actinium radium ratio in the Wilberforce uraninite as four percent (pri. com.)

and

$$C = 7.175 \times \text{antilog } 9 \text{ years} \qquad (6)$$

It should be remembered, however, that in this case we are including all U, both Urradium (U 238$^+$) and Uractinium (U 239) and that T_u thus obtained is an average figure. It will be very desirable to get the disintegration ratio of the uranium of the kolm which gives such pure radium lead (At. wt. = 206.01)[1] although it seems not probable that it will give a materially different value of T_u.

Again, as regards the disintegration ratio of thorium and its half life T_{th}, we are not yet prepared to accept the value based on the scintillation counts by Rutherford and Geiger, which is extensively used, since it is at hopeless variance with certain results from mineral analyses. One can (as one of us has already explained elsewhere)[2] derive a value for the ratio of the disintegration constants of uranium and thorium from the composition of such radioactive minerals as contain both elements in comparable quantities (table I) or rather in quantities corresponding to comparable rates of lead production, provided that the amounts of both kinds of lead $RaG+$ and ThD actually present in the minerals have been found by atomic weight

TABLE I.—Analyses of Thorium Uranium Minerals.

		Thorianite		Thorite	
	I	II	III	Ceylon	Brevik
Kovarik's No. A. J. S., 1930, p. 90	9	10	11		2
Uranium %........	11.8	20.2	26.8	1.62	0.45
Thorium %........	68.9	62.1	57.0	54.4	30.10
Lead %...........	2.34	3.11	3.5	0.36	0.35
At. Weight of lead..	207.27	206.91	206.84	207.77	207.90
† $RaG^* : U$.........	0.0815	0.0861	0.0780	0.0282	0.0474
$ThD : Th$..........	0.0200	0.0218	0.0246	0.0058	0.0109
$\dfrac{\dagger RaG^* : U}{ThD : Th}$..........	4.07	3.95	3.17	4.86	4.35
k..	0.246	0.253	0.315	0.206	0.230
$\lambda u/\lambda th$ after Kovarik:	3.78	3.68	2.91		3.50

† RaG^* means here all lead derived from Uranium.

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[1] Baxter and Bliss, Jour. Am. Chem. Soc. 52 (1930) 4848.

[2] Wien. Berichte, Vol. 131, p. 551 of 1922; Mitt. Inst. f. Radium Forschung, No. 150.

determination, and provided there has not been selective removal or addition of these elements.

The values of k are not exactly the reciprocals of those given by Kovarik in his table II, partly because there has not been used[1] the even atomic weights, e. g., 206, but the most accurately known (at present) which allow for a packing factor, e. g., 206.05 and 208.02.

The average value of k, however, that we get (0.25) differs very widely from that used by Kovarik based on scintillation counts (0.38) by an amount largely in excess of the margin of error in the analytical results.

A conclusive proof of the correctness of the value $k = 0.25$ seemed[2] to be offered by the first analysis of thorianite made by Hönigschmid. For here, assuming the atomic weights of uranium lead and of *ThD* to be known, we know the exact value of the ratio between the two sorts of lead present in the analyzed specimen, since by lucky chance the ratio to be inferred from the proportions of *U* and *Th* present, happens to agree with that corresponding to the atomic weight of common lead. Therefore the content of the latter is not significant in the calculation of the ratio of uranium lead to *ThD*.

As Holmes has suggested, we find that an attempt to maintain the old value of k (0.38) leads into a serious conflict with ascertained analytical results.

There still exists, however, a possibility of doubting the disintegration constant of thorium as derived from this analysis of thorianite by assuming that the mineral might have contained originally about 6% more of uranium which has *quite recently* been lost without losing *lead* or *helium!*

However, since the arguments are not absolutely convincing a new direct determination of this important constant (k) would be most welcome. Accurate measurements of the number of alpha particles emitted per second by uranium and thorium are in progress in the Radium Institute in Vienna but will still take some time to complete, but a preliminary experiment has been made which, although attaining an accuracy of only a few percent, owing to the small number of particles counted, shows clearly that the value of the constant k lies in the vicinity of 0.25 and certainly not near 0.38[3].

We used two preparations, one containing 1.392 mg. radio-active pure U_3O_8 spread out as uniformly as possible over a disc 2 cm. in diameter, while the second preparation consisted of 2.086 mg. ThO_2 distributed over the same surface. The quantities of radioactive substances used were determined on a microbalance sensitive to 10^{-6} gm. The thorium preparation was purified from mesothorium in 1914 and should therefore lack some 18–20 percent of the equilibrium amounts of the disintegration products. This again was checked by comparison measurement of this same sample of thorium with an analyzed thorite and pitchblende of known composition.

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[1] Kirsch, Geologie und Radioktivität, pp. 150–154.

[2] Geologie und Radioktivität, p. 150.

[3] Physikalische Zeitschrift 31 (1930, Nov. 15) p. 1718.

Directly before each preparation was put a system of apertures consisting of a metal disc 4 mm. thick, which contained 12 holes of diameter 4 mm. Only a fraction of 2.25 percent of the total radiation emitted by the preparations passed through the apertures. This system was closed by a very thin foil of gold and formed one of the electrodes of an ionization chamber, while the second electrode, connected with the grid of the entrance valve of the valve electrometer, was placed at a distance 4 mm. from the first.

In 307 sec. were counted 225 α particles emitted by the uranium preparation, while the number counted without the preparation (the zero effect due to traces of contamination) was six in 117 sec. With the thorium preparation the count gave 188 particles in 276 sec., the zero effect being 19 particles in 235 sec. The net effect following from these counts was for uranium 41 particles per min., for the thorium preparation 36. Knowing that uranium emits about 2.6 $(10)^4$ particles per gram and sec. we should have expected an effect of about 40 per min. with the experimental arrangement used. This surprisingly good agreement at any rate proves that the substance was in fact spread out in a uniform manner, so that no particles were lost by absorption in the substance.

In case thorium alone without its disintegration products emits the same number of particles per gram and sec. as UI, we should expect from our thorium preparation 3/2 of the number from the uranium preparation per element of each series because of the greater quantity of the substance used. Further, owing to the presence of 80% of the disintegration products in equilibrium with thorium themselves giving off particles we should expect 5/2 times the number of particles from uranium preparation (UI UII), finally we must add 3.5 percent because of the smaller content of oxygen in the ThO_2 compared to U_3O_8. We should therefore get

$$41 \times 3/2 \times 5/2 \times 1.035 = 159 \text{ particles per min.} \tag{5}$$

Now our thorium preparation yields only 36 particles per min., that is 4.4 times less than the value 159. From this it follows that the disintegration constant of thorium is about 4.4·232/238, i. e., 4.3 times smaller than that of uranium and furthermore that *k*, the ratio of the rates of lead production of both the elements must be roughly 0.24. Again if the thorium should happen to contain traces of ionium or on the other hand hold more than 80% of its equilibrium quantity of disintegration products—for it cannot contain less—this would lead to a still smaller value of *k*!

On the whole, we can say that a reasonable agreement exists between these direct measurements and the calculations from the mineral analyses above. Rutherford and Geiger in their classical pioneer work on this subject do not claim a high degree of accuracy; their paper contains no description of the way in which the thorium was prepared, nor are the results of measurements on thorite films mentioned or discussed.

Pending more accurate measurements, we are thus entitled to assume as the values most suitable for calculating the ages of minerals:

$$C = 7100 \quad \text{and} \quad k = 0.25$$

and accordingly, for the interpolation method before mentioned, the half lives:

$$T_u = (.693/1.62 = .428)10^{+10} = 4.28 \times \text{antilog } 9 \text{ years} \qquad (7)$$

and

$$T_{th} = (.693/3.9 = .18)10^{+11} = 1.8 \times \text{antilog } 10 \text{ years} \qquad (8)$$

$$= 18 \times \text{antilog } 9 \text{ years}$$

in remarkable agreement with the measurements of H. N. MacCoy who found $T_{th} = 1.78 \times$ antilog 10 years (Phys. Rev. (2) 1 (1913) p. 403); cf. also B. Heimann Wien. Berichte *123* (1914) p. 1369, and L. Meitner Phys. Zs. 19 (1918) p. 257. The *U* in eq. 7 includes all the *U* isotopes.

PART II.

APPLICATION TO WILBERFORCE URANINITE.

Canada has for years been famous for its pegmatites and pegmatite minerals on which H. V. Ellsworth has done such fine work[1] and more recently H. S. Spence.[2]

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[1] Canada Department of Mines, Summary Report 1921, Part D, and the American Mineralogist, a number of papers.

[2] American Mineralogist, Sept. Oct. 1930,—Also Dec.

Special attention has been given by Ellsworth to those minerals to which one might look for sources of radium, and recently deposits approaching the commercial have been found.

Years ago, Lane presented a discussion of the possible solution of Laurentian (pre-Cambrian) problems by means of the relative age of minerals, estimated by the extent to which the disintegration of uranium, thorium, actinium and radium to lead had gone on. Some recent newspaper notices[1] have rather overestimated the exactitude with which such estimates may be made (1852 million years), and Hevesy's recent lecture,[2] being to a general audience, is a bit too positive.

It is the object of this section to show by application to the Wilberforce uraninite that though there are uncertainties and greater exactitude is much to be wished for and is indeed in sight, one can obtain results already which are of geologic value, and the facts and research needed for closer estimates will be pointed out.

Thanks to H. V. Ellsworth and the officers of the Ontario Radium Corporation the committee on the Estimation of Geologic Age by Atomic Disintegration were enabled to purchase some twenty pounds of excellent material for research. This had already been analyzed three times so that it has as yet been deemed needful to analyze the material used for atomic weight determination only for *U*, *Th*, and *Pb*. H. S. Spence and R. K. Carnochan have described the occurrence fully and given two of the analyses early in 1930.[3] So we need give only a third analysis made more recently by H. V. Ellsworth and the figures for *U*, *Th* and *Pb* obtained by Wells on the very same material as that from which the atomic weight of the lead was determined, as follows:

Sp. G. = 9.062 at 20°.57

UO_2	39.10
UO_3	32.40
ThO_2	10.60
PbO	10.95
CeO_3	1.88
Y_2O_3	2.14
Fe_2O_3	0.43
Al_2O_3	0.09

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[1] Boston Post, Nov. 23, 1930, p. 11 of color-feature section.

[2] Science Nov. 21, 1930, p. 509.

[3] Transaction Can. Inst. Mining and Metallurgy for 1930.

SiO_2	0.19
CaO	1.01
MgO	0.08
MnO	0.03
H_2O	0.05
H_2O at 100°	0.65
	0.31
	100.06

The U. Th. Pb were:

	I E. W. Todd	II H. V. Ellsworth altered	III H. V. E.	IV R. C. Wells
U	60.56	55.26	61.44	53.52
Th	10.02	11.92	9.32	10.37
Pb	9.65	10.25	10.19	9.26
$Pb/U + 0.25Th$	.153	.176	.16	.165
Age in millions of years	1092	1258	1142	1178

Using the formula and the constants above given equation 1 becomes

$$\text{Age} = 7.14 \times \text{antilog } 9 \text{ years} \cdot \frac{Pb}{U + 0.25Th} \tag{7}$$

and the various analyses give the figures for the age of the last row. Following the suggestion given above (p. 359), analysis IV gives an age to be interpolated between 1090 and 1160 years.

We see that the approximate formula gives us ages between 1100 and 1300 million years. But we can make a closer estimate than this, since Baxter and Bliss have determined the atomic weight of the lead of Analysis 4[1] as 206.19(4). We can tell about how much of the lead is derived from Radium, how much from thorium, etc. Kovarik assumes four kinds of lead

$Pb(206)$ RaG from Radium
$Pb(207)$ AcD from Actinium
$Pb(208)$ ThD from Thorium
$Pb(207.2)$ ordinary lead. E. g. (Pb 209).

But there are also possibilities of other leads, and ordinary lead has been shown by Aston to consist of all three leads $Pb(206)$ (207) and

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[1] Journal Am. Chemical Society (Dec. 1930) 52: 4852.

(208), part of which may be thought to have formed from *U* and *Th* before the uraninite was crystallized. It can be shown that there is but little of this ordinary lead in the Wilberforce Uraninite anyway, however:

(1) In the first place, by the atomic weight of the lead which is so largely *RaG*, corresponding to the large amount of *U* in the mineral.

(2) In the second place, an x-ray examination by John T. Norton shows only the lattice of UO_2, 5.45 Ångstrom units, which would not be likely if there were much original lead.

(3) In the third place, the amount of extra oxygen as shown in Ellsworth's analysis by the UO_3 is ample to consider the *Pb* as having been derived from UO_2 with *O* set free in accordance with Ellsworth's autoxidation theory. No original lead is needed.

(4) In the fourth place, the amount of *He* is the highest Bennett and Moore have found (10.39 cc per gram, p. 372, below), and this again points to a large amount of radioactive lead.

(5) As will be seen, whatever values we take for *k* or for the amount of actinium lead, there must be some actinium and thorium lead and to that extent the possibilities of ordinary lead are diminished.

(6) There is a sixth check theoretically possible which has not yet been made.

Whenever lead is separated from a mineral like uraninite there are isotopes of lead which are radioactive separated with it.

RaB, half gone in 26.8 minutes; the quantity in equilibrium with 1 gram *U*. 1.02 antilog—14 grams.

RaD, half gone in 22 years; the quantity in equilibrium with 1 gram *U*. 3.1 antilog—9 grams.

AcB, half gone in 36 minutes; the quantity in equilibrium with 1 gram *U*. 2.5 antilog—17 grams.

ThB, half gone in 10.6 hours; the quantity in equilibrium with 1 gram *Th*. 6.41 antilog—14 grams.

These are in too small quantities to be recognized by ordinary chemical methods, but they could be recognized by the electroscope and by very prompt repeated examination of the decay curve an idea as to whether the actinium series was at present contributing a substantial amount as a check on direct actinium determinations.

The proportion of protactinium to radium[1] is not yet determined. On that James E. Wildish and A. V. Grosse are at work. Grosse and Kurbatow report actinium to radium as four percent.

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[1] Cf. Jour. Am. Chemical Soc., Jan. 1930, p. 175.

(7) Finally there is the consistency of the age results obtained with the geological conditions and the age of other minerals. But of course in applying this test there is great danger of reasoning in a circle.

These minor methods should and will be left to the last.

Grouping all the lead that is not *RaG* and *ThD* formed since the mineral crystallized, together as y we have

$$RaG + y + ThD = 9.26 \qquad (8)$$

We include in y: (1) Any actinium lead; (2) Any *RaG* and *ThD* formed before the mineral crystallized and present in solid solution, not a result of the decomposition of the uranium and thorium segregates in the crystallization of the mineral; (3) Any ordinary lead or *Pb*(209) that may be present; and (4) The insignificant amount of *RaB*(214) *AcB*(213?) *ThB*(212) and *RaD*(210).

From the atomic weight of the lead we shall have:

$$\frac{RaG}{206} + \frac{y}{207} + \frac{ThD}{208} = \frac{9.26}{206.195} \qquad (9)$$

There is a question here whether we should take an even 206 or allow for a packing effect. Hönigschmid hardly thinks that the determination of the atomic weight of *Ra* should put it down to 205.95, and the atomic weight of the kolm lead 206.01 may be expected to have something subtracted for actinium lead. There is an uncertainty of about .01 which becomes of considerable importance if we eliminate *RaG* and determine *ThD*. Eliminating *ThD* from these two equations, we can get

$$RaG = \frac{206}{2}\left(9.26 \cdot \frac{1.805}{206.195} - \frac{y}{207} \right) = 8.3493 - y(.4976) \qquad (10)$$

$$ThD = 0.910 - (.5024)\ y \qquad (11)$$

Whence *ThD* cannot be greater than 0.91, since y is not less than O, and y cannot be greater than 1.82 — and this y must include both *Act.D* and any ordinary lead. Thus *RaG* must be between 8.3473 and 7.44. (Fig. 1.)

These equations are nearly like those used by A. Holmes in Nature, Sept. 6, 1930. If we can add one more equation, the equations can be solved definitely and then by equations given below we could get the age in three ways. If we can assume either that y is all *Act D* and for instance three percent of *RaG* or that as Holmes assumes *ThD* is 0.38 or .25 of $RaG + y$ or of *RaG*. But none of the uncertainties as to how much of y there is or what its atomic weight may be can effect the maximum value for x. It cannot be more than 8.3492. The

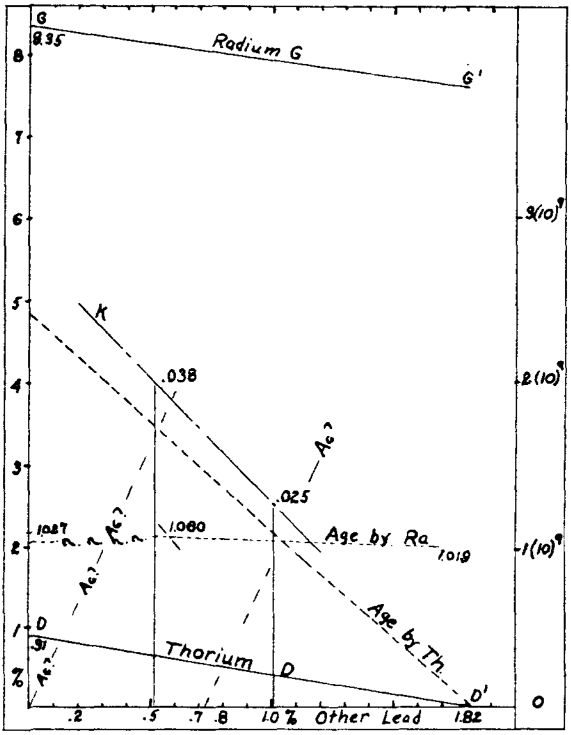

FIGURE 1. Illustrates the computation of all possible ages of the Wilberforce uraninite consistent with the amounts of uranium, thorium and lead and the atomic weight of the latter. Possible amounts of lead not derived from radium are laid off as abscissas. Full lines join the tops of the ordinates which show the Radium G and Thorium G that correspond and the scale is on the left. Lines with dashes show the ages corresponding. Their scale is on the right. The relative lead producing capacity of thorium to uranium is shown by the line marked k.

minute we assume, however, a rate of production of thorium or actinium, its value becomes definite. But instead of beginning with any such assumption, we plot the values of *RaG* as ordinates with the various possible values of y as abscissas (Fig. 1). The exact atomic weight of y to be taken whether 207 or 207.2 will make but a very slight difference in the slope of this line. In view of the age which we shall find must come to the mineral under any supposition, and the fact that there is certainly some thorium lead and actinium lead and plenty of oxygen for autoxidation as Norton has shown and the original UO_2 lattice is unaltered, we believe we have been safe in agreeing with Kovarik[1] for similar minerals, that the lead is practically $Pb206$ or $Pb207$ or $Pb208$ and in using 207 for the denominator of y. It must be remembered, however that in estimating the age if we assume y to be all actinium lead we may make a more substantial error and get the age too high or conversely.

In Fig. 1 with $Pb207$ as abscissas, the lines representing the corresponding properties of RaG and ThD are straight.

If we assume that there is a factor k as used in equation (1) of part (1) such that the age so obtained will be the same as that obtained from the quotient of RaG/UI, then if UI is urradium the parent of radium and AcU the parent of Actinium $U = UI + AcU$

$$\frac{RaG}{UI} = \frac{Pb}{UI + AcU + kTh} = \frac{RaG + y + ThD}{UI + AcU + kTh} \tag{12}$$

In equation (12) it is assumed, as in equation (1), that AcU is so small that a factor, call it j, dependent on the relative rate of disintegration of UI and AcU need not be introduced. If it is, then in equation (14) we must write $jAcU$.

$$\frac{AcU + UI + kTh}{UI} = \frac{RaG + y + ThD}{RaG} \tag{13}$$

$$\frac{AcU + kTh}{UI} = \frac{y + ThD}{RaG} = \frac{Pb - RaG}{RaG}$$

$$\therefore k = \frac{UI \cdot (Pb - RaG)}{Th \cdot RaG} - \frac{AcU}{Th} \tag{14}$$

The last term will often be small. If we neglect it and if we take k as having a value 0.25 or 0.38, we will find certain values of ThD and RaG are implied. So then we have another curve Fig. 1 for values of k, and we may pretty surely assume that k lies between 0.38 and 0.25. Piggott has pointed out that according to this test, Aston's estimates of the isotopes in Bröggerite lead are substantially out. Aston did not claim high accuracy.

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[1] Science, Aug. 1, 1930, p. 122–125.

Now as to age we may rewrite Kovarik's equations[1] which are essentially the usual ones,[2] using decimal logarithms instead of exponentials and the weights actually found in analysis rather than numbers of atoms, and the half lives instead of the disintegration constants, as follows:

Age by uranium in millions of years

$$= 14{,}290 \log\left(1 + 1.156\,\frac{RaG +}{U - AcU}\right) \qquad (15)$$

This is if we use for the half life of uranium a value (4300 million years) which allows for the amount of actinium supposed to be in uranium, and the proportion of disintegration in a given time that is supposed to belong to it. In the ordinary chemical analysis it is not possible to separate the U which changes to radium from that which changes to actinium. If we assume that the Ur-radium parent of radium has an atomic weight of 238.07 and the Ur-actinium is U_{239} and that ordinary uranium has an atomic weight of 238.14, then[3] about six percent of the U is U_{239} and we should have

$$U\ 238.08 - AcU = .94 \cdot (53.52) = 50.3 \qquad (16)$$

$$\text{Whence Age} = 14{,}290 \log.\left(1 + 1.156\ \frac{RaG}{50.3 + \text{some part of } 3.2}\right) \quad (17)$$

When we make *RaG* as great as possible in the equations above, there would be no *AcD*, hence no Ur-actinium, and we should have

$$\text{Age} = 14{,}290 \log.\ (1 + 1.156\ (8.349/53.52)\ ) \qquad (18)$$

(in millions of years) = 14,290 log. (1.180) = 1027 million years if $y = 0$. Or assuming T_u is not reduced for the proportion of actinium it would be 45/43 greater, i. e. 1075 million years. On the other hand, when we assume that all the y lead is Ac-lead, an obviously impossible value since thorium is present, we may then have, as a maximum

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[1] Am. J. Sci., 1930, p. 98.

[2] Geologie und Radioktivität (p. 129)

[3] Geologie und Radioktivität (p. 138). The usual atomic weight of U is given as 238.17, but 238.14 is the latest by Hönigschmid, used in his new edition of Landolt–Börnstein's tables. It will be interesting to know the exact age and source of the different uraniums, as the proportions of Uractinium may differ.

$$\begin{aligned} \text{Age} &= 14{,}290 \text{ log. } (1 + 1.156\ (7.59/50.3)\ ) \\ &= 14{,}800 \text{ log. } (1.175 = 1050 \text{ million years} \qquad (19) \end{aligned}$$

For intermediate values we may assume $U(238)$ to be 50.3 as soon as we have $y = .322$ and we have, assuming $U(238) = 50.3$ (by slide rule)

and $y = 5$ and 1.0, the ages (20)

1061 and 1031 million years respectively.

Now as to the age by the thorium ratio, if we have

$$\text{Age} = 3.323\ T_{th} \text{ log. } (1 + 1.115\ (ThD/10.37)\ ) \qquad (21)$$

(In millions of years) $= 59{,}800$ log. $(1 + 1.115\ (0 \text{ to } .91)\ (10.37)\ )$.

Then the range must be from 2400 million years (if $ThD = .91$ as shown in the figure 1) down, and if we assume $y = 1$ $ThD = .408$; $RaG = 7.85$; $Th = 10.37$ and U is 53.52 to 50.3 and $k = ThD.U/Th.RaG = 24.8$ to 26.8—consistent with what was found in Part I.

As to Actinium, Kovarik, following Aston, makes its rate of decay such that $Tacu = 2.7 \times$ antilog 8 years.

But if about four percent of the atoms disintegrating are actinium while six percent of the uranium is the percent of uractinium, the rate of disintegration must be less rather than more, and A. Holmes came to the same conclusion (Nature, Sept. 6, 1930).

We can give more definite figures when Baxter has determined the atomic weight of the very same uranium. But to show how the calculation would be:—

If the actinium lead is three to four per cent of the radium lead, i. e., 0.27 (about a third of y) and *if* the rate of decay of the Ur-actinium (239) is two thirds that of the uranium and *if* the percent of Ur-actinium is about six percent of the U since the half life of Ur-actinium would be three halves that of $U(238)$, then

$$\begin{aligned} \text{Age} &= \text{about } 22{,}000 \text{ log. } (1 + 1.153 \times .27/3.22) = 22000 \times .04 \text{ to } .06 \\ &= 880 \text{ to } 1320 \text{ million years} \end{aligned}$$

Very possibly the checking of the age may be the most accurate control of the rate of decay of the $U239$. If all the y is actinium lead, we should have to reduce the half life more to get accordant ages, but the half life assumed by Kovarik would not agree at all with any reasonable suppositions. If we assume that all the y is Pb and that one-sixth of the U is $U(239)$ and there cannot be much more, then with the half life assumed, we should get 1420 million years.

It is not worthwhile to go into further detailed calculations. But we can see how unlikely it is that the decay of the original ur-actinium should be as rapid as Rutherford and Kovarik assume, and we see the import of the work upon which A. v. Grosse, Wildish and Baxter are engaged for the Committee, while we can see also that it is unlikely that the Wilberforce uraninite is 10% away from 1030 million years old. In the calculations a slide rule has been used.

We can clearly see from Fig. 1 that unless there is some great error in the data given us by the physicists, this age of 1030 million years is not far from the truth, and that when the physical data are well enough known, we shall have a triple check!

W. R. Bennett reports May 7, 1931, the following results on three 1.5 gram portions of the same sample of Wilberforce Uraninite on which the atomic weight determinations were made:

15.605, 15.543, and 15.620 cc of helium

This gives an average amount of 10.39 cc He per gram of ore. Whence (I. C. T. Vol. 1, p. 381) the ratio and age is

$$(10.39/(53.52 + 2.9) = .185) \times 910 = 168 \text{ million years,}$$
$$\text{or, (using Kirsch's } k\text{) } 10.37 \times .19 = 1.97) = .187$$

The age is much less than the age by the lead ratios, even when multiplying by 3 to allow for diffusion.[1] But since there is a larger amount of helium per gram than in any mineral cited in the International Critical Tables, one would naturally expect a greater amount of diffusion.

If we compare with a similar and similarly rich mineral,—the bröeggerite of Ånneröd, we find that the relations of the lead ratios, —for Wilberforce (.165), for Ånneröd (.139); and the helium ratios—for Wilberforce (.187), for Ånneröd (.14) are similar. Both indicate that the Wilberforce material is older.

III. Interpretation of the *Pb* Ratio.

By G. Kirsch.

We shall now proceed to a discussion of the important question of how the results from age determinations based on the *Pb* ratio are to be interpreted. The radio-active minerals belonging to the pegmatite dikes from a certain region and generated during a given period of intrusion (leaving aside the question whether the intrusion period was really single or subdivided into a sequence of repeated intrusions) give as a rule the following picture:

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[1] Kirsch, Geologie und Radioaktivität, p. 191.

First such minerals as have been found by X-ray analysis to possess crystal structure without having had their temperature raised to incandescence, or the Wilberforce Uraninite even give a Laue diagram, i. e., possess a crystal structure of unchanged orientation. Such minerals are found to have their ages comprised within a time-interval of about 200 millions of years. Taking a few examples, we have the Canadian pitchblendes (*Pb* : *U* 0.146 to 0.179), the Ceylon thorianites (0.056 to 0.088 leaving aside some very few specimens of still greater age) or the bröggerites from Raade and Moss in Norway (0.112 to 0.148, Morogoro 0.091) Wilberforce and allied occurrences (0.15 to 0.176.) This limitation to a certain time interval we may take as established, for the very few exceptions do not necessarily belong to the main intrusion period.

One alternative view, viz. that the bröggerites are all of the same age and their different *Pb* ratios are to be explained by accidental additions of ordinary lead or chemical errors, is made less likely by atomic weight determinations with chemical analysis, notably these made by Honigschmid (206, 06). For with the same material that Honigschmid used, a considerable number of single crystals have been submitted to measurements of the *Pb* ratio with results spread over the range before-mentioned. Another alternative explanation is that the lead has been extracted from these minerals to a variable extent. But these are analogous variations shown by thorianites from Ceylon. The large amount of helium contained in these latter minerals makes it most improbable that they should have been altered to the requisite extent. Another argument against this second explanation is the subdivision of the bröggerites according to their *Pb* ratios into well-defined groups[1] as such a distribution must necessarily have been effaced even by slight alterations.

Summing up, we may say that the minerals here considered must be assumed to have ages corresponding to the values found for their *Pb* ratios.

In any case, we must not assume that the age thus defined is identical with the age of the pegmatite dike in which the mineral was found.

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[1] Forschungen und Fortschritte, 10 Nov. 1929, p. 372.

For, as has repeatedly been shown, minerals from the same locality, which may be assumed to have been found in the same dike, give age values differing considerably from each other, as is indicated in the following table.

TABLE II.—BRÖGGERITES FROM KARLSHUS.

	Pb ratio
1. Anal. C. N. Fenner and C. S. Piggot	0.129
2. Anal. E. Gleditsch	0.137
3. Anal. E. Gleditsch	0.133
4. Anal. W. Riss	0.125
5. Anal. W. Riss	0.129
6. Anal. W. Riss	0.138

We must then either assume (that the analytical methods are accurate only to some ten per cent or) that bröggerite crystals may be generated during large intervals of time while the pegmatite dike has solidified (or that there is a zonal variation in composition and the material analyzed is not from the same zone), or we have to ascribe a greater age to the crystals than to the dike itself. They must therefore have been formed beforehand at different times and at greater depth and have been carried upwards with the molten magma when the dike was formed. The ages of the different crystals then would only signify at what times the magma was fluid and for the present nothing else.

A second group of minerals, silicates and other salts where the anion is the radical of a weak acid, which under the action of the radio-active rays are transformed into amorphous glassy substances in the so-called metamict state, give *Pb* ratios that range from the highest values found with the former group down practically to zero. Thus for example minerals of this type from Ontario in Canada have given the *Pb* ratios 0.014, 0.023, 0.024, 0.030, 0.035, 0.042, 0.046, 0.088, 0.115, 0.149, 0.158, 0.198; again there are thorites from Ceylon with the *Pb* ratio 0.024, 0.043, 0.063, 0.094.

Furthermore, the minerals of the thorite type from the Middle Devonian at Brevik, Norway, known from Holmes' and Lawson's investigations belong to this group.

We turn to the possible interpretations of the variability in the *Pb* ratio amongst the minerals of this second group. The assumption that for these thorites the age should be taken equal to that calculated from the actual value of the *Pb* ratio involves very serious difficulties as has already been stated by Holmes. Thus one would have to postulate that the surges of hot solutions which at intervals of time may be assumed to have passed through the dikes as veins have carried a certain, not too small, amount of dissolved thorium, and, displacing other crystalline matter, have deposited large single individual crystals of thorite, etc. However, as the thorites considered are unable to regenerate their crystal lattices, once they have been destroyed by the action of the rays, one can hardly ascribe to them this faculty of displacing other crystallized substances. Considering also, that quite often crystals are found having an age equal to that of the oldest minerals in the first group and therefore of an undoubted magmatic origin, one can hardly avoid the conclusion that the minerals of the second group are practically all of them quite as old as those of the first group. Besides, if one should accept the admixture of ordinary lead in a variable amount to the crystals of the second group as an explanation for the variability of their *Pb* ratio, one would be led to the improbable conclusion that all minerals of this kind all over the earth should be considered of quite recent formation.

It must always be remembered that ordinary lead is a stranger to the pegmatitic dikes. In those cases where the atomic weight of the lead in such minerals has been determined (note the two thorites in Table I), the results are in favor of a purely radio-active origin.

We have now to discuss as a last alternative the explanation of Ellsworth that the differences in the *Pb* ratios of the metamict minerals are due to losses of uranium or lead in variable amounts. In this connection it should be noticed that, even though the interiors of many crystalline phases are not accessible to the different mineralizing agencies occurring in the dikes, this is not necessarily the case with the amorphous phases, indeed the opposite may be assumed as true. Further, it seems obvious that especially lead, which apart from helium is the only substance falling out of line with those other elements which have spontaneously crystallized together, will be extracted in the first place.

Bearing in mind that the physical conditions (temperature) which may lead to a revival of pneumatolytic or of hydrothermal activities within a dike, owing to the large dimensions of the system considered and owing also to the low thermal conductivity, must change very slowly indeed, it seems obvious that the span of time during which the crystals are liable to undergo chemical changes must be counted in hundred thousands if not in millions of years. Now the immense length of time with which we may here count makes it very probable that the possible reactions will proceed until a certain final state of equilibrium has been attained. This view is also supported by the remarkable similarity in composition of such metamorphosed minerals derived from the most different localities. Thus an analysis of the gummite from North Carolina proves its content of silicon dioxide to be five percent, whereas the water content is 10.5%[1] and for the gummite from St. Joachimstal the same results are found. Also the gummite from Katanga has the same water content. The water content of thorites is generally between six and seven percent whether they are from Scandinavia, from Asia Minor, or from Ceylon (Doelter, Hb. d. Min.-Chem. III, p. 228, anal. 2–7) whereas the uranothorites have in general 11 to 12 percent of H_2O (l. c. anal. 9–13). It is therefore probable that under the action of chemical agencies a metamict mineral may readily move its lead. Granted this, the age found for a certain metamict mineral would simply denote the length of time elapsed since chemical agencies were last active having free access to the crystal in question.

Summing up the contents of the proceeding paragraph we may say: In the values found for the *Pb* ratio in non-metamict minerals (oxides, salts of strong acids with a crystalline structure), we have in all probability indications of magmatic activity in the depth at the corresponding times. On the other hand, the *Pb* ratio of metamict minerals (salts of weak acids) indicates the time when the last pneumatolytic or hydrothermal activity occurred.

So far we have only dealt with pegmatite minerals. Now there are also radio-active fissure minerals of pneumatolytic or hydrothermal origin, to which we may apply the conclusions drawn from the preceding reasoning. The *Pb* ratio for such minerals, whether they are pitchblendes or metamict minerals must necessarily correspond to epochs of increased activity and mineralization in the dikes. However, as an increase in the dike activity in pegmatites as well as in fissure must be taken as a sign of increased magmatic activity in greater depths, one must infer that the ages of radio-active minerals in the large majority of cases are to be considered as a sign of magmatic events occurring at the times derived from these data.

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[1] See paper presented to the Mineralogical Society at the Toronto meeting, 1930, by E. P. Henderson, E. Posnjak, and C. S. Ross, Am. Mineralogist, 1931, p. 213.

RELATIONSHIP BETWEEN RADIO-ACTIVE AGE DETERMINATIONS AND THE STRATIGRAPHIC TIME SCALE.

The task of fitting in a pegmatite dike into the stratigraphic time scale does not fall on the physicist but on geologists. We shall here limit our discussion to a brief indication of how the age of pegmatite dikes can be estimated from the age of the minerals it contains.

Taking as an example the pegmatite dike at Divino de Uba in Brazil, from which the two minerals investigated by C. N. Fenner, monazite and samarskite, were derived, we may say that most probably its age is at least that of the samarskite and certainly not greater than that of the monazite. Its actual age cannot be more exactly defined than by these limits, for the monazite might possibly have been injected with the magma as a preformed crystal, whereas the samarskite, being a metamict mineral, may have been subjected to mineralizing influences during a later period, in the course of which part of its lead may have vanished. Possibly more exact estimates of the age might have been derived from a closer examination of its paragenesis, provided the sample affords any information of this kind. Thus if the monazite should prove to be intergrown with minerals undoubtedly formed during the solidification of the dike, or else to show arrest of growth due to such minerals, one would infer that the age of the dike coincided with that of the monazite.

Turning again to the pegmatite dikes of the Fredrikshald granite, the age of the dike at the Karlshus quarry cannot be taken as greater than that which corresponds to a *Pb* ratio of 0.125 : vide Table II. Bröggerites found at Annerod have given *Pb* ratios of 0.136, 0.124, 0.124, and 0.112. Provided all these crystals were obtained from the same dike, its age cannot be higher than the *Pb* ratio 0.112 indicates. Only in cases where from one and the same dike ten or more crystals from non-metamict minerals have been analysed can its age be stated with an accuracy of a few percent. Either the age is equal to that of the youngest mineral it contains or may be some few million years younger. That dikes differing in age by quite considerable amounts exist near Fredrikshald is not altogether impossible. In the majority of cases the dikes were most probably formed already at a period indicated by the *Pb* ratio 0.012, whereas a few dikes containing bröggerites may have been formed at a period later in time, possibly as late as the upper Paleozoic age, although these intrusions seem, so far, not to have been laid bare by subsequent erosion.

We cannot well take an intrusion to have remained in the fluid state for as long a period as 200 million years (the span of differences between indicated ages of different bröggerites). The formation of these bröggerites therefore may have occurred at recurrent epochs of liquidity. In the course of such repeated fusions leading to differentiation, the conditions for the formation of bröggerites must be expected to show systematic variations. This is borne out by the analyses of the bröggerites. The oxygen content is liable to change, rising from the oldest to the youngest crystals as is seen from figure 2,

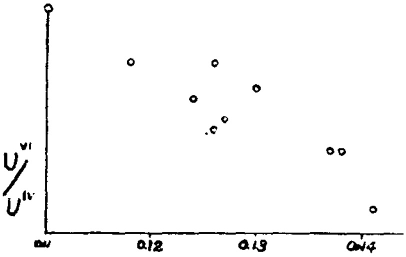

Figure 2. Oxygen content indicated by U^{VI} to U^{IV} and lead uranium ratio indicating age of the bröggerites.

where the ratio $U^{VI} : U^{IV}$ is plotted against the *Pb* ratio. This is the reverse of the effect of autooxidation! Here the lead is brought in as uranium corresponding to the original composition of the mineral. Also the thorium content falls from the oldest to the youngest bröggerites: Among some 60 analyses 11 have *Th* : *U* 0.12 all of which have $Pb : U > 0.130$; three analyses gave *Th* : *U* 0.01 with the *Pb* ratio 0.112, 0.118, 0.125. Further this decrease in the thorium content is not regular: from a maximum at *Pb* : *U* 0.139 the thorium content falls steadily to a minimum at *Pb* : *U* 132 rising again abruptly to a new maximum at *Pb* : *U* 0.131 falling again to a second minimum at *Pb* : *U* 0.125. Outside these limits the number of analyses is too restricted to allow of any definite conclusions regarding a similar periodicity, which is nevertheless intimated. Regarding the distribution of the bröggerites over different *Pb* ratios, the following remarks should be made. After a certain period of time from which only a few crystals have been analyzed the formation of bröggerites seems to make a vigorous start at the value *Pb* : *U* 0.139 and goes on with at least two culminations down to the value *Pb* : *U* 0.125 after which again a few crystals are spread out over a larger interval of time.

The only other region from which a large number of radio-active crystals of the same species have been analyzed (some 20) is Ceylon, where the thorianites investigated may really be considered as pitchblendes abnormally rich in thorium. As far as can be judged from this limited material the same facts that were found with the bröggerites are again found here. Following on a few single crystals of greater age, we notice a formation of thorianites on a larger scale lasting over a span of time comparable in length to that found with the bröggerites. Within this interval a decrease in the *Th* : *U* ratio is found to occur twice. Finally there are again a number of single crystals spread out over a larger span of time as with the bröggerites. Among these last named thorianites there is in analogy with the three bröggerites of abnormally low thorium content an example of thorianite of exceptionally low thorium content, i. e., a pitchblende having only 7.8 per cent *Th*. Thus these results offer a very close analogy to those obtained with the bröggerites from the Fredrikshald granites saving a general shift of the ratio *Th* : *U* to higher values differing from the former in a ratio of about 10 : 1. One can hardly avoid the conclusion that this fact must be due to a low content of uranium in the magma, as has already been found by Smeeth and Watson working with rocks from Mysore.[1] This is probably characteristic also for Ceylon possibly also for Madagascar and Mozambique.

The next step to be taken will be to study on a broad basis all analogous occurrences. In this paper we hope to have shown how far distant we are still from an accurate (i. e., within the limits of several tens of million years) correlation between stratigraphic and radio-active chronology, while yet in a broad way the ratios are useful and reliable.

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[1] Phil. Mag. (6) *28*, 44, 1914; *35*, 206, 1918.

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BERMUDA DURING THE ICE AGE.

WITH THIRTEEN PLATES AND EIGHTEEN TEXT FIGURES.

BY ROBERT W. SAYLES.

BERMUDA DURING THE ICE AGE.

By Robert W. Sayles.

(Read by title October 14, 1931.)

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INTRODUCTION.

Inception of the Theory.

In March 1923, I visited Bermuda for the first time. Although this was primarily a vacation trip, a casual study of the geology was made. Very soon two facts forced themselves upon my mind. One was the luxuriant vegetation at present, the other the tremendous amount of dune formation in the past in contrast to the almost total absence of dunes now forming. I was also puzzled to understand how such great dunes could develop on such a small island, only 19 miles long and averaging less than 2 miles wide.

During the last few days of the stay at St. George's, I noted a deep red stratum eighteen inches to two feet thick intercalated between eolian limestones. Further study convinced me that this was an ancient buried soil, for it had all the essential characteristics of a soil and had many fossil terrestrial gastropods. It could be called a fossil soil. Such a soil, with thick eolian limestone below and above it could not form unless there had been a long interval between two periods of dune formation, an interval measured probably by thousands of years. Was it possible that the dunes below and above this fossil soil had formed when the island was larger and perhaps during a time of stronger winds? I did not know then, anything whatever of the literature on Bermuda, and of course was ignorant of the fact that Verrill and others had recognized the necessity of a larger Bermuda to account for the dune formation, due to uplift of the islands.

In reflecting upon the present small size of Bermuda, it was natural to conclude that Bermuda must have been larger at one time and had subsided. A subsidence of sixty feet would change the area from about two hundred square miles to the present size of about twenty square miles. As I was very familiar with the glacial control theory of coral reefs advanced by Daly, it was most logical to explain a raising of water-level by deglaciation of the Pleistocene ice caps. It was at this point in the reasoning that it occurred to me that the buried soil I had seen and puzzled over might mean an interglacial episode of the Pleistocene, when the position of sea-level and the type of climate would be very different presumably from that existing during a glacial age, and would be more like conditions existing today. On the other hand, while the northern continents were buried under ice the climate of Bermuda ought to be stormier, and probably colder than the present climate, and furthermore Bermuda should be larger, due to subtraction of water from the oceans, and a larger Bermuda would explain the great dune formations better than the smaller Bermuda of today. A colder Bermuda would also reduce vegetation and this in turn would aid dune formation. During such a time soil formation might be very much restricted if not entirely prevented.

If the fossil soil found really meant an interglacial interval, there should be more than one, but there was no time left to investigate further. On my return to Boston I read one of Verrill's[1] papers on Bermuda and found that he spoke of several fossil soils in the eolian limestones. This encouragement made it possible for me to present a brief paper before the geologists at the meeting of the Geological Society of America in December, 1923.

HISTORY OF THE FIELD WORK AND ACKNOWLEDGMENTS.

With the realization that there were several distinct fossil soils in the formations at Bermuda, it was evident that a study of the paleontology of the soils might make it possible to distinguish each soil. With this thought in mind, I invited Dr. Thomas H. Clark, now of McGill University, to accompany me to Bermuda in January 1924. We had just nine days of field work during this visit, but in that time found over fifty fossil soil localities and at least three and possibly four distinct soils in several places. We presented a brief paper to the Geological Society of America in 1925. In 1926 we visited the islands again for another short field trip of two weeks during the latter part of May. Clark has made a number of contributions to the investigation. He discovered many of the fossil soil localities and recognized the importance of accretion, or addition to soils by wind action, in the formation of the lighter colored soils. Clark studied the collection of fossil snails and submitted to me a list of the fossils found in each locality.

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[1] A. E. Verrill. The Bermuda Islands, Part IV. Trans. Conn. Acad. Arts and Sciences, Vol. 12, June, 1905.

In April, 1928, Professor Kirtley F. Mather of Harvard University, visited Bermuda with me. He contributed to the problem in several ways, but especially in finding marine fossils in the two important marine formations. In August of the same year Professor A. C. Swinnerton of Antioch College visited Bermuda to make a study of the caves. This study, together with his general observations on the geology, have been of much value in corroborating my own observations on several points as well as adding to our knowledge. In 1929, during March and part of April, I visited Bermuda accompanied by my family and made a more thorough study of the geology. It was during this visit that I met Professor Charles Schuchert of Yale University and was able to show him the formations in several parts of the islands, and to convince him of the reality of the different formations. Professor Schuchert made collections of fossils, especially of the fossil snails, and also aided me in many other ways. In November, of the same year, Professor R. A. Daly accompanied me to Bermuda and we saw a great deal of the islands in a few days, and discovered more fossil soil localities.

I am greatly indebted to many who have not been to Bermuda with me. First of all I wish to express my gratitude to Assistant Professor Marland P. Billings of Harvard University, who has edited the manuscript with much skill. In addition he has brought up to date the correlation of the Bermuda formations with the Pleistocene formations of North America and also added a chapter on the planation of the Bermuda group in the late Tertiary or early Pleistocene. At a time when unavoidable responsibilities made it impossible for me to revise and edit the paper, his help is deeply appreciated.

I wish to express my gratitude to Professor Emeritus E. L. Mark of Harvard University for aiding me from the very beginning of the problem, but especially for advising me to investigate the quarry at Shore Hills in St. George's. Mrs. R. D. House, formerly of the Harvard Mineralogical Laboratory, made several chemical analyses of the soils. The petrology of the soils and eolianites was done by Professor Esper S. Larsen of Harvard University and more than fifty analyses were made, a long and arduous undertaking. For the time and thought he has given to this problem I cannot give enough credit. A study of the foraminifera found in the fossil soils was made by Professor Joseph A. Cushman of Harvard University. A study of the foraminifera in the eolian formations below and above the fossil soils was made by Mr. Lothrop Bartlett at Professor Cushman's laboratory, at Sharon, with important results. Professor Cushman also reëxamined the samples from the deep boring made at Bermuda in 1914, and came to conclusions different from those arrived at from the first examination made many years earlier. Dr. M. A. Howe, of the New York Botanical Garden, examined the marine algae found in a few of the fossil soils. Professor Leon W. Collet, of the University of Geneva, Switzerland, visited Bermuda in December 1926, alone, and was favorably impressed with the theory outlined in this paper. I wish to express my thanks to Mr. Freemont Rider, Editor of The Rider Press, Incorporated, for permission to copy in part the outlines of the map in their guide book. Mr. Edward A. Schmitz has skilfully made the soil map and cross sections.

PREVIOUS GEOLOGICAL WORK.

After returning from my first visit to Bermuda in 1923, I studied the literature on the geology of the islands. I discovered that others had made observations similar to mine, but no one had considered the possible relation of glaciation to the evolution of Bermuda. The first real contribution to the geology of Bermuda was published by R. J. Nelson (*1840*). He made a distinct advance in demonstrating that modern Bermuda is composed largely of wind-blown fragments of shells and is not a true atoll. Papers by J. J. Rein (*1870* and *1881*), and Sir Wyville Thomson (*1877*) should be mentioned. W. N. Rice (*1884*) saw the necessity of a greater Bermuda to explain the great eolian deposits. He assumed a former higher stand of the island in contrast to the idea of the lowering of sea-level adopted in the present article. The papers by J. W. Fewkes (*1888* and *1890*) and Heilprin (*1889*) appeared shortly thereafter. Alexander Agassiz (*1895*, p. 220) doubted the need of appealing to subsidence or elevation. "Whenever the accumulations from the reef were sufficient to build up a beach reaching the surface, all the conditions necessary for the formation of sand dunes existed and we need not call upon either a subsidence or an elevation to account for the existing condition of the Bermudas." A paper by Stevenson (*1897*) appeared two years later. The most important papers are those by A. E. Verrill (*1903* and *1907*). He recognized a basal Walsingham formation, composed of eolian limestones and red soils, a younger marine limestone, called the Devonshire formation, and the youngest of all, the eolian limestones and red soils, called the Paget formation. The present investigation confirms this stratigraphy in a general way, but the relations are much more complicated than Verrill supposed. He was also the first to describe and discuss the origin of the red fossil soils and went so far as to estimate the amount of weathering and length of time necessary for them to form. The paleontology is well described. Verrill states that many of the species of snails are now extinct, which suggested the possibility of developing a stratigraphic paleontology for Bermuda. Pilsbry's (*1888*) description of the fossil land molluscs should be mentioned, for although strictly zoölogical, it has formed the basis for later paleontological work. Pirsson (*1914*) has given an excellent description of the deep boring in the island. It proves the current hypothesis that the eolian limestones of Bermuda form a thin veneer a few hundred feet thick resting on a volcanic platform. I have already published four brief notes on the fossil soils of Bermuda. (Sayles, *1924*, *1926*, *1928* and *1930*.)

TOPOGRAPHY.

The Bermuda Islands lie 675 nautical miles southeast of New York City. They are included within the rectangle bounded by N. Lat. 32° 14′ and 32° 24′ and W. Long. 64° 38′ and 64° 53′. Modern Bermuda consists of over 150 islands and islets grouped together in a great hook trending northeast-southwest (see Fig. 1). The total area is about 19¼ square miles. In general the islands consist of rolling hills rising from one to two hundred feet above sea-level. At present the islands are sparsely wooded, although locally the vegetation is very luxuriant.

There are no surface streams in Bermuda. The drainage is entirely underground. The valleys, therefore, are not the product of running water. They are in part true sink-holes, formed by the collapse of the roofs of caves, and in part residual depressions left by the construction of sand dunes. Some of the depressions are occupied by peat bogs, the most conspicuous of which are Devonshire Marsh and Pembroke Marsh. The peat in the former is forty feet deep. Modern sand dunes are rare, although drifting sand is present in the vicinity of Elbow Bay and also near Tucker's Town. (Stevenson, *1897*, pp. 99–101.) In both areas, however, there is evidence demonstrating that the formation of these small, local dunes was the result of deforestation. From the descriptions given by Verrill (*1903*, part III, 26, pp. 181–210; *1907*, part IV, 23, pp. 107–113), as well as from the early records,

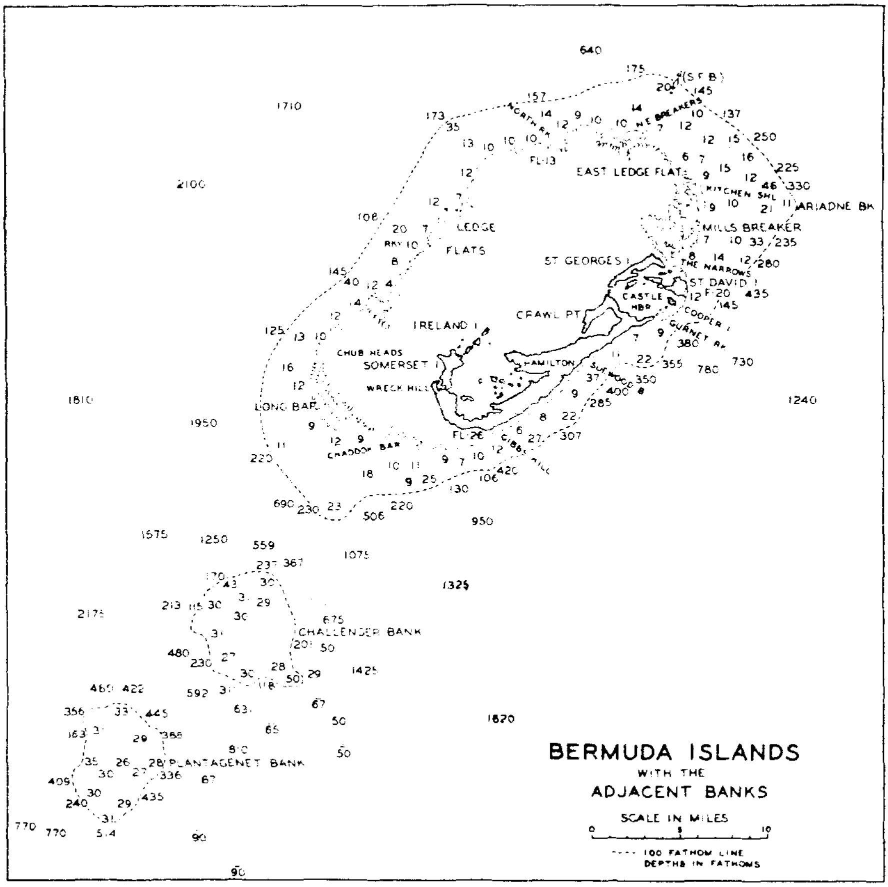
BERMUDA ISLANDS
WITH THE
ADJACENT BANKS
SCALE IN MILES
0
5
10
---- 100 FATHOM LINE
DEPTHS IN FATHOMS
EAST LEDGE FLATS
KITCHEN SHL
ARIADNE BK
MILLS BREAKER
THE NARROWS
ST GEORGES I
ST DAVID I
CASTLE HBR
COOPER I
GURNET RK
CRAWL PT
IRELAND I
LEDGE FLATS
CHUB HEADS
SOMERSET I
WRECK HILL
HAMILTON
LONG BAR
CHADDOCK BAR
CHALLENGER BANK
PLANTAGENET BANK
NE BREAKERS
640
1710
2100
1810
1950
1240
1575
1250
559
1075
1325
950
2175
1425
1820
690
506
220
780
730
435

FIGURE 1.[1]

it is clear that before man invaded the islands they were heavily wooded and quite free from drifting sands.

The islands are perched on the southeast side of an elliptical submarine platform (see Fig. 1). The major diameter of the ellipse, trending northeast-southwest, is about 22½ miles; the minor, about 10½ miles. The total area is somewhat more than 200 square miles. Resting on this platform, the top of which is about 60 to 70 feet below sea-level, is an elliptical reef ranging in width from one to two miles. Its outer edge is from one to three miles inside the margin of the main platform. Most of this reef is submerged beneath very shallow water, averaging from ten to twenty feet in depth, but on the southeast side portions of the reef project above sea-level. Within the reef there is a lagoon in which the water is about sixty feet or less in depth. The configuration of this reef is thus suggestive of an atoll. As Verrill and others have pointed out, however, Bermuda is not a true atoll, for it is composed of eolian limestone. The corals and associated forms have contributed only a thin veneer. Beyond the edge of the platform the sea floor drops off rapidly to abyssal depths. The slope averages about 10 degrees and the surrounding ocean is about 16,000 feet deep (Fig. 2).

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[1] On account of the general use of the name "Argus" in the literature, the new name "Plantagenet" has not been adopted in this paper.

Fourteen miles to the southwest of the Bermuda platform is the Challenger Bank, the area of which is about twenty square miles and the depth approximately 180 feet. Still further to the southwest is the Argus Bank of similar size and depth.

IGNEOUS ROCKS.

It had long been supposed that the Bermuda platform is volcanic. The first proof, however, was obtained in 1912, when a deep boring was made in an unsuccessful attempt to obtain an adequate supply of fresh water for the Princess Hotel. L. V. Pirsson (*1914*) has fortunately recorded the data obtained from the well. Its site is in Southampton parish on the slope of a hill about a mile west of the Gibbs' Hill lighthouse. It began at an elevation of 135 feet above sea-level and was 1413 feet deep. The boring was made by churn drill and only powder and small grains were available for study. Pirsson gives the following section:

Eolian limestones—surface to 245 feet below sea level.

Brown, deeply weathered volcanics—245 feet to 455 feet below sea-level.

Sand and gravel of volcanic material—455 feet to 560 feet below sea-level.

Basaltic lavas—560 feet to 1278 feet below sea-level.

The petrography of the lavas was studied by both Pirsson and Thomas.[1] The dominant type is a melilite basalt, composed of microphenocrysts of augite and altered olivine set in a groundmass of augite, melilite, apatite, analcite, biotite, magnetite, titanite, perovskite, nephelite, and sanidine. Although brown garnet is not mentioned, it is very probably present in small quantities, for, as will be pointed out later, it has been found in the fossil soils. A second group of lavas are of lamprophyric or monchiquitic types. The essential constituents are augite, biotite, and analcite; the minor acessories, perovskite and apatite. The groundmass is presumably nephelite, sanidine, and analcite, with such secondary minerals as titanite and calcite. Dr. Thomas[1] also reports one specimen of a keratophyre, in which microphenocrysts of albite and biotite are set in a groundmass composed of albite and microperthite.

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[1] Pirsson quotes in his paper, already cited, a letter from Thomas.

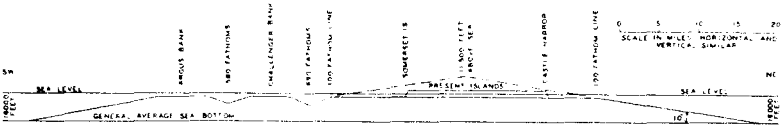
SW
SEA LEVEL
16000 FEET
GENERAL AVERAGE SEA BOTTOM
ARGUS BANK
580 FATHOMS
CHALLENGER BANK
893 FATHOMS
100 FATHOM LINE
SOMERSET IS
PRESENT ISLANDS
CASTLE HARBOR
100 FATHOM LINE
0 5 10 15 20
SCALE IN MILES HORIZONTAL AND VERTICAL SIMILAR
NE
SEA LEVEL
10°
16000 FEET

FIGURE 2. Cross section of Argus, Challenger, and Bermuda banks in a S.W.-N.E. line. (After Pirsson.)

The true interpretation to be given to the geologic section is still debatable. The presence of sand and gravel beneath the brown weathered volcanics implies that everything between the eolian limestones and the solid basaltic lavas is a product of subaerial weathering reworked by the ocean. It does not appear to be an autochthonous residual soil. Another problem concerns the depth of the igneous platform below sea level. If the boring is on the flanks of the platform, the surface is at some depth less than 245 feet below sea level, but when all the factors are considered, it seems legitimate to assume that the surface of the igneous platform of Bermuda is about 245 feet below sea level.

The elliptical shape of the Bermuda platform suggests that the volcano had at least two vents aligned in a northeast-southwest direction. The Challenger and Argus banks lie on a similar trend line and are the eroded stumps of two volcanoes. At some time in the past, therefore, Bermuda was the site of a line of volcanoes. They are certainly pre-Pleistocene and may be pre-Tertiary. Cushman informed Pirsson that certain foraminifera brought up in the drilling were Eocene and the latter concluded that the volcanoes were pre-Tertiary. Cushman has kindly reëxamined the material at my request and now feels that the foraminifera are more likely Miocene or later.

SEDIMENTARY ROCKS.

Rock Types.

Reposing on the igneous platform of Bermuda are the calcareous sedimentary formations. The dominant type is an indurated eolian limestone. In my 1929 paper I proposed the name *eolianite* for all consolidated sedimentary rocks which have been deposited by the wind. If the rock is composed largely of calcareous material it would be termed a *calcareous eolianite;* if of quartz grains, a *quartz eolianite*. The Bermuda rock is thus a calcareous eolianite. In addition there are fossil soils, marine organic limestones, "sand-rock," and various present-day deposits such as calcareous ooze, beach sands, and dunes.

The Bermuda *eolianite* is a white or cream-colored, wind-deposited, more or less consolidated, rock composed chiefly of fragments of calcareous shells. The individual fragments vary greatly in shape. Most of them are rounded, others sub-angular, and still others very angular. The average size is half a millimeter, but the grains range in diameter from a tenth of a millimeter to two millimeters. Pelycepods have supplied most of the material, but foraminifera, crabs, and algae are also present. The eolianite is almost invariably cross-bedded, although in the more indurated varieties this is sometimes difficult to observe. The degree of induration varies greatly. The younger formations are loosely cemented, but the older ones are commonly compact and crystalline. Proximity to the present surface is also a controlling factor. In quarrying operations, after a thin veneer about 15 feet thick of relatively compact eolianite has been removed, the poorly cemented sands are encountered. After roads are cut down into the unconsolidated sands, rapid induration sets in and a compact eolianite soon develops.

The *fossil soils* are considered at much greater length in a later chapter. A few words, however, are necessary at this point. The soils vary greatly in physical and chemical composition. Most strikingly they are a deep-red or chocolate-brown color, but they are often pink or even cream-colored and sometimes gray. The former are composed largely of silica, alumina, and ferric iron; the latter are highly calcareous. Ordinarily the soils are only a few inches thick, but locally they fill great pockets in the underlying eolianite and may attain at maximum a thickness of twelve feet or more. Fossil snail shells are abundant in the soils, particularly those of lighter color.

Two processes are involved in the formation of the soils. One is long-continued weathering, which leaves the insoluble residue of the eolianite as a soil. A second process is the deposition by the wind of a finely pulverized material. After the deposition of such material it may be more or less deeply weathered. In any particular soil either one of the two processes may predominate. The absence of any evidence of earthworms, ants or other soil mixers would indicate that the making of these Bermuda soils was different from that which prevails on the mainland.

A *soil-base*, described in greater detail on a later page, is frequently present beneath the fossil soils of Bermuda. It is usually from ½ to two inches thick and is a compact, subcrystalline, cream-colored limestone.

The marine organic limestones are composed of well-cemented marine shells and fragments of shells.

Collophanite ($CaO \cdot P_2O_5 \cdot CO_2$), a product of guano, is found on Stokes Point (see p. 401). It is a hard, fine-grained grayish rock resembling quartzite.

STRATIGRAPHY.

Stratigraphic Column. In the course of the present investigation the following stratigraphic column has been established. Due to the eolian nature of the deposits, overlaps and unconformities are abundant. In many sections some of the formations may be completely lacking. These relations are brought out in the structure sections (see Figs. 6, 7, 8, 9, 10, 11, 16, 17).

TABLE I. STRATIGRAPHY OF BERMUDA.

Present soil	Usually gray or light-colored pink soil; red in small pockets.
Southampton eolianite	White loosely consolidated eolianite; compact only near surface.
McGall's soil	Mainly a light-colored calcareous soil of accretion; red only locally.
Somerset eolianite	White loosely consolidated eolianite; compact only near surface.
Signal Hill soil	Gray to pink soil, in large part due to accretion.
Warwick eolianite	White loosely consolidated eolianite; compact only near surface.
St. George's soil	Thick deep-red soil in depressions; pink to red on slopes; from ½ foot to two feet thick; a soil of weathering.

Pembroke eolianite	White to cream-colored eolianite; not as compact as Walsingham, but better consolidated than the younger eolianites.
Harrington soil	Light-colored to red soil, four feet thick at maximum; mainly a soil of accretion.
Devonshire limestone	Loosely consolidated or compact organic limestone composed of marine mollusc shells and a few corals.
Shore Hills soil	Deep-red or brown soil from one to four feet thick; mainly a residual soil.
Belmont limestone	Compact limestone composed mainly of marine mollusc shells.
Walsingham eolianite	Usually a cream-colored compact crystalline eolianite; in places loosely consolidated.
(Great unconformity)	
Volcanic platform	Melilite basalt, monchiquite, and keratophyre flows. Also altered volcanic material deposited in the sea.

Walsingham eolianite. A. E. Verrill (*1907*, p. 24) proposed the name Walsingham formation to designate that portion of the older Bermudian strata of limestone and red clay characterized by containing several species of extinct land snails, of which the largest and most abundant is *Poecilozonites nelsoni*. I propose to re-define the term Walsingham so as to restrict it to the eolianite beneath the Belmont marine limestone. As thus re-defined the Walsingham is a compact, crystalline eolian limestone in which the cross-bedding is commonly masked by induration and re-crystallization. Locally it is loosely consolidated. The formation may be seen to best advantage in the Government Quarry at Paynter's Vale, on the west shore of Castle Harbor (132),[1] where the rock is a compact crystalline eolianite. It is also found in all the caves in the type locality, the Walsingham district, which is the isthmus separating Castle Harbor from Harrington Sound. The stratigraphic relations of the Walsingham formation with the overlying horizons may be seen best along the stretch of road extending for half a mile west of Shark Hole (123). Other localities are Tucker's Town Cave (131), Tucker's Island (130), Crystal Cave, Walsingham Cave, Leamington Cave, and other caves in the Walsingham district.

Belmont limestone. (Fig. 6, 7, 8, and 9.) The Belmont limestone is a compact, cream-colored, marine formation, composed dominantly of well-preserved mollusc shells and some foraminifera. Rounded eolianite grains are common. The type locality is a quarry on the Shore Hills Hotel golf course (89). About 15 feet of gently inclined marine strata are exposed. The top of the formation is about twenty-five feet above sea-level, but the base is not exposed. Overlying this marine limestone is the Shore Hills soil. Other localities where the Belmont limestone may be seen are: Grape Bay (130), Wilson's Island (129), Hinson Island (105), Marshall Island (104), Hotel Inverurie (102), Belmont Hotel (37), McGall's Bay (69), Stokes' Point (24), and possibly Ruth's Point (60), Mullet Bay (10), and Surf Bay (21).

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[1] Numbers in parentheses refer to the locality numbers on the map, Plate XIV.

Shore Hills soil. (Fig. 5, 6, 7, 8, 9, 14, 17.) For the fossil soil lying stratigraphically above the Belmont limestone, I propose the name Shore Hills from the type locality at the quarry on the Shore Hills golf course, now the property of the Bermuda Biological Station for Research, Inc. This is the thickest fossil soil in Bermuda. It is deep red in color and averages four feet, in some pockets at least twelve feet, in thickness. This formation is also well exposed at Grape Bay (130), where it rests directly on the Belmont marine limestone. Much of the soil at the latter locality has been destroyed by recent erosion, but the deep solution-pockets in the marine limestone are filled with the fossil soil and demonstrate a long interval of weathering. The deep cylindrical solution-pockets projecting downward into the limestone have been called "palmetto stumps" by the natives, on the assumption that they are the casts of stumps of that tree. Wherever such forms are found they may be used as evidence of the former presence of a fossil soil. Other localities where the Shore Hills soil is found are: Hinson Island (105), Marshall Island (104), Darrell Island (128), Hotel Inverurie (102), Belmont Hotel (37), marine bench to the west of the McGall's Bay locality (69), "palmetto stumps" at Spanish Rock (13), and possibly the fossil-soil horizon just above the cave at Tucker's Island (127). The lower soils at Surf Beach (21) and Ruth's Point (60) may be of this age.

Devonshire limestone. (Fig. 17.) A. E. Verrill (*1907*, p. 32) proposed the name Devonshire formation (from Devonshire parish) for the marine limestones, or, as he called them, "beach-rocks." He recognized only one such horizon, but it is clear that there are two, one above, the other below the Shore Hills soil. For the marine limestone beneath the Shore Hills soil I have already introduced the name Belmont. I propose to restrict the term Devonshire to the marine limestone overlying the Shore Hills soil. The type locality is at Devonshire Bay, where the formation is exposed under the cliffs in front of the old fort site. The lower portion belongs to the Belmont formation and is composed of compact, creamy, organic limestone rich in pelecypod shells. Resting unconformably on this lower member is the Devonshire formation, a conglomerate several feet thick, composed of flat pebbles four or five inches long. Although the Devonshire formation is well indurated at the type locality, it is frequently only loosely consolidated, as at Hungry Bay (133). At Stokes Point, the fossil corals which are present in the limestone, suggest climatic conditions similar to the present. The formation is also exposed at McGall's Bay (69), Spanish Rock (13), Elbow Bay (50), Stokes Point (24), western end of Pink Beach (107), and possibly Surf Bay, Ruth's Point, and Mullett Bay (10).

Harrington soil. (Fig. 8, 10, 14, 16, 17.) For the fossil soil developed directly above the Devonshire marine limestone I propose the name Harrington, from the type locality on the southeast shore of Harringon Sound just west of Shark Hole (123) where it may be followed for several hundreds yards on the north side of the road until it reaches sea-level. About one hundred feet farther to the west the "palmetto stump," indicative of the Shore Hills soil, may be seen. The Harrington soil at this locality is light colored, four feet thick in places, and composed of wind-blown materials, greatly weathered in some places but fairly fresh in others. The Harrington soil thus differs greatly from the Shore Hills soil. The former may be largely a soil of accretion, the latter a soil mainly of weathering. The significance of this fact is discussed on later pages. The extinct land snail *Poecilozonites nelsoni* and many other species of snails are abundant. Other localities are McGall's Bay (66–78), Devonshire Bay (79–81), Hungry Bay (133), Grape Bay (130), Elbow Bay (50), Hinson Island (105), Wilson's Island (129), both sides of Pembroke Marsh, north of Hamilton (93), and probably most of the low-lying fossil soils around Hamilton (56–58). Some of the fossil soils northwest of Hamilton probably belong to the Harrington. It is also possible that the Shore Hills and Harrington soils have locally merged into one, as at Hinson Island. Along the north shore some of the soils appear to be of this age, for example the Shelly Bay racecourse (61), Cottage Hill (19), Bailey's Bay (20), Mullet Bay (10 and 11), Ruth's Point (60), and Sandys Parish (46 and 121).

Although the fossil land snail *Poecilozonites nelsoni* is not always present in the Harrington soil, it has not been found in any higher horizon. Along the south shore this species has been recognized only at Spanish Rock. Some of the fossil soil outcrops inland may be of Pembroke age, but at the present moment definite data are lacking.

Pembroke eolianite. (Fig. 6, 7, 10.) For the eolian limestone overlying the Harrington soil the name Pembroke is proposed because of its abundance in the parish of that name. The type locality is on the east end of Reid Street, Hamilton, near the junction with East Broadway, and about one hundred feet south of the livery stable (57). At the base of the section is a compact eolianite, probably the Walsingham. The deep-red soil over this is assigned to the Harrington-Shore Hills, above which is the Pembroke eolianite. A deep-red soil overlying the Pembroke can be seen in a cliff near the east end of Reid Street (58). This probably represents the interval of time from the Pembroke to the present. On the Middle Road the relations of the soils and eolianites are excellently exposed. Nearly all the islands in Hamilton Harbor, Great Sound, and Little Sound are capped by Pembroke eolianite. Other localities are Wilson's Island (129), St. George's (3 and 4), McGall's Bay (69), Devonshire Bay (79), and Hungry Bay (133). Locally the Pembroke is absent, as at Elbow Beach (50) and Grape Bay (130).

St. George's soil. (Fig. 3, 6, 7, 10, 11, 12, 13.) The term St. George's is suggested for the fossil soil overlying the Pembroke eolianite. The type locality is on Ferry Road, St. George's (1), where it has a deep-red or chocolate-brown color and at a maximum is two feet thick. The St. George's soil is also well exposed at McGall's Bay (68), Simmons Beach (108), Elbow Beach (50), Hamilton (25), in the new cut on Eliot Street in Hamilton, and at the four corners on the Paget-Warwick line (113).

Warwick eolianite. (Fig. 3, 7, 10, 13.) For the eolianite overlying the St. George's soil, I propose the name Warwick. The type locality is in the parish of Warwick. It may also be seen at McGall's Bay (68), Simmons Beach (109), Elbow Bay and other places. On the road to Gibbs' Hill light it is probably the eolianite between the two soils on the west side of the road. The Warwick eolianite is a white to cream-colored, loosely consolidated eolianite. It is indurated for only a few feet below the surface.

Signal Hill soil. (Fig. 7, 10.) This name is proposed for the soil overlying the Warwick eolianite, from the type locality at Signal

Hill, St. George's (5), where it is pink and about two feet thick. Throughout the islands as a whole it is a soil of accretion, although in local pockets the products of residual decay have accumulated. The Signal Hill soil may be well seen at McGall's Bay (69), Simmons Beach (109), Elbow Beach (50), and at Khyber Pass (33).

Somerset eolianite. (Fig. 7, 10, 12.) For the eolianite overlying the Signal Hill soil, I suggest the name Somerset from the village of that name in Sandys Parish. As a rule it is white to cream-colored, thin, and loosely consolidated. Only near the surface is it well cemented. Dr. Thomas Clark first recognized it as a distinct formation in 1926 at McGall's Bay, where it lies in the Signal Hill soil (69). It has also been found at St. George's (6) and at Simmons Beach, and other places.

McGall's soil. (Fig. 4, 7.) This name is proposed for the soil overlying the Somerset eolianite. The type locality is at McGall's Bay, but it has also been found at St. George's (6), Elbow Bay (50), and Simmons Beach (56). It is very similar in appearance to the Signal Hill soil. It is usually very light-colored, although in local pockets it may be rather pink. This soil is largely a product of accretion rather than of decay.

Southampton eolianite. (Fig. 4, 7, 10.) The Southampton eolianite, named from the parish of that name, does not differ essentially from the Somerset eolianite. Both are white, loosely consolidated eolianites, being cemented into a compact rock only near the surface. The Southampton is well exposed at McGall's Bay (69), St. George's (6), and Simmons Beach (108). It undoubtedly occurs in numerous other localities.

Recent soil. (Fig. 5, 6, 7, 10.) The recent soil, developed since the deposition of the Southampton eolianite, is comparatively thin. It is gray to brown in color. On level ground it is not more than six inches thick, which conforms with the thickness of post-Wisconsin soils in the limestone regions of the United States under similar topographic conditions. On hill-sides the soil may be completely lacking. The recent soil may be seen on the golf course at St. George's (6), at Shore Hills (89), where it is developed on the Pembroke or later eolianite, at Spanish Rock, and many other places.

In speaking of the recent soil it must be noted that the deep soils in many places are the result of weathering for a much longer time than the post-Wisconsin or Recent episode. In many places dune formation has not gone on for a very long time. For example, in much of the Walsingham district, where the Bermuda caves are mostly located, dune formation has not gone on since the Harrington or Pembroke stages. In many depressions the surface soils are very thick, and it is in such places that the productive truck and flower gardens are found. The thick soil in low land or depressions is the result of long weathering and also of the wash of rains, which carry weathered materials to lower levels about as fast as they are formed on the slopes. Earthworms do not seem to have played any part in the formation of the fossil soils of Bermuda. At least there is no evidence in the soils that they existed. The common earthworm, *Lumbricus terrestris*, evidently did not find its way here. Whether or not there were ants, we do not know.

Sand Dunes. Modern sand dunes have been discussed in the section dealing with the topography of Bermuda.

TYPE LOCALITIES AND SPECIAL AREAS.[1]

The stratigraphic relations of the eolianites and fossil soils of Bermuda are best displayed at St. George's, McGall's Bay, and on the Warwick-Paget line. These localities will therefore be described in considerable detail, in order to demonstrate the stratigraphic column already given. A brief description will also be given of the sedimentary rocks in the Hamilton region, the Walsingham cave district, and some of the islands.

St. George's. St. George's parish may well be considered the type locality of the fossil soils of Bermuda. Four of the five soils are exposed in such relations as to preclude any possible doubt that each soil represents a distinct and separate horizon. About 300 feet west of a residence on Ferry Road called "Glen Duror" the Harrington soil may be seen on the north side of the road near the level of the gutter. It is about two feet thick and of a light color. No fossils were found. It is covered by Pembroke eolianite which evidently came from an easterly direction, as indicated by the dip of the cross-bedding. On the same road several hundred feet east of Glen Duror the St. George's soil is exposed in the cliff on the north side of the road

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[1] It is desirable to place on record a detailed description of the many localities upon which are based the conclusions reached in this paper, particularly for the benefit of geologists who may visit Bermuda in the future. Many of my readers, not being interested in the details of Bermuda geology, may prefer to omit these long descriptions and pass at once to the section entitled "Fossil Soils."

(Fig. 3). Here it has a deep-red or chocolate color, characteristic of residual soils. It is two feet thick at maximum. This soil has no complete fossils here, but toward the east it rises at a small angle, becomes lighter in color, and encloses many snails. Erosion has cut it off at a point about one hundred yards east of the deep-red part of the exposure. West of the locality first mentioned, where the soil is so red, it may be followed in the cliff for several hundred feet. This soil is clearly younger than the Harrington. The Warwick eolianite, which overlies the St. George's soil, appears to have come from the north.

Leaving Ferry Street near the eastern limit of the exposure of the St. George's soil and going up Signal Hill by the pass, the Signal Hill soil may be seen near the top of the pass. It dips north and is plainly a higher soil than the St. George's, for the Warwick eolianite intervenes. The soil has a pale color, is about two feet thick, and slopes to the north. Fossils are plentiful in both exposures on either side of the pass. Vegetation must have been thick enough to protect it from rain-wash, for no soil could form at such an angle unless it was so protected. The overlying eolianite, which seems to have come from the north, is the Somerset.

A still higher soil, the McGall's, may be seen at a number of places around Signal Hill, but the best locality is on the golf course (6) about two hundred yards northeast of the locality last described (Fig. 4). It is a moderately consolidated, dark layer about a foot thick, overlain by several feet of Southampton eolianite, which in turn is overlain by the recent soil. The McGall's soil at this locality is full of fossils and carries fragments of carbonized wood. On exposure to the air for a few months this soil becomes so hard that a hammer is required to break it. Because of deforestation at St. George's in the early days, when St. George's was the principal settlement at Bermuda, the recent soil has been completely eroded in most places. The resistant soil base, however, has protected the underlying eolianite. From this instance we may conclude that when the fossil soils were forming, vegetation must have flourished, for otherwise no soils could have formed on the hill slopes.

The important Stokes' Point (24) and Shore Hills (89) localities, are also included in the St. George's area. At both of these places the Belmont, Shore Hills soil, Devonshire marine formation, and Pembroke eolianite are found. The Harrington is not a distinct formation at Shore Hills, and at Stokes' Point it is uncertain.

No geologist who has visited Stokes Point has come away satisfied

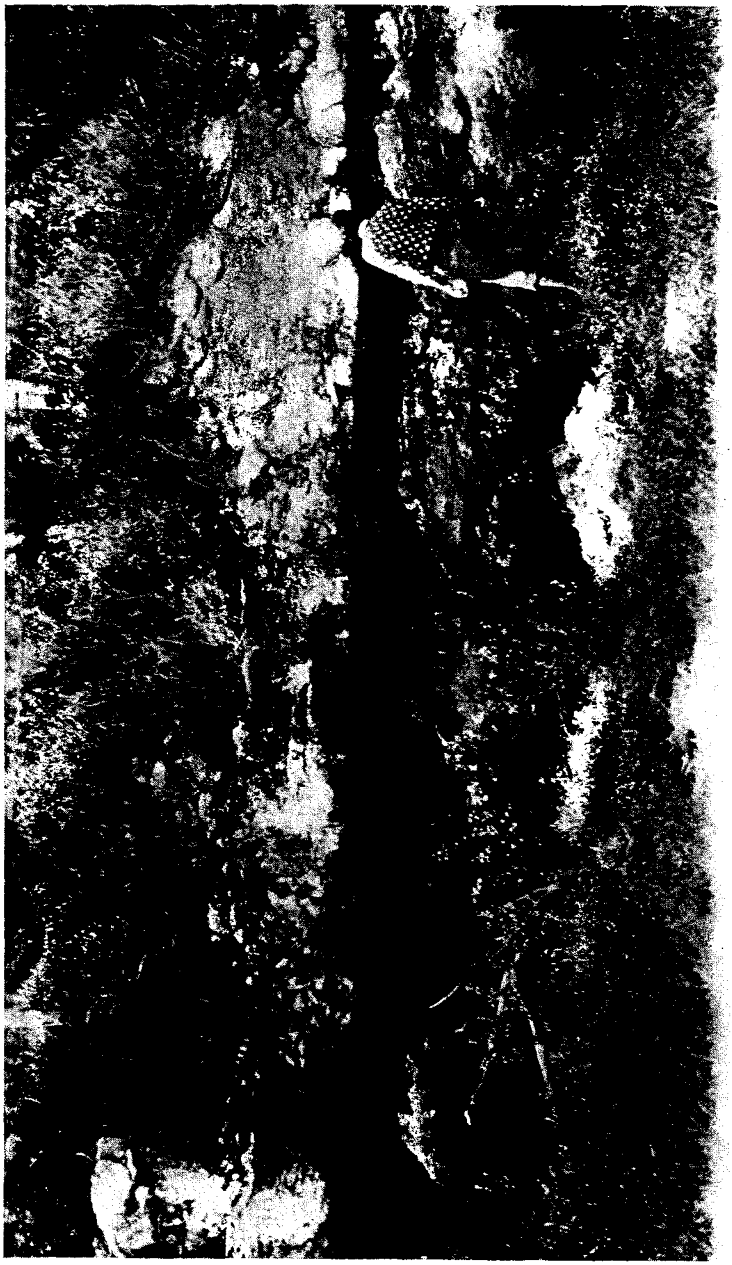

FIGURE 3. Deep red phase of the St. George's soil at St. Georges. Thickness 18 inches to 2 feet at deepest parts. Photo. by author. Locality 1.

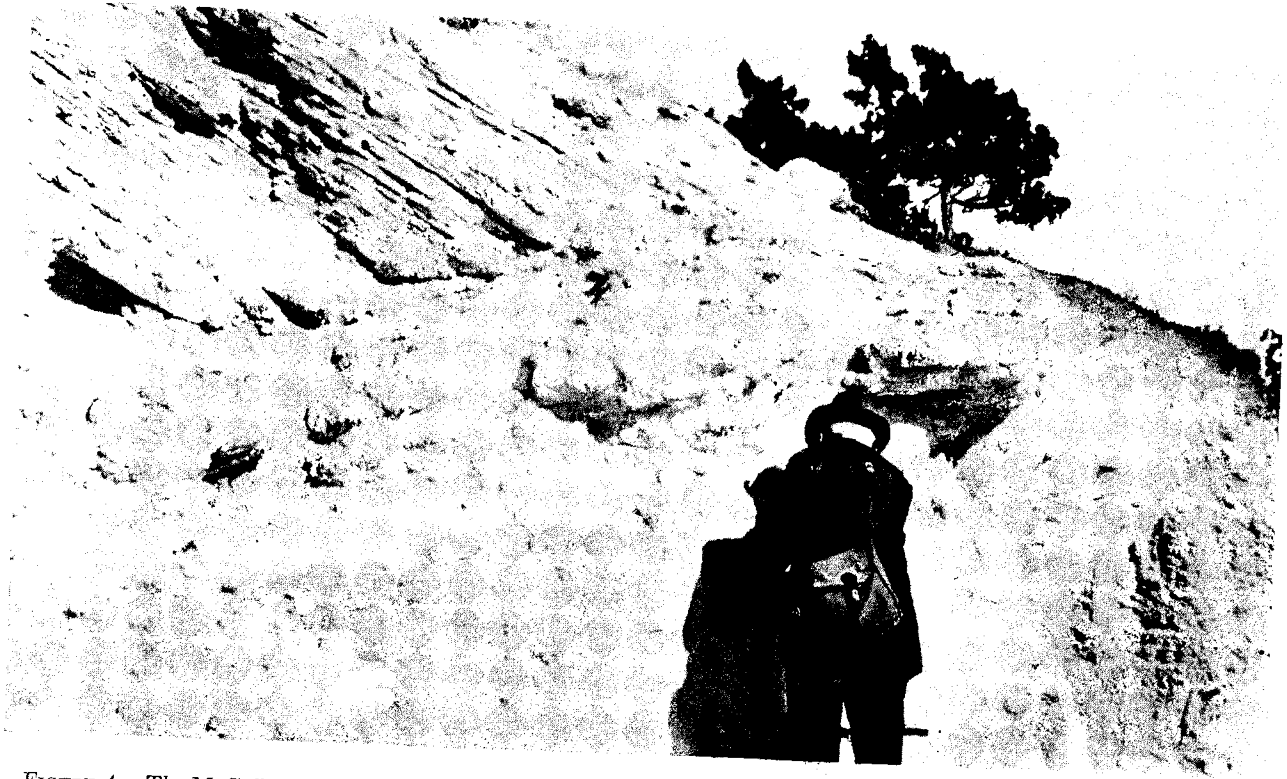

FIGURE 4. The McGall's soil at St. George's. Note the transition beds above the head of Professor Schuchert, and the dune structure of the Southampton eolianite lying on the transition beds. The soil is very compact and when exposed to the air for a short time can be broken only with a hammer. Many individuals of *Poecilozonites bermudensis* var. *zonatus* were found and a number of fragments of carbonized wood. Photo. by author. Locality 6.

as to just what has happened there. It is still an unsolved problem. The underlying rock here is a reddish, very compact crystalline limestone An examination of a thin section of this rock shows it to be composed of well-rounded calcareous grains of foraminifera. Both the eolian grains and the foraminifera are much decomposed. The decomposition is so great that I have concluded that this underlying rock, which I correlate with the Belmont, has been exposed to the air for a long period of time and might almost be called a soil. If so, it is part of the Shore Hills soil and the overlying rubble soil must be Harrington. This soil lying on the red rock is about 18 inches thick near the water-level and decreases in thickness upwards in the cliff. Solution holes penetrate the red formation, but they are not deep as at other places where the Shore Hills lies on the Belmont. The soil itself has evidently been worked by the waves in its lower part. Fragments of the red rock, reaching a diameter of a foot, are found in it, and some of these also show solution cavities, as noted by Swinnerton. Marine shells are common in the lower part of the soil, but in the upper part there are no marine shells. Fossil land-snails are abundant in this upper part. Above the soil comes the Devonshire marine formation, with a plentiful assortment of marine forms including corals and pelecypods. The thickness of the Devonshire here is about five feet. Above the Devonshire comes the Pembroke eolianite.

There is a cliff at Stokes Point, where these geological relations can be seen. This cliff, about fifteen feet high, shows distinct evidence of a higher sea-level. It is hollowed out so as to produce what might almost be called a sea-cave. On the inside wall there is a mass of marine rubble, rounded pebbles, and marine fossils in a consolidated mass. That the sea has been at work here is evident. The old water-level is well shown about twelve feet above the present water-level. Among the ancient pebbles found at Stokes Point are some which are distinctly oölitic. The nature of this oölite has not been determined. There are also pebbles of a rock that looks very much like quartzite. Larsen analyzed this rock and found it to be collophanite, which is a phosphate rock containing in addition calcium, carbon dioxide and water.

If the red base-rock here is of Shore Hills age, and the soil lying on it Harrington, the Devonshire is younger than the Harrington. This does not fit the facts along the south shore and elsewhere. At Swing Bridge (12) there is a soil which has every appearance of being Harrington in age, and this locality is only a short distance from Stokes' Point. Just what the relations at Stokes' Point are is not clear at present, but I believe the red rock is of Shore Hills age and the rubble soil with the marine and land forms to be a part of the Shore Hills soil, and that the rising water-level of Devonshire time was interrupted long enough to permit the development of the upper part of the soil containing the land forms. The level then rose again and covered the soil and the Devonshire marine limestone was laid down. Locally the Harrington appears to be missing. It may have been eroded by the winds which marked the advent of the Pembroke eolianite, for this took place at McGall's Bay, also locally. Stokes Point requires more study, and when correlations over the rest of Bermuda are more certain it will be possible completely to solve this difficult problem.

At Shore Hills, the type locality of the Shore Hills soil, there is much of fundamental interest. In the first place, the rock at the quarry on the golf course is a true marine formation of crystalline limestone containing many marine forms. These included fossils have been examined by Dr. Carlotta J. Maury, who finds none to be pre-Pleistocene. Verrill did not mention this formation, and, as far as I can discover, it has not been noted before. Shore Hills was off the main line of my first visits to Bermuda, and I am indebted to Professor Mark for suggesting that I visit this locality. I discovered at this time, in company with Professor Mather, that the formation contained many marine fossils. This is the highest known outcrop of the Belmont marine formation, and the highest marine formation at Bermuda. Fifteen feet of this Belmont is in sight (Fig. 5). On the Belmont lies the Shore Hills soil with an average thickness of four feet and pockets twelve feet or more deep. Above the soil lies an eolianite, four feet thick, which has an older appearance than the Warwick and later eolianites. It is thought, provisionally, to be Pembroke in age. On the Pembroke lies a more recent soil, the present soil. It is about a foot thick. Like some other parts of Bermuda this locality escaped much of the later dune action. The great thickness of the Shore Hills soil indicates, however, that there must have been a considerable thickness of rock at one time between the Shore Hills and the Belmont. The nature of this rock is not known. On account of this uncertainty the exact relations here await further research. The fossils of the eolianite above the Shore Hills soil indicate little. There is, therefore, a possibility that this may be one of the younger formations, and that the soil lying on it is entirely post-glacial in age.

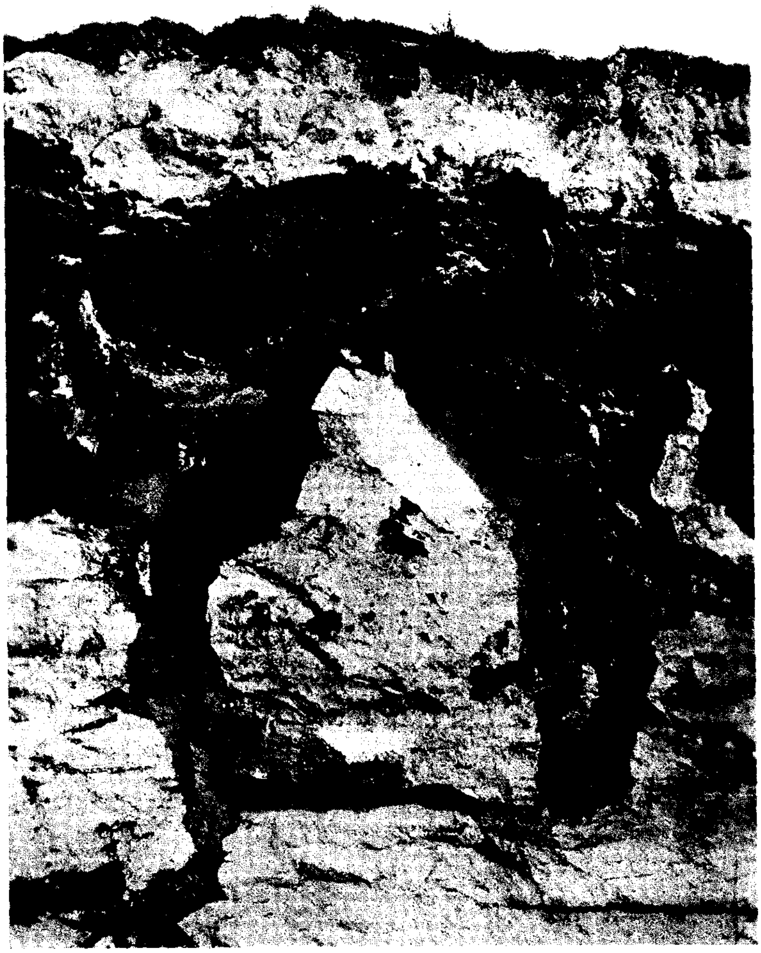

FIGURE 5. The highly weathered soil at Shore Hills, St. George's, known in this paper as the Shore Hills soil. Average thickness above pockets 4 feet. Depth of pockets about 12 feet. Lies on Belmont marine limestone 25 feet above tide. Eolianite on soil, of Pembroke or later age. The thickness of the soil on the eolianite, about 1 foot, would suggest a length of time longer than post-Wisconsin, and the appearance of the eolianite itself suggests that it is one of the older eolianites. Fossils do not help in this case. Photo by A. C. Swinnerton. Locality 89.

It is most probable that in other parts of Bermuda this high marine formation lies buried under later eolianites.

The St. George's region was studied first and it is thought best to describe it first, inasmuch as it is the type region of the fossil soils, though logically the Stokes' Point and Shore Hills might well have been treated first. The main point to establish is that here there are undoubtedly five distinct fossil soils in addition to that now forming.

Surf Bay (21-22). At the west end of the beach at Surf Bay there is a bench of rock, the upper surface of which is about ten feet above sea-level. The upper part of the bench is composed of an indurated fossil soil which extends some distance up into the cliff and contains many shells of the larger species of *Poecilozonites bermudensis*. Near the top of the cliff there is another fossil soil about six inches thick with many fossils, mostly *P. bermudensis*, and in the sands just above this soil are more fossils of the same species. This upper soil is of a light color and fine grain. The lower soil is probably of Harrington age. The upper soil may be of St. George's age. This locality has not been visited since 1924 and no further details are available.

Pink Beach (107). At the western end of Pink Beach (Fig. 6) a thick fossil soil is seen just above the sands of the beach and in the cliff. The formations noted here occur as follows. The lowest formation is a marine limestone with many marine fossils. It is very compact, although much pitted by marine corrosion (see plate 10). The exact contact with the overlying soil is not visible. The soil is about two feet thick, about five feet above sea-level, red in color and carries many fossils, poorly preserved. The fossils are mostly land snails, but the species were not determined. There are also fragments of crabs in the mass. Above this soil comes eolianite, about five feet thick, and lying on this another red soil, about eighteen inches thick, with fossils, and divided formerly by a soil base. This soil base has been destroyed by weathering and only fragments are now to be seen, as proof of its former existence. Above the upper soil is a thin soil composed of fragments of rock, forming a kind of rubble, the recent soil.

It is supposed that the marine formation underlying this series is of Belmont age, that the lower soil is of Harrington or St. George's age, and that the upper double soil consists of St. George's, Signal Hill or McGall's soils. The uncertainty of the age of the lower soil makes any definite determination impossible at present. The age of the eolianite between the two soils cannot be definitely determined. By the edge of the water there is a small stack of eolianite. It is firmly, but unconformably, connected with the underlying Belmont marine limestone. The base is as compact as the Belmont, while higher up it is much less resistant, showing the effect of marine conditions on consolidation. In general the shore-rocks close to the sea are very much indurated, while higher up they are much softer. There is no soil between this stack and the Belmont formation, and it has been concluded that the eolianite of this stack is younger than the formations noted above. I believe that it is the remains of a late dune which was formed and then submerged by the more recent rise of sea-level, and all but destroyed during this rise and fall of the sea. The marine pitting in the Belmont formation is noteworthy when compared with the much deeper pitting in the shore-rocks back of Walsingham House. It is evident to me that the uncovering or removal of dunes at Pink Beach, of which this stack is the last survival, is a fairly recent event, and that at Walsingham House there were no dunes to remove.

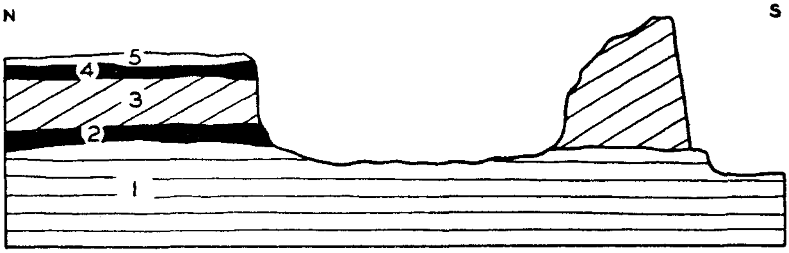

FIGURE 6. Pink Beach in section, looking east.

1. Belmont marine formation.
2. Shore Hills-Harrington or St. George's soil.
3. Pembroke eolianite.
4. St. George's or later soil.
5. Recent rubble soil.

On the right a stack, the remnant of a late dune, the erosion due to a fairly recent higher stand of the sea. Locality 107.

Albouy's Point (15-17). Here there is a problem which remains to be settled. In a quarry on the north of the road a deep-red soil appears about midway in the face of the quarry. The soil has many fossils, mostly *Poecilozonites bermudensis*. It is about a foot thick. The rock is a very much indurated eolianite both below and above the soil. Little time was available for the collection of fossils, so, it is not known definitely whether or not the species *P. nelsoni* is here. This soil has every appearance of being the Harrington, or even Shore Hills soil, with Walsingham eolianite below the soil and Pembroke eolianite above. More study must be given to this important locality before anything very definite can be said.

Spanish Rock. One of the best places to see the relations between the Belmont, Shore Hills, and Devonshire formations is at the Spanish Rock locality (13). At the west end of Spittal pond, and among the rocks near the ocean may be seen the familiar "palmetto stumps" which so often locate the former position of the Shore Hills soil. The soil itself has been eroded away. In the cliff to the east, not over 30 feet away, is the Devonshire formation, which at this point is full of fossils. Above the Devonshire comes an eolianite. At this point the Harrington soil appears to be missing. However, on the ocean side, in the cliff at a point 100 yards or perhaps more to the east, there is a soil on a level with the floor of the sea-cave found there. This cave cannot easily be reached from the "palmetto" locality, on account of the steep cliff on the ocean front at this point. By going eastward, however, several hundreds of yards, a way can be found to the ocean and then it is comparatively easy to make one's way back toward the west, where the cave is found at about 15 feet above sea-level. At this point a single *Poecilozonites nelsoni* was found. It is concluded that this soil here is the Harrington. The erosion of the Harrington soil locally has been noted at McGall's Bay and at Stokes Point and it appears to have occurred also at this Spanish Rock locality. The importance of this locality (13) is evident. The close proximity of the Belmont and Devonshire marine formations in many places makes it difficult to distinguish between them. The Shore Hills soil is sometimes found between them but not always, and this is due to the erosion of the Shore Hills by the rising waters of the Devonshire. In places, as at Grape Bay, the Devonshire does not appear, but the Shore Hills and Belmont are excellently shown. In cases like this dunes must have covered the Shore Hills while the Harrington was forming in other places, and not until wave action had swept away the covering of the dune, many years later, was the Shore Hills again exposed to the air.

McGall's Bay (plate 7). The McGall's Bay locality (66–78) is the only place other than St. George's where four distinct fossil soils may be seen together. At the base of the exposed section is the Belmont marine limestone. On top of this a few isolated patches of Shore Hills

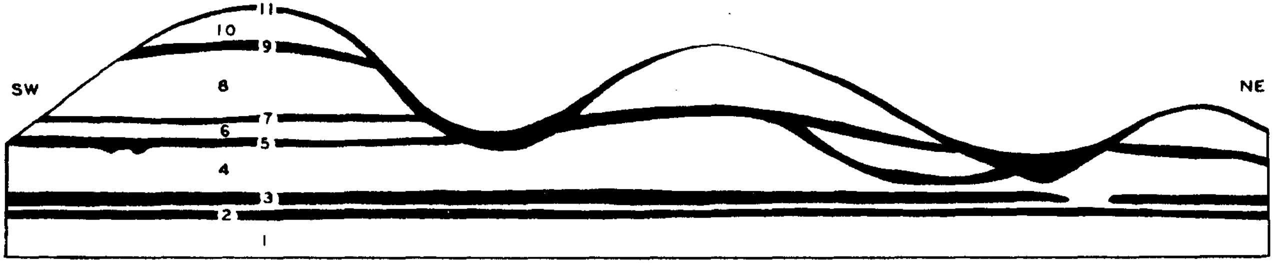

FIGURE 7. A rough field sketch of McGall's Bay section.

1. Belmont marine limestone.
2. Shore Hills soil (a patch only).
3. Harrington soil.
4. Pembroke eolianite.
5. St. George's soil.
6. Warwick eolianite.
7. Signal Hill soil.
8. Somerset eolianite.
9. McGall's soil.
10. Southampton eolianite.
11. Recent soil.

Localities 66–75 (Clark and Sayles).

soil suggest its former presence in greater quantity. About 100 yards west of the most westerly part of this exposure in the marine bench occur the "palmetto stumps," or solution holes, with remnants of an indurated soil as further evidence of the former presence of the Shore Hills soil. Above this is a slight thickness of sandstone and then the Harrington soil, which is relatively thick, clay-like in its constituency, and light in color. Next in the section is the Pembroke eolianite, which along this shore ranges in thickness from three to twenty feet. The overlying St. George's soil is red and varies from eight inches to a foot in thickness. Above this comes several feet of Warwick eolianite, and still higher the Signal Hill soil, which is red but only six to eight inches thick. The Somerset eolianite overlying the Signal Hill soil ranges from ten to twenty feet thick. Next is the McGall's soil, which is light in color, little weathered, full of fossils, and clearly a soil of accretion. It is only eighteen inches thick. At the top of the section is the Southampton eolianite, scarcely a foot thick, which is overlain by a few inches of recent soil.

Devonshire Bay (80–81). Starting at the water's edge under the cliff at the point, it is found that the lowest formation is a compact marine limestone, made up as usual of eolian sand grains and containing many fossils. Above the limestone, and lying unconformably on it, is a conglomerate made up of pebbles, presumably of the hard underlying limestone, and with a matrix of sand (See plate 8). Above the conglomerate is a formation of sandstone several feet thick. Lying on the sandstone and merging with it is a fossil soil of light color and fine texture and unconsolidated. There are numerous fossil snails in the soil, mostly of the species *P. bermudensis*. Above the soil is a thick eolianite formation dipping at an angle of 30° north. On both sides of the path which traverses the remains of the old fort there is a soil between two and three feet thick, of a deep red color, dipping about 20° southwards towards the ocean, and disappearing from view. This soil has very poorly preserved fossils, and cuts the underlying almost horizontally stratified beds. Remains of a soil base may be seen in scattered pieces. Above the soil are eolianite beds, dipping at 30° N., the transition beds showing very well. The soil merges above with the more recent soils.

The ages of the beds in this section are thought to be as follows. The lowest marine formation is considered of Belmont age. The conglomerate and overlying sands are without much doubt the Devonshire formation. The lower fossil soil is probably the Harrington,

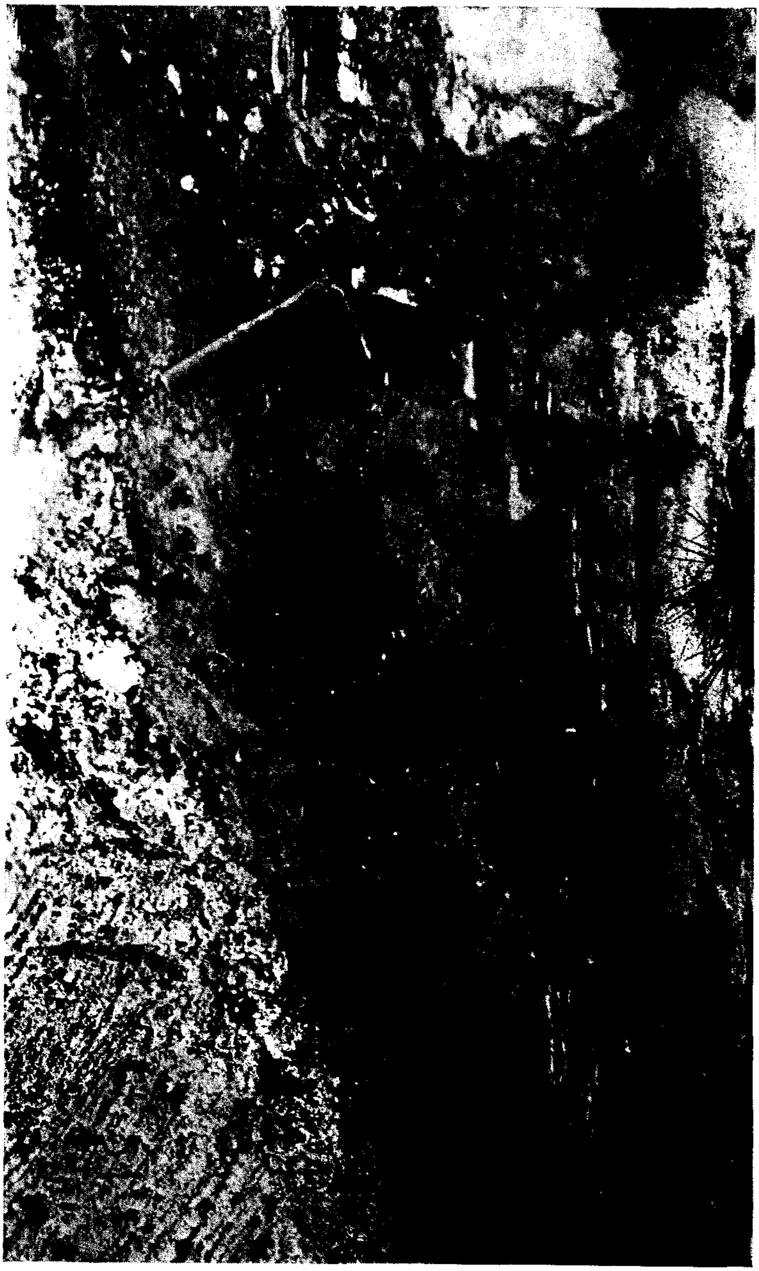

FIGURE 8. A view of what is probably the Shore Hills-Harrington soil at the old fort site at Devonshire Bay. Note remnant of soil base under hammer, reduced by weathering of underlying horizontal formation. Also note thin transition beds and northward dipping eolianite. Fossils are rare and poorly preserved. Photo. by author. Locality 79.

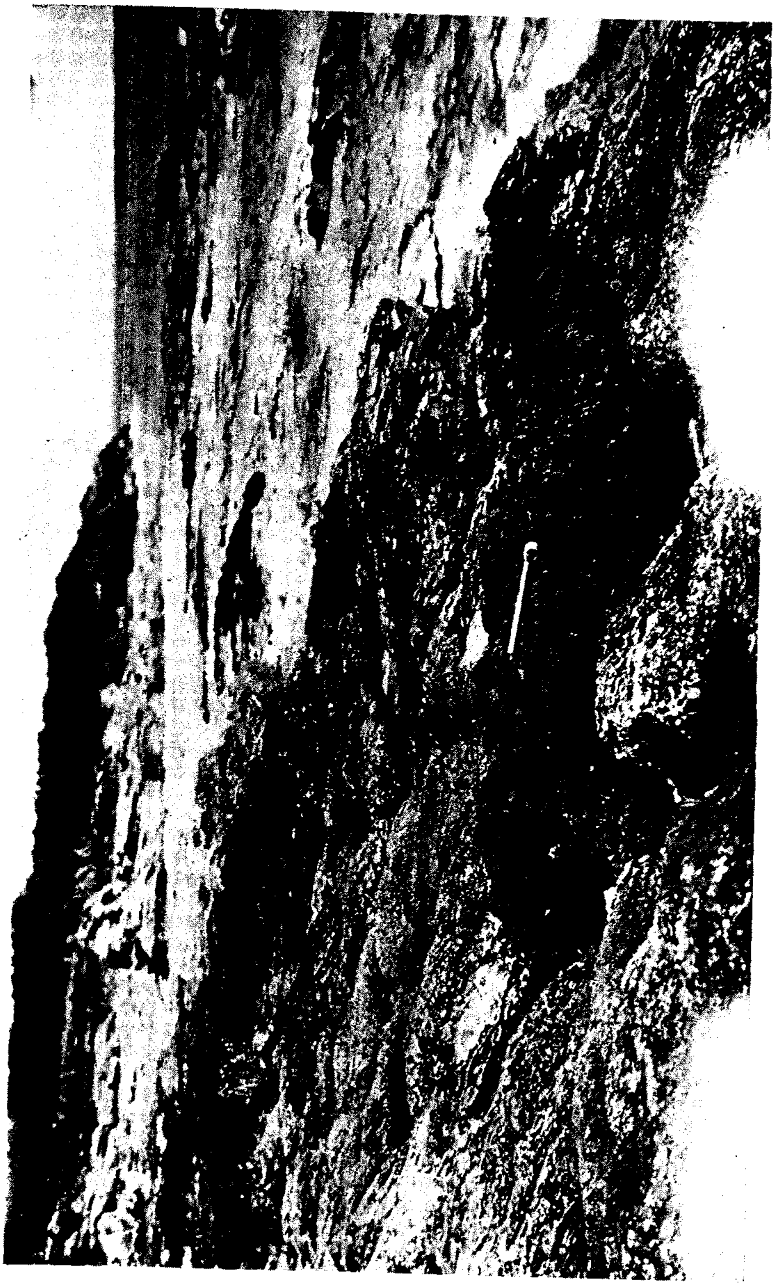

FIGURE 9. The Shore Hills soil lying in patches on the Belmont marine formation at Grape Bay. Note the uneven surface of the Belmont where it has been stripped of the soil, and the solution holes in the Belmont. Hammer lies on soil. Photo. by author. Locality 130.

preserved by dunes soon after its formation, and hence kept in a fresh condition. The eolianite on the Harrington is of Pembroke age. The soil described in the old fort is not so easy to explain. On account of its extremely weathered condition and great thickness and its position on nearly horizontal beds, similar to the conditions at Wilson's Island, it might very well be of Shore Hills age. On the shore to the west of the lowest formations described above, are some holes in the Belmont which would appear to be of the "palmetto" type, and these in turn would indicate the former presence of the Shore Hills soil. It is most probable that the Shore Hills soil was here washed away before the deposition of the Devonshire formation, and that higher up the Harrington appears as probably a double soil with Shore Hills, and that the soil in the old fort is truly of Shore Hills age. Future study of this most important locality must be made before we can arrive at any definite conclusion. The location of the soil in the fort on beds very gently dipping towards the ocean, is so like the conditions at Wilson's Island, that it is logical to place this old soil provisionally in the Shore Hills-Harrington group of soils.

Hungry Bay (see plate 9). Excavations by the Road Department of the Bermuda Government have been going on here for the last few years to such an extent that to describe Hungry Bay as we saw it, would not help a visitor of today to any great extent. However this may be, an attempt will be made to describe this Hungry Bay locality as Clark and I first saw it in 1926.

The hard marine limestone is, as so often the case, the lowest member. Above this marine formation comes a soil about 18 inches thick, of a gray color and fine texture. The bottom of the soil has many marine forms and about midway these marine forms disappear and land forms take their place. This soil rises to the east at an angle of about 10° and as it rises the color changes to red and it becomes somewhat thinner. The soil rests on the marine formation at the lowest part of the exposure, a few feet higher up it rests on thin bedded sand rock not so completely consolidated, as it rises in the cliff, and cuts this sand rock. Eolianite dipping northeast rests on the fossil soil.

Of the formations described here, the marine formation at the base is without doubt the Belmont marine. Above this comes what must be the Devonshire marine formation, and the marine shells in the lower part of the soil are undoubtedly of this age. The Devonshire rises in the cliff under the soil. The fossil soil is the Harrington without much doubt. The eolianite on the soil is of Pembroke age.

The Shore Hills soil was not visible at the time we visited this locality, having been washed away when the waters of the Devonshire rose. This was also the case at Devonshire Bay.

Verrill described a section west of Hungry Bay, more accurately known as Grape Bay. I have not exactly located the section, but have seen one very similar at Grape Bay. The Spanish Rock, Devonshire Bay, Hungry Bay, and Grape Bay localities when studied comparatively, give a clear idea of the order of the formations along this south shore, and other places. Specially important is the fact that the Devonshire formation is entirely distinct from the Shore Hills, although belonging to the same long period of submergence, correlated in my table of formations with the Pleistocene, Yarmouth interglacial stage.

Grape Bay (Fig. 9). Here there is the usual resistant marine limestone at the base of the section. Lying on it is the Shore Hills soil in patches, the solution holes being filled with the soil. Above the soil comes a few inches of sand rock, which is undoubtedly of Devonshire age. Resting on this Devonshire is a red soil about a foot thick with fossil land gastropods, mostly *Poecilozonites bermudensis*. Resting on the soil are a few feet of eolianite of Pembroke age.

The importance of this Grape Bay locality cannot be over-estimated. Without it the problem of the Shore Hills and Harrington soils would not be clear. This is the only place I have found on the south shore where the Shore Hills is preserved to any extent. At Spanish Rock and at other places it may be found in "palmetto stumps," but here the soil itself, several inches thick, is seen lying on the surface of the Belmont and also in the solution holes. When the Devonshire submergence took place this Shore Hills soil escaped destruction on higher ground and merged with the Harrington, so that for the localities on the higher ground there was no interruption of the Shore Hills-Harrington episode, which was brought to a close with the advent of the Pembroke eolianite episode or stage. At Wilson's island the thick red soil called the Shore Hills, may be of Harrington age. The conditions here appear to be very similar to those at Hungry Bay. If such are the relations, the underlying thin-bedded rock at Wilson's Island must be of like age with the similar rock at Hungry Bay, which would make it Devonshire in age.

Elbow Beach (50a and 50b). The Elbow Beach locality displays at least four soils. The formations in order are as follows: a hard marine limestone at the bottom, undoubtedly of Belmont age, followed, in ascending order, by a gray-brown soil 2 feet thick with a few fossil shell fragments; a layer of sand about 3 feet thick with *Poecilozonites bermudensis* var. *zonatus;* and a heavy red soil bearing fragments and fossil land snails, mostly *P. bermudensis* var. *zonatus.* This soil outcrops on the beach for over 200 feet and is very conspicuous. Northwards for about 200 feet and only slightly higher than the top of this fossil soil, is a wave-cut bench. At the inner margin of the bench, about 100 feet west of the path to the bath-houses, I saw the red loam of a soil recently excavated in the bench, and this was undoubtedly the same soil as that found on the beach. About 50 feet above this bench at the eastern end of the Elbow Beach Hotel, there was a very hard fossil soil about one foot thick, with a few fossils, and above this soil eolianite dipping east or northeast. Still higher up, at the top of the hill, there is a thin fossil soil, with many individuals of *P. bermudensis* var. *zonatus.*

In this section the lowest formation is Belmont marine limestone. Following this is the bleached Shore Hills-Harrington soil. Sands above the latter are of Harrington-Pembroke age. The thick, red soil is without doubt the St. George's soil. Lying on the St. George's soil is the Warwick eolianite. The soil at the east end of the hotel is the Signal Hill soil. Above the Signal Hill soil is the Somerset eolianite. At the top of the hill is the McGall's soil. The Elbow Beach section is important for comparison with other sections along the South Shore, and also with the sections about Hamilton. It is evident that wave action has cut back the section in more recent times, as is indicated by the bench, which is about eight to ten feet above sea-level.

Warwick-Paget line section (Fig. 10). This most important section in Bermuda gives an excellent exposure along the road that follows the boundary between the parishes of Paget and Warwick. On the south side of Hamilton Harbor, under the ferry waiting-room just west of the Inverurie Hotel, the Shore Hills soil, resting on the compact Belmont marine limestone, is well exposed. The soil is about eighteen inches thick and deep red. The overlying eolianite, the cross-bedding of which dips southward, is considered to be the Pembroke, the Harrington soil and the Devonshire limestone both being absent. On the road south from the Inverurie Hotel a red soil is exposed at the first house on the west side of the street, less than one hundred feet from the hotel. This is probably the Harrington soil, and the overlying eolianite Pembroke. About two hundred yards further south on the east side of the road this eolianite is in turn overlain by a fairly thick

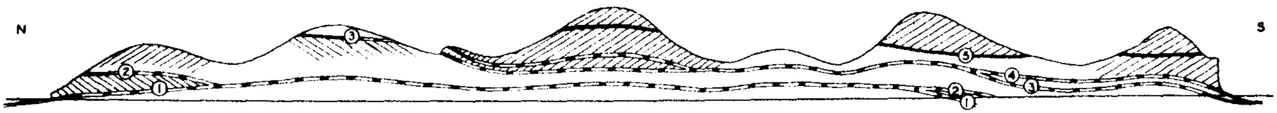

FIGURE 10. Partly hypothetical section across island on Paget-Warwick line.

1. Shore Hills soil.
2. Harrington soil.
3. St. George's soil.
4. Signal Hill soil.
5. McGall's soil.

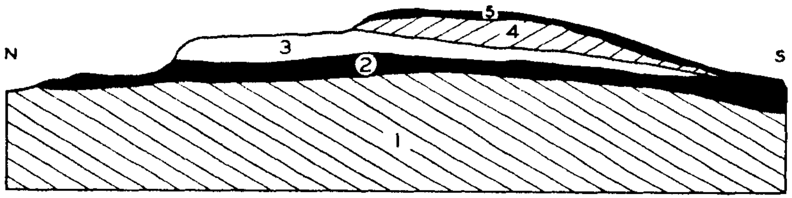

FIGURE 11. Section on Paget-Warwick road, about 200 yards south of Inverurie Hotel.

1. Pembroke eolianite.
2. St. George's soil.
3. Sands on soil.
4. Warwick eolianite.
5. Present soil.

Locality 125.

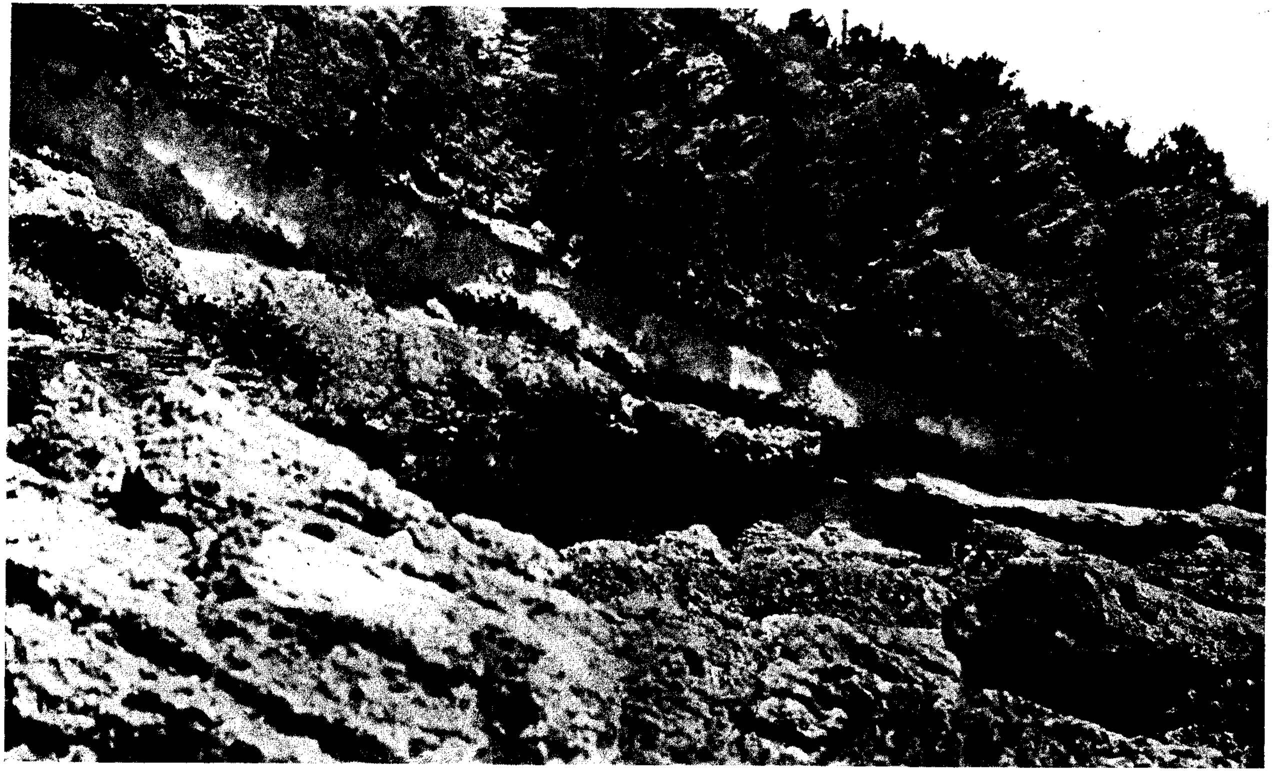

FIGURE 12. View of union of St. George's and Signal Hill soils at Simmons Beach. Note thickness of both soils, the transition beds above the Signal Hill soil, and the northward dipping Somerset eolianite. The lower parts of both soils, are weathered more deeply than the upper parts and probably represent a much longer time interval than the much thicker upper parts. Both soils have many fossils, mainly of the species *Poecilozonites bermudensis* var. *zonatus*. Photo. by author.

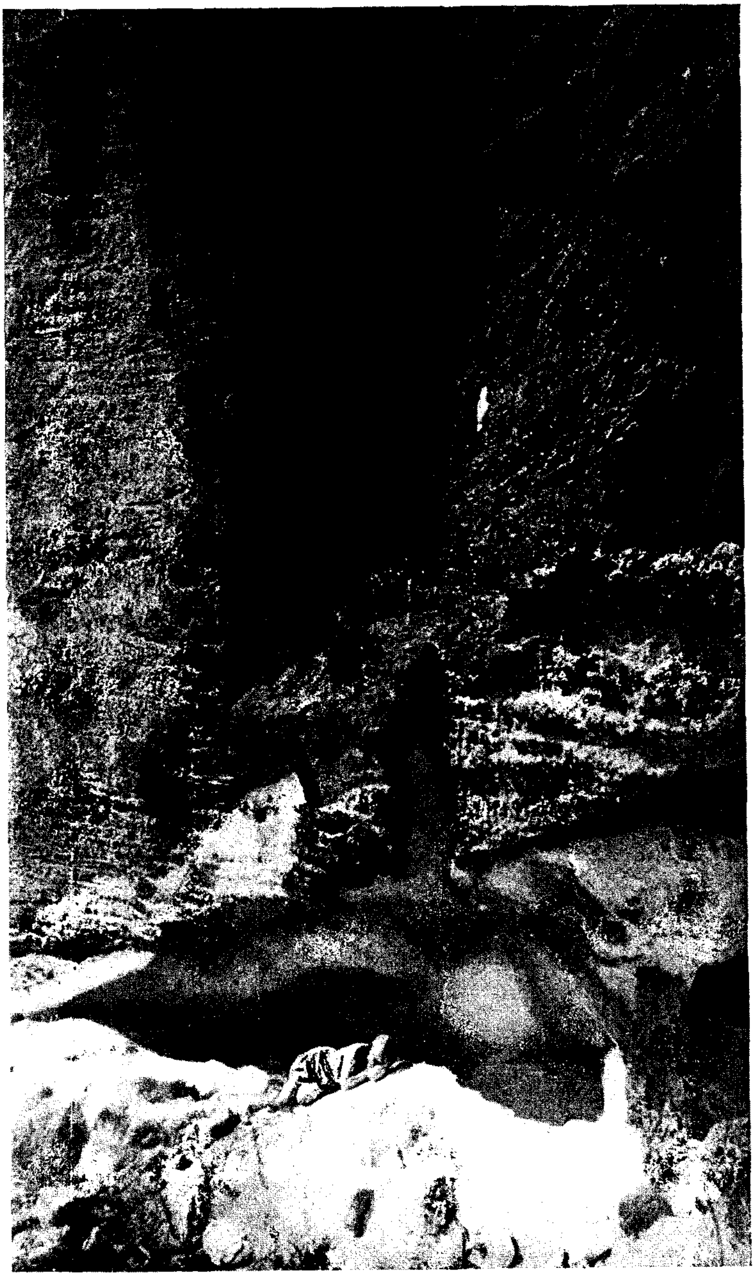

FIGURE 13. View of St. George's soil overlain by Warwick eolianite at Simmons Beach south shore. The thickness of the soil here is about 5 feet, and although it is largely a soil of accretion rather than weathering, weathering has progressed to a considerable extent, especially in the lower part. The beds of eolianite below are cut off. This soil, like all those at Simmons Beach, is full of the fossil *Poecilozonites bermudensis*, var. *zonatus*. The Warwick eolianite dips away from the observer at the common 30° angle of repose. Photo. by the author.

soil, which is correlated with the St. George's (Fig. 11). This soil dips gently to the north and has been cut off by erosion. A short distance to the south the north-south road is crossed by an east-west road, the "middle road." About one hundred feet east of the four-corners the St. George's soil, about three feet thick, is again exposed on the south side of the road; it dips southward. About fifty feet south of the four-corners the same soil is exposed a third time, although here it is not over nine inches thick. The underlying eolianite is considered to be the Pembroke; the superjacent eolianite, the Warwick. A still higher soil, the Signal Hill, is exposed at the crest of the next Hill, Mount Royal (see plate 6). It conforms to the surface of the hill rather closely, is about two feet thick, and shows considerable weathering. About three hundred yards still further to the south the Signal Hill soil, completely lithified, is once more exposed, at a point on the east side of the road. To the west of the road is a high hill of eolianite in which a quarry is located. This eolianite probably belongs to the Southampton formation. Further to the south there is a declivity in the road. In a pasture about fifty feet east of the declivity the Signal Hill soil is exposed as an indurated red rock. Crossing the South Shore Road and following a foot path past a small house on the east, a fossil soil underlying the recent soil is well exposed in a fresh cut. It may be the Signal Hill. The whole section may be seen in the cliff at Simmons Beach, where three soils are exposed (Figs. 12 and 13). The highest is correlated as McGall's, the lowest as St. George's. The soils are very thick and are underlain by soil bases. The lowest soil dips below sea level about three hundred yards west of the Paget-Warwick line, demonstrating that when it was formed sea-level was lower than now. Fossil land molluscs are extremely abundant and well preserved in all the soils in the cliff.

The North Shore. Along the North Shore road there are a number of outcrops of a thick red soil rising from sea-level and usually not far above it. Near Bailey's Bay on the south side of the road, the very characteristic old topography and deep weathering can be seen. In a 10-foot road cut about 200 feet west of Bailey's Bay, there is one of these thick soils. Here *Poecilozonites nelsoni* was found; hence it is safe to place this soil in the Harrington stage. At Cottage Hill on the side of the road is a thick soil, about fifty feet above sea-level. The fossil *P. nelsoni* was found here also, and this soil is without doubt Harrington in age. At the Shelly Bay race course in the rock under the grandstand is a thick soil about 25 feet above sea-level. Although *P. nelsoni* was not found here, the general resemblance of this soil to those just mentioned, and its position would place it as Harrington. The eolianites overlying this Harrington horizon have not been searched for soils. Along the North Shore, from the Hotel Frascati to Spanish Point soils appear to be absent, but this may be because of the fact that this part of Bermuda has not been investigated thoroughly. It may be that all the eolianites and soils are represented here. Localities 83 and 85, north of Devonshire Marsh, prove that some, at least, of these later formations are present. On account of the absence of soils along this part of the North Shore, and the recent aspect of the hills, which still show dune contours, this part of Bermuda is considered to be of Wisconsin age, underlain by the older Pembroke—St. George's topography. Pembroke Marsh separates the older topography on the south from this younger topography on the north.

Hamilton Region. One of the most important regions in the study of the formations at Bermuda is that around the city of Hamilton, but it is frequently difficult to assign each soil to the proper stratigraphic horizon. On Reid Street, Hamilton (57) the Walsingham eolianite, very firm and compact, is exposed beneath a compound soil six feet thick (Fig. 14). The lower part of this soil is red and two feet thick; the upper part is pink, about four feet thick, and distinctly a soil of accretion, indicating a return to dune conditions. The lower part is of Shore Hills age, the upper of Harrington. At this locality the *Poecilizonites nelsoni* was found. A soil may be seen in the cliff near the Reid St.-E. Broadway junction. Continuing further east and bearing left up the hill called Middle Road, a pink soil may be seen on the left about half way up the grade. This is probably the Harrington. It is not over a foot thick. Above it is the Pembroke eolianite. At the top of the hill may be seen a thick red soil, two feet deep, which is provisionally assigned to the St. George's. Fragments of fossils have been found in these soils, but they have not been carefully studied. A. E. Verrill (*1907*, p. 74) cites Goldie (1867) as follows: "Prof. T. W. Goldie, in his printed lecture on the Geological Formation of Bermuda, reprinted in 1893, pp. 14, 15, mentioned a 'belt or layer' of 'red clay' soil, eight inches thick, underlying the eolian limestones near Hamilton. This layer was about 60 to 70 feet above sea-level and at about 130 feet below the surface of the hill. It was found in making a boring for a well at the military establishment on Prospect Hill. The layer of red clay was underlaid by strata of compact limestone. Perhaps this was part of the Walsingham formation."

This soil might be either the Shore Hills-Harrington or the St. George's soil.

The soil on Elliott Street is probably the most interesting in the city because of the deep pockets of weathering. On the south side of Elliott Street this soil is fully four feet thick and of a deep-red or chocolate-brown color. One of the pockets is four feet deeper, making the total thickness of the soil at this point eight feet (see plate 2). Obviously a long period of weathering is involved. Under this soil many fossils have been found, the most common species of which is *Poecilozonites bermudensis*. On the north side of the street the same soil can be seen near the top of the cut, but it is not over a foot thick. Evidently the soil on the south side accumulated in a hollow. The same soil may be followed northward on the west side of Cedar Avenue. It contains many specimens of *P. bermudensis*, but *P. nelsoni* has not been observed. This soil is placed provisionally in the St. George's.

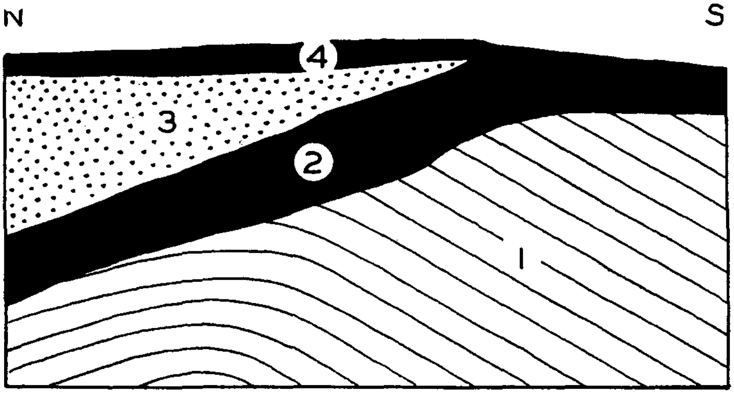

FIGURE 14. Reid Street section.

1. Walsingham eolianite.
2. Shore Hills-Harrington soil.
3. Sands belonging to late Harrington time.
4. Present soil merged with Harrington.

Note erosion of Walsingham under soil. Locality 57.

Further east in the new cut on Elliott Street there is a splendid exposure on both sides of the road. The soil may be either the St.

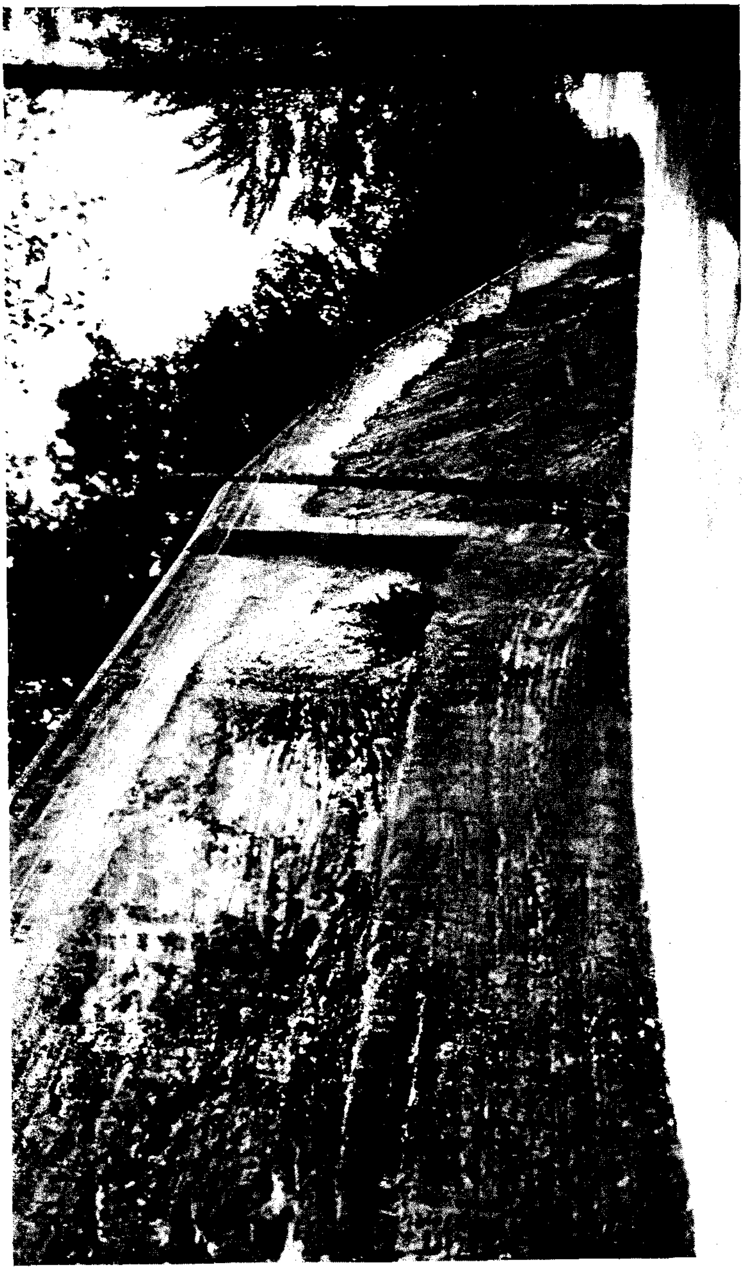

FIGURE 15. Section of a dune of eolianite north of Pembroke marsh, near Hamilton. The wind direction was north, or from left to right in the view. The middle and back of the dune are shown, the middle beds about horizontal and the back beds dipping at an angle of about 30°, the angle of repose for this material. Photo. by author.

George's or Signal Hill (see plate 4). On Pitts Bay road near the entrance to the Princess Hotel there is a thick soil only a few feet above sea-level exposed in the cliff under the road (55). This soil is about eighteen inches thick, of a red color, and contains many small species of fossil snails which are commonly found in the Harrington-Shore Hills soil (to which it is assigned); but none of *P. nelsoni*.

About one hundred yards north of the entrance to the Princess Hotel a street connecting Pitts Bay road with Serpentine road leads to a very good soil exposure about half-way between the two roads (59). The soil is exposed on the west side of the street. On account of its fossil content, its thickness, and the large amount of erosion that has taken place since its formation, it is considered to be St. George's. It is without much doubt the same soil as that found on Elliott Street (98).

On the southwest border of Pembroke Marsh, along the southwest side of Serpentine road there is a soil exposure (31) four feet thick about ten feet above sea-level. Because of its great thickness and position it is assigned to the Shore Hills-Harrington horizon. The same soil may be seen again at locality 30 further west. At locality 27, just before crossing the small bridge to the Grasmere Hotel, there are two soils on the south side of the road, a lower one about two feet thick devoid of fossils and an upper thicker soil. A soil-base divides the two soils. The age is without much doubt Shore Hills-Harrington, and the upper soil may represent all post-Harrington time.

There are a number of other fossil soils in the region which should be noted. The two soils back of the Opera House on Victoria Street deserve mention (see plate 5). The section here in downward succession is: (1) recent soil; (2) white eolianite; (3) upper soil of light pink color between three and four feet thick, dipping gently northward, and with an irregular lower surface due to solution cavities—a soil base present; (4) about three feet of white eolianite; (5) a red soil one foot thick dipping gently northward; even soil base without solution cavities; consolidated sand-rock above and below the soil. The age of this group is difficult to determine without further study of the Hamilton region. The lower soil is probably Shore Hills, the upper Harrington or St. George's. There is a thick soil about 300 feet east of the Opera House, but the age is unknown.

In a general way the parish of Pembroke may be divided into two parts, an area south of Pembroke Marsh, which is underlain mainly by the Pembroke and older eolianites, and an area of higher land to the north, which is underlain by the younger eolianites. These relations are discussed more fully in the section on physiography.

Walsingham Cave District. The Walsingham Cave District, located between Castle Harbor and Harrington Sound, is specially important because of the excellent development of the Walsingham eolianite. This eolianite, which forms the base of the stratigraphic column in Bermuda, reaches higher elevations here than elsewhere. It is compact, well jointed, and deeply pitted. Since this formation is found at elevations as much as one hundred feet above sea-level and inasmuch as the Belmont and Devonshire marine limestones are not found at elevations greater than twenty-five feet, we may conclude that this particular part of Bermuda has stood above water ever since Walsingham time.

The foreset beds of these cross-bedded eolian limestones dip steeply to the east and northeast, demonstrating that the dunes migrated from the west or southwest, that is, out of what is now Harrington Sound. This body of water may fill a limestone sink or it may be an inter-dune depression. The paucity of younger eolianites in the Walsingham Cave District confirms the observation already made in Pembroke parish, that the younger dunes are confined to the coast.

At Crystal Cave the relations of the rocks can be seen very well. Just inside the iron gate of the cave, on the north wall, a soil with large *Poecilozonites nelsoni* may be seen. This is considered to be the Shore Hills soil. The underlying eolianite is Walsingham, the upper eolianite Pembroke. In the government quarry near Paynter's Vale the Walsingham formation is well exposed. There are many fossils in the overlying soil, among them *P. nelsoni.* Around the large sink hole near the center of the quarry are many kinds of fossils, including crabs, bird bones, and snails. A complete study of this locality would undoubtedly yield a great deal of information.

The caves of this area are described in a later page dealing with the physiography of Bermuda.

The Islands near Hamilton. The islands near Hamilton and as far away as Little Sound and Southampton furnish a problem so closely connected with the fossil soils found on them that a brief word must be said about their origin and history.

There are two groups, an outer group and an inner group. Hawkins Island, Long Island, Agar's Island and a number of smaller islands form a crescent, and may be considered the outer islands. I have been unable to find evidence of fossil soils on any of these. The inner group comprises many, but it is unnecessary to name them all. The more important ones are, Hinson, Marshall, Darrell, Burt, Grace, Wilson's, Morgan's and Tucker's. The Belmont marine is surely found at Marshall Island and at locality 37 north of the Belmont Hotel. As the soil on Darrell Island was seen from a boat only, on the last visit to Bermuda, I cannot say whether the Belmont occurs there. The same is true of locality 101 near Burgess Point. The Devonshire marine formation is found on Agar's Island, Marshall Island and on Wilson's Island.

These inner islands are all the product of erosion. None of the original dune forms are preserved, while in western Pembroke dune-shapes can be seen, the result of Wisconsin dune-building, at least on the tops of the hills. The Devonshire formation on Agar's Island is not even well consolidated, while the Belmont formation noted above is very resistant and has a decidedly old appearance. It does not seem possible that these can be the same marine formation. In any event, erosion is responsible for the shapes of the inner islands and erosion went much deeper than the present sea-level, since several of the fossil soils dip well below sea-level, as at Hinson Island and Marshall Island. It is evident that the soils were to a large extent formed during a time when the water-level was lower than it is today, and as the water-level was considerably higher during the Shore Hills stage than now, all that can be said is that the Shore Hills soil in these islands was formed when the water-level had subsided considerably from its highest point. Of course this was true of the Shore Hills locality and was probably true of all the places where the Belmont and Shore Hills formations are seen together. A cave was found by divers in the ship channel in Hamilton Harbor 30 feet below the present sea-level, with stalactites and red soil. Hence it was during a time of low water that this region was eroded. What prevented dune formation here at that time? Cressy has shown that dunes always follow a retreating body of water, as in the case of the dunes at the southern end of Lake Michigan. He found that the recent dunes there were formerly near the lake at a higher stage and that the old dunes were far away, and well set. The Hamilton region during this time of low water was protected from dune activity, as was the case about the Walsingham cave district. Dunes did not enter there. The erosion of these islands continued through the Pembroke, the St. George's, and all later stages of the Pleistocene, and the result is as we see it today. I have supposed that this whole island region was continuous land during the time of the formation of the Shore Hills soil, and that valleys were eroded in it fully as much during a stage or stages of low water, as of high water, and that when the sea-level rose again these valleys became filled by marine waters as we see them today. Actual subsidence of this inner group of islands is the other alternative. Harrington Sound might be explained in the same manner. It is not impossible that certain areas had subsided, but if this actually happened, the main hypothesis of this paper would not be essentially affected.

Several localities among the inner islands are very important: Hinson Island, Marshall Island, Wilson's Island, and Tucker's Island, and along the shore of the main island, likewise the Salt Kettle exposure, the Inverurie and the Belmont shore exposures. These will be taken up in order.

Hinson Island (Fig. 16), the most easterly of the few displaying soils, was visited in 1926 by Clark and the writer. At the prominent soil exposure on the west side of the island (105), at the water's edge, Clark found a *P. nelsoni* in sands just above the red soil, and I found a specimen of the same species, poorly preserved, in the deep-red soil itself. Eolianite underlies the deep-red soil. The soil is divided by a soil base and thus is really a double soil. This fact may not be clear near the water's edge, but on the side of the hill, about 100 feet north, this soil base is well seen between the two soils. Complete

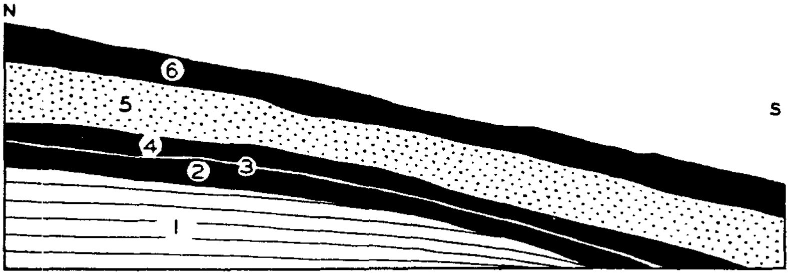

FIGURE 16. Hinson Island section.

1. Walsingham eolianite.
2. Shore Hills soil.
3. Soil base.
4. Shore hills or Harrington soil?
5. Sands with fossils.
6. Harrington soil?
Locality 105.

consolidation makes both of them very hard above the sea-level outcrop. Overlying are sands about 3 feet thick with fossils, and above the sands, another soil with fossils. All the soils and intercalated sands dip about 10° to the southwest. Following the lower soils northward they are seen to reach the uppermost part of the island and encircle it, forming a kind of crown about the top.

I believe that the underlying eolianite is of Walsingham age, that the lowest soils are of Shore Hills age and that the sands and overlying soil are of Harrington age. The Pembroke formation is missing here, although there may be a slight remnant of it near the top of the island. This locality closely resembles that at Wilson's Island. There is no record of *P. nelsoni* having been found among these islands until Clark and I found it there in 1926. Like the other inner islands, Hinson Island is a remnant of the old land of Bermuda and a most important place for study.

Marshall Island (104) is the next island northwest of Hinson. At its eastern end, near Timlin's Narrows, a deep red soil can be seen close to the water's edge. It is almost horizontal but dips slightly southward. Here the order of the formation is as follows: a very hard limestone extending below sea-level and containing marine fossils; above it loosely consolidated sand rock about 3 feet thick, possibly of marine origin, and dipping gently east; on this sand rock a deep red soil about 2 feet thick with *Poecilozonites nelsoni* to the southwest, eolianite lying on the soil. Mather found fossils in the marine limestone and I found several specimens of *P. nelsoni* in the soil. The soil appears to be single. It dips gently below sea-level.

The marine limestone is without doubt of Belmont age. The sands above are probably of Devonshire age. The Shore Hills soil does not appear on the Belmont, as it does in most places, and was probably eroded. The soil is without much doubt of Harrington age.

The soil at Inverurie, (102), has already been described under the section called the Paget-Warwick line. The Salt Kettle locality 45, and the Belmont locality 37, also face the inner islands on the south, and present a common problem.

The Salt Kettle (45). About 200 feet south of the ferry station on the east, and forming part of the wall of a very small house, is a hard pink soil about one foot thick, dipping north. The eolianite under this soil dips south, a relation like that represented at the Inverurie locality. Hence the Salt Kettle soil is probably of Harrington age. About 200 feet south of this soil the eolianite also dips south. It is probably the same eolianite found under the soil, at the house.

The Belmont soil, (locality 37), can be found at an opening in the wall north of the Belmont Hotel and close to sea-level. The soil here is underlain by the Belmont formation, in which I found several marine fossils. This is a very compact rock. The soil itself—the Shore Hills soil—is about three feet thick and dark red. On account of its thorough weathering, fossils are scarce. At the bottom of the soil is a breccia resembling that at Stokes Point.

Wilson's Island, (locality 129) (Fig. 17). The next section in order would be that just east of Burgess Point in the cliff, and that on Darrell Island. Unfortunately there was not time to visit these exposures, which were seen only from a motor boat.

The good section at the small Wilson's Island, will be described next. The island lies about 100 yards off-shore at the base of Gibbs Hill. The lowest formation here is a thinly stratified marine or eolian sand-rock dipping very gently northwards. If it is marine, as is most probable, it is of Devonshire age; if eolianite, it is of Walsingham age.

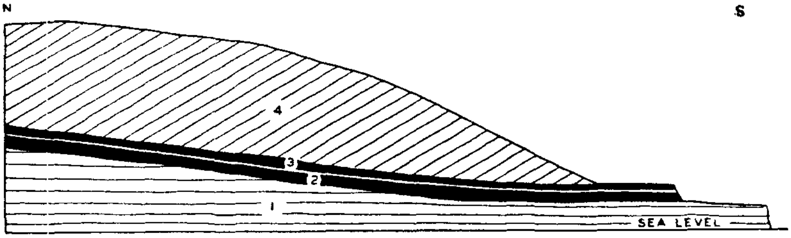

FIGURE 17. Wilson's Island section.

1. Devonshire marine limestone?
2. Harrington soil?
3. Harrington soil?
4. Pembroke eolianite.

Locality 129

On a shelf cut into this formation, sloping gently northward and upward, is a deep-red soil about 2 feet thick. The soil cuts the underlying formation. A poorly preserved specimen of *P. nelsoni* was found here. Above the deep-red soil come sands, as at Hinson Island, then a light colored soil with many fossils, mostly *P. bermudensis*. Above this soil comes northward dipping eolianite at the foreset angle of 30°. The lowest formation here, as noted above is of Devonshire or Walsingham age. The deep-red soil is undoubtedly of Harrington age, and the sands and soil above them of Harrington age. The eolianite resting on the Harrington is of Pembroke age. It can be seen readily that the shape of the island is due entirely to erosion.

Only those islands of the inner group have been mentioned where fossil soils have been found.

Tucker's Island (locality 127), the most westerly of the inner group, has long been noted for its cave. Before the other caves were discovered many people visited Tucker's cave, and a reminder of their visits has been left in the blackened roof and stalactites caused by the smoke from torches used for illumination. It is impossible to see this cave without a boat. Verrill speaks of finding some *P. nelsoni* fossils just above the entrance to the cave. This must indicate the former presence of the Shore Hills soil and remnants of this soil can still be seen. The rock in which the cave was formed is probably of Belmont age, but this point awaits further investigation. The top of this formation is about 15 feet above sea-level. No eolianite was seen and it is evident that any eolianite that may have been formed here was eroded, together with most of the soil.

FOSSIL SOILS.

CHARACTER AND ORIGIN.

R. J. Nelson (*1840*) noted that the soils now forming on Bermuda are gray on hillsides, but red in valleys and hollows. The light-gray or pink soils found on the slopes usually have a large percentage of lime carbonate, whereas the deep-red or chocolate-colored soils found in the depressions are composed largely of silica, alumina, and iron. Similar relations are found in the fossil soils. The St. George's soil displays these features in the exposure on Ferry Road, St. George's, just east of the railroad cut, in the cliff on the north side of the road. Where the soil is at its lowest elevation it is deep-red and about eighteen inches thick. It rises both to the east and the west, becomes thinner and lighter-colored, eventually attaining a pink tinge. In several places it has been eroded away. The pink portion contains many fossil snails, which are completely lacking in the deep-red part. It is evident that as fast as the oxidized material formed it was washed into the hollows by the run-off.

Three processes have been active in the production of the soils of Bermuda. Much of the material is obviously residual, the insoluble residue of large amounts of eolianite. Soils of this type are well oxidized and either deep-red or chocolate-brown. A second process is by accretion from above. Fine calcareous sediment blown around by the wind has frequently contributed a large quantity of material to the soils. Such soils are very light-colored, either cream, pink, or gray. Thirdly, the material formed by either of the two processes just outlined may be concentrated in hollows by rain-wash. It is frequently difficult to evaluate these three processes. In general, however, a soil of accretion will be much lighter in color than a residual soil. Moreover, it would be difficult for a strictly residual soil to form on a steep hill-slope. Rain-wash would convey the minute particles of decomposition to lower levels about as fast as they formed, leaving on the hill-sides a relatively thin soil composed of fresh material. The fossil soils of Bermuda show such relations. Wherever they rest on a relatively steep slope they are pink or gray rather than red, indicating a greater percentage of unoxidized material. In the hollows the fossil soils are red or chocolate-brown. The color of a soil at any particular spot is therefore a result of a number of variables, among which should be mentioned position, method of formation, and length of time involved in accumulating the soil.

A. E. Verrill (*1907*, p. 57) made a careful estimate of the time necessary to form a strictly residual soil. From a study of the rate of disintegration of the old stone forts on Castle Island and other islands, some of which were probably built before 1620, he concluded that it "would require 120,000 years for the destruction of the two hundred feet of hard limestone necessary to form one foot of soil." He states that other evidence checked with this conclusion. The petrologic study in this paper also shows that Verrill's estimate of the amount of eolianite that must be destroyed to form a foot of soil is of the right order of magnitude. Our work suggests that one hundred feet of eolianite would give one foot of soil. If so, a foot of soil might accumulate in 60,000 years. Post-glacial soils in limestone regions of the United States are about six inches thick. If we take post-glacial time as about 25,000 years,[1] a foot of soil might accumulate in 50,000 years.

A soil developed on low flat land would probably form at some such rate, that is, one foot in from 50,000 years to 120,000 years. But a soil concentrating in a hollow would increase much more rapidly, due to the large amount of material washed in from the surrounding slopes.

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[1] G. F. Kay, The relative ages of the Iowan and Wisconsin drift sheets, Am. Jour. Sci., vol. 21, pp. 158–172, 1931.

For example, it has already been mentioned that the St. George's soil on Ferry Road is eighteen inches thick in the bottom of the depression. But much of this has been washed in from the nearby slopes. Finally, the soils of accretion, would accumulate much more rapidly, perhaps within a few thousand years.

The estimates may be applied with confidence, however, to soils formed on a low land-surface which had been reduced by long erosion. The Shore Hills soil on Marshall's Island was formed under such conditions and is about two feet thick. It would take from 100,000 to 240,000 years for such a soil to accumulate. At the Shore Hills quarry the deep residual soil, representing Shore Hills and Harrington time combined, is about four feet thick and in certain pockets twelve feet thick. The smaller figure suggests that it took from 200,000 to 500,000 years for this soil to accumulate. Since the Harrington is usually a soil of accretion, it probably accumulated in a relatively short time and most of the figures given above should be assigned to Shore Hills time. A fair average for the length of Shore Hills time would be perhaps 200,000 years ± 50,000.

On flat surfaces the St. George's soil is about two feet thick, indicating an interval of 100,000 to 240,000 years. It never attains the great thicknesses noted for the Shore Hills-Harrington soil. A fair average for St. George's time would be about 150,000 years ± 50,000. Unfortunately the upper two soils, the McGall's and the Signal Hill, are never found in suitable topographic positions for making accurate estimates of the length of time involved in their accumulation. Moreover, they are largely soils of accretion. They undoubtedly accumulated much more rapidly than the residual soils, perhaps within a few thousand years. The modern soil has probably accumulated within post-Wisconsin time, that is, within 25,000 years, for its thickness, six inches, is of the right order of magnitude.

All these estimates are admittedly crude. The errors may be considerable, but the figures nevertheless are probably of the right order of magnitude. They indicate that several hundred thousand years were necessary for the production of the fossil soils of Bermuda. In addition, periods of unknown length were involved in the accumulation of the eolianite. Moreover, the figures clearly show that the soil-making ages were of vastly different lengths. Shore Hills-Harrington time lasted 200,000 years, St. George's time 120,000 years, Signal Hill and McGall's time a few thousand or tens of thousands of years each, and post-eolianite time 25,000 years.

SOIL BASES.

Between each soil and the underlying eolianite there is frequently a thin layer of cream-colored, compact, finely crystalline limestone from one-half inch to two inches thick. Similar material coats the surface of many of the hills in Bermuda at the present time, due to the removal of the soil because of deforestation. At several localities the fossil soil is not only underlain by a soil base, but a similar layer divides the soil into an upper and a lower part. I interpret such compound soils as indicating two distinct soil-forming episodes separated by a long interval of time. At Pink Beach, for example, there are two soils separated by five feet of eolianite; the lower soil is four feet thick, the upper two feet. The upper soil, however, is actually compound, for in addition to the soil-base beneath it there is a much broken soil-base dividing the soil into two parts.

PALEONTOLOGY.

INTRODUCTION.

A detailed paleontological study of the fossil soils was made in the hope that the relative ages of the formations might be determined, and also, that index fossils might be established for distinguishing the various soils. Unfortunately the results are largely negative, or at the best inconclusive, at the present time. None of the snails, foraminifera, or algae is of any help in determining the age of the formations, other than that they are either Tertiary or Quaternary. The snails, however, may prove useful as index fossils. *Poecilozonites nelsoni* is confined to the Harrington soil and older formations. Many other species of snails are now extinct, but sufficient data were not obtained to determine the exact horizon to which each is limited. What relevant suggestions can now be made are indicated in Table III. This may be a profitable method of attack in the future.

TABLE II. PALEONTOLOGY OF THE FOSSIL SOILS.

Identifications by T. H. Clark.

Locality Nos.		
1.	Poecilozonites bermudensis var. zonatus	St. George's soil Ferry Road.
	" reinianus	St. George's.
	" circumfirmatus	
	Succinea somersensis	
5.	Poecilozonites bermudensis var. zonatus	Signal Hill soil.
	" circumfirmatus	
	Succinea somersensis	

Locality Nos.		
6.	Poecilozonites bermudensis var. zonatus " circumfirmatus	Soil in cliff north of Signal Hill, by shore. McGall's soil?
7.	Peocilozonites circumfirmatus	About 1500 ft. west of Tobacco Bay.
10.	Poecilozonites reinianus " circumfirmatus	Mullet Bay. At sharp curve in road south of Davis' Hole. Thick soil 18″ with marine fossils at bottom and land snails above. About 6′ above sea-level. Harrington soil.
13.	Poecilozonites nelsoni " " var. callosus " circumfirmatus " bermudensis var. zonatus " reinianus Strobilops hubbardi Succinea somersensis	Spanish rock. Soil 12″-18″ light colored. Many fossils, but soil discontinuous laterally. Situated in sea-cave about 15 feet above sea-level. Harrington soil.
15.	Poecilozonites bermudensis var. zonatus " reinianus " circumfirmatus Vertigo numellata Carychium bermudense Succinea somersensis	Albuoy Point. This is probably the Harrington soil.
16.	Poecilozonites bermudensis " circumfirmatus " cupula " dalli Succinea somersensis	Albuoy Point. Near 15 and probably the same soil.
17.	Poecilozonites cupula " dalli Succinea dalli	Albuoy Point. In the same cut, and probably the same soil (17).
18.	Poecilozonites bermudensis var. zonatus " circumfirmatus Succinea somersensis	Shelly Bay race course. Harrington soil.
19.	Poecilozonites nelsoni " bermudensis var. zonatus " reinianus " circumfirmatus	Cottage Hill. Outcrops on side of road cut. About 50′ above sea-level. Harrington soil.
20.	Poecilozonites nelsoni " bermudensis var. zonatus " reinianus " circumfirmatus	Bailey's Bay. In 10′ road cut just west of Bailey's Bay. Dips S. W. 30. Harrington soil.

Locality Nos.		
21.	Poecilozonites bermudensis " circumfirmatus	Surf Beach. Fossils found on sea-cut shelf about 10′ above sea-level. Sea caves along shore at same altitude. Deep red soil. Harrington.
22.	Poecilozonites bermudensis " circumfirmatus Vertigo numellata Carychium bermudense Succinia somersensis	Surf Beach. Half way up concrete steps. St. George's soil.
23.	Poecilozonites bermudensis var. zonatus	Town Cut. Soil on each side of cut, each about 1′ thick and dipping gently northeast and west. Higher on south side of cut. St. George's or Signal Hill?
24.	Poecilozonites reinianus	Stokes Point. Soil lying on Shore Hills marine. In lower part many marine fossils and in upper part many land forms but mostly one species. Shore Hills soil.
33.	Poecilozonites bermudensis " reinianus " circumfirmatus Thysanophora hypolepta Carychium bermudense Succinea somersensis " barbadensis	Serpentine Road, Hamilton (59). Shore Hills-Harrington soil.
42.	Poecilozonites bermudensis " circumfirmatus	Soil on road south of Jews Bay. Signal Hill or McGall's soil?
43.	Poecilozonites bermudensis var. zonatus Euconulus turbinatus Succinea bermudensis	About ¼ mile farther west than locality 42 and probably the same soil.
46.	Poecilozonites bermudensis var. zonatus " circumfirmatus	At head of Mangrove Bay near the Somerset-Watford bridge. Marine forms in lower part and land forms in upper part. About 6′ above sea-level. Harrington?
55.	Poecilozonites bermudensis var. zonatus " circumfirmatus	Princess Hotel Harrington soil.

Map Nos.		
57.	Poecilozonites bermudensis var. zonatus " circumfirmatus	Reid Street soil. Very red and thick at bottom. Almost no fossils found. Shore Hills-Harrington.
66.	Poecilozonites bermudensis var. zonatus P. circumfirmatus Succinea bermudensis Euconulus turbinatus Vertigo numellata Carychium bermudense Bifidaria jamaicensis Thysanophora hypolepta Zonitoides bristoli	Lowest soil exposed along the cliffs at McGall's Bay locality. Pale color, horizontal, and continuous with the soil of Loc. 69 and 78. Many fossils, especially small species. Harrington soil.
67.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. blandi Succinea bermudensis Carychium bermudensis Zonitoides bristoli Bifidaria servilis	Fossils found in sand lying on soil 66, and part of it.
68.	Vertigo numellata Carychium bermudensis Bifidaria jamaicensis Thysanophora hypolepta Zonitoides bristoli	A red soil above 67. This soil is continuous to the east with 77, and to the west with 70 and 72. Fossils all small species. St. George's soil.
69.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. vanattai angulifera Succinea bermudensis Euconulus turbinatus Vertigo numellata Carychium bermudensis Bifidaria jamaicensis Thysanophora hypolepta Zonitoides bristoli	Lowest soil seen in the cliff at western end of McGall's Bay locality. Stratigraphically continuous with 66. Many fossils. Harrington soil.
70.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. vanattai	A red soil lying above 69. In places cavities filled with fossils and boulders. Filling may be of recent date. Fossils scarce. St. George's soil.
72.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. vanattai	A red soil lying not far below 75. It is the equivalent of the soil of 70 and also 68 and 77. St. George's.

Map Nos.		
74.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. gulicki P. reinianus Succinea bermudensis Euconulus turbinatus Bifidaria jamaicensis	Soil continuous with 73. Fossils of 73 and 74 recorded together. McGall's soil.
75.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. reinianus	A pink soil of great horizontal regularity, continuous with soil 71. Along the cliffs to the east this soil is less regular. Few fossils. Signal Hill soil.
76.	Poecilozonites bermudensis var. zonatus	Fossils from sands lying in the lowest soil, about 1000′ east of loc. 66. Harrington sands. Transition.
77.	Poecilozonatus bermudensis var. zonatus P. circumfirmatus	This is a very strong red soil with a pink one above. No sands lie between them. These two soils are indisputably the red and pink soils of localities 70 and 71. The fossils were found only in the red lower soil. St. George's and Signal Hill soils.
78.	Poecilizonites bermudensis var. zonatus P. circumfirmatus Succinea bermudensis	This soil lies on a bench about 6′ A. T., in precisely the same position as does the same soil at loc. 66 and 69. Fossils are rare. Harrington soil.
79.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. blandi Vertigo numellata Carychium bermudense Thysanophora hypolepta	This is a very strong red soil, one to two feet thick, on the path through the old fortifications at Devonshire Bay. Downwards it grades into disrupted eolianite. It is covered by about 4″ of transition sands. This zone carries many *P. zonatus*. It dips under the hill in a southwesterly direction, and is probably continuous with the soil of loc. 81 and 82. This

Map Nos.		
		would make it of Harrington age. It is possible, however, that it overlies that soil, in which case it would be St. George's in age.
80.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. blandi Vertigo numellata Carychium bermudensis Thysanophora hypolepta Zonitoides bristoli	A pale soil under the cliff at Devonshire Bay. Below it is a coarse sand rock of the Devonshire-Harrington soil.
81.	Poecilozonites bermudensis var. zonatus Succinea bermudensis	A red soil on both sides of a small hill east of Cox's Bay. Age not determined.
83.	Poecilozonites bermudensis var. zonatus P. circumfirmatus var. discrepans P. blandi	This is a light colored but thick soil in a large quarry in the northern end of Devonshire marshes. It contains many P. zonatus fossils mostly with traces of their original coloring. It dips steeply to the south. Age not certain, but on account of the great number of P. zonatus fossils it should be one of the later soils.
84.	Poecilozonites bermudensis var. zonatus P. circumfirmatus P. blandi Vertigo numellata Carychium bermudensis Thysanophora hypolepta Zonitoides bristoli	Transition sands immediately above 83. Above this sand was found by Clark, a hermit crab shell, Livonia pica, at least 100 feet A. T. These sands belong in age to the soil underneath.
86.	Poecilozonites bermudensis var. P. circumfirmatus var. discrepans P. reinianus P. blandi	Soil in pits near road level of the big quarry of loc. 83, and possibly the same soil as 83.

FOSSIL SNAILS.

After accompanying the writer to Bermuda twice, Clark studied the collections and prepared the locality list (Table II). Twenty-six species of snails have been found. The most abundant and interesting genus is Poecilozonites, which is indigenous to Bermuda. Twelve species of this genus, some of which are extinct, are found in the fossil soils of Bermuda. *Thysanophora hypolepta* is also confined to Bermuda. *Strobilops hubbardi*, although extinct in Bermuda, is still living in the Gulf states and Jamaica. *Bifidaria jamaicensis* lives not only in Bermuda but also in the Greater Antilles; *B. servilis* lives in Cuba and other West Indian islands.

TABLE III. STRATIGRAPHIC PALEONTOLOGY.[1]

Species	S.H.	H.	St.G.	Sig.	M.	Present
Poecilozonites bermudensis var. zonatus Verrill		x	x	x	x	?
P. reinianus (Pfeiffer) Pilsbry	x	x	x	x	x	Living
P. circumfirmatus (Redfield) Pilsbry		x	x	x	x	Living, very common
P. nelsoni (Bland) Pilsbry		x				Extinct
P. nelsoni var. callosus Gulick		x				Extinct
P. cupula Gulick		x				Extinct
P. dalli Gulick		x				Extinct?
P. vanattai angulifera Pilsbry		x				Living
P. vanattai Pilsbry			x			Living
P. gulicki Pilsbry					x	Extinct
P. blandi Pilsbry		x	?			?
P. circumfirmatus var. discrepans (Pfeiffer) Pilsbry						Living, not Common
P. bermudensis (Pfeiffer) Pilsbry		x	x	?	?	Living
Succinea somersensis Verrill		x	x	x		Living
S. bermudensis Pfeiffer		x		?	x	Extinct?
S. barbadensis Guilding						Living
Strobilops hubbardi (Brown)		x				Not living in Bermuda
Vertigo numellata Gulick		x	x			Extinct?
Carychium bermudense Gulick		x	x			Extinct?
Thysanophora hypolepta (Shuttl.) Pilsbry		x	x			Living
Euconulus turbinatus Gulick		x		?	x	Extinct?
Bifidaria jamaicensis C.B.Ad.		x	x		x	Living
B. servilis, Guilding		x				Living
Zonitoides bristoli Gulick		x	x			Extinct

[1] x indicates form present. ? indicates soil not definitely identified, S.H. Shore Hills, H. Harrington, St.G. St. George's, Sig. Signal Hill, M. McGall's.

Table III shows in what soils each species has been found and also indicates whether they are living or extinct. The soils are arranged chronologically, the oldest at the left. Unfortunately the collections were largely confined to the Harrington soil. In future work, more extensive collections should be made from all the soils and a set of index fossils established from known localities. At the present time all that we can say with confidence is that *P. nelsoni* is confined to the Harrington and older formations and that *P. bermudensis* var. *zonatus* is very abundant locally in the St. George's, Signal Hill, and McGall's soils.

FORAMINIFERA.

The writer is greatly indebted to Professor Joseph A. Cushman and Mr. Lothrop Bartlett for studying the foraminifera. The former has studied the foraminifera from the fossil soils. Most specimens were rounded by wind action and are somewhat decomposed.

Cushman states that these are living forms which live in water not over twenty fathoms deep and under climatic conditions similar to those which now prevail on the islands. Since these genera have varied but little since Pliocene time, they can not be used as horizon-markers. Evidently the foraminifera have been blown inland and incorporated in the soils.

Mr. Bartlett, when a student at Professor Cushman's laboratory, determined the foraminifera of the sand-rock. It will be noted from Table IV that there are very few species in the soils themselves and the immediately underlying coarse-grained sand-rock, but that in the much finer-grained, overlying eolianite many species are present. The greatest number of species found in any soil was eight, and usually there were fewer than four. The same general features exist in the coarse sand-rock. But in the fine-grained sand-rock thirty-four species were found in one specimen. The larger number of species found in the fine-grained eolianite would suggest the lowering of sea-level and exposure of the fine-grained mud to wind action. This might well indicate renewal of glaciation on the continents. The species are listed in Table IV.

TABLE IV. FORAMINIFERA.

Foraminifera from fossil soils of Bermuda. (Determined by Joseph A. Cushman).

1. Alveolina pulchella.
2. Amphistegina lessoni.

3. Carpentaria.
4. Gypsina.
5. Massilina.
6. Orbiculina adunca.
7. Orbiculina pulchella.
8. Orbitolites.
9. Quinqueloculina dilatata.
10. Texularia agglutinans.
11. Triloculina oblonga.

Foraminifera from coarse sand below soil like beach sand. Locality 46. (Determined by Lothrop Bartlett).

1. Asterigerina carinata d'Orbigny.
2. Archais aduncus (Fichtel and Moll).
3. Quinqueloculina sp.
4. Amphistegina lessonii d'Orbigny.

Foraminifera from fine-grained eolianite above soil, like material from lagoon bottom. Locality 86. (Determined by Lothrop Bartlett).

1. Textularia agglutinana d'Orbigny.
2. Quinquelo culina agglutinana d'Orbigny.
3. " auberiana.
4. " circularis.
5. " parkeri (H. B. Brady) var. occidentalis Cushman.
6. " polygona d'Orbigny.
7. " sp.
8. " sp.
9. Hauerina bradyi Cushman.
10. Triloculina carinata d'Orbigny.
11. " fichteliana d'Orbigny.
12. " linneiana d'Orbigny.
13. " oblonga ? (Montagu).
14. " planciana d'Orbigny.
15. " sp.
16. Biloculina denticulata (H. B. Brady).
17. " subsphaerica d'Orbigny.
18. Vertebralina cassis d'Orbigny.
19. Elphidium lanieri (d'Orbigny).
20. Peneroplis pertusus (Forskal).
21. " planatus (Fichtel and Moll.)
22. " proteus d'Orbigny.
23. Archais aduncus (Fichtel and Moll.)
24. Sorites duplex (Carpenter).
25. Discorbis sp.
26. Eponides sp.
27. Asterigerina carinata d'Orbigny.

28. Amphistegina lessonii d'Orbigny.
29. Globigerina sp.
30. Anomalina ammonoides (Reuss).
31. " sp.
32. Cibicides lobatula (Walker and Jacob).
33. " sp.
34. ?

ALGAE.

Dr. Marshall A. Howe who identified some of the algae found in the soils, states in a letter: "The fragments of algae shown in your sections appear to belong to genera such as occur in the Bermudian sea at the present time, e. g., *Neomeris*, *Halimeda* and *Amphiroa* and *Lithoporella melobesioides*. Other genera are doubtless present,—such as *Lithophyllum*,—representatives of which are now widely distributed from the tropics to the Arctic regions."

PETROLOGY AND CHEMISTRY.

PETROLOGY.

The writer is greatly indebted to Professor Esper S. Larsen of Harvard University for a careful and detailed petrographic study of the fossil soils and eolianites. This investigation has consumed many days of Dr. Larsen's time, and I wish to take this opportunity of thanking him.

The petrologic study was undertaken for several reasons. In the first place, the nature of the insoluble residue in the soils is of interest. Secondly, it was thought that the mineralogical composition of this insoluble residue in the various soils might vary sufficiently to be an aid in correlation. Thirdly, the ratio of the weight percentage of insoluble residue in the soils to the weight percentage of insoluble residue in the eolianites gives, on certain assumptions, the number of feet of dissolved eolianite necessary to furnish one foot of soil.

Eolianites. Nine specimens of eolianite were studied and the results are compiled in Table V.[1] The insoluble residue ranges from 0.0004 % to 0.002 % of the total rock; the average is 0.0008 %.

The following procedure was employed in making the petrologic analyses of the soils and eolianites. From five to one thousand grams of the specimen—the exact amount depending on the character of the sample and the amount available—were treated with dilute hydrochloric acid until all the carbonate, and as much as possible of the iron oxide and other colloidal products, were dissolved. In the soils the insoluble residue consisted of sand grains and red clay. The amount of residue from the light-colored, purer soils was small, but from the red soils it reached fifty percent or more. The amount of sand was also greater in the red soils. The insoluble residue was rubbed between the fingers and the clay washed away with water. The sand grains were then studied under the microscope by the immersion method. For many samples the sand was divided into three or more parts with a strong electro-magnet before the microscopic study was undertaken. The method employed determines only minerals that are insoluble in dilute hydrochloric acid and coarser than silt size. The results of the petrographic study are shown in Table V. The estimated percentages of minerals are only rough approximations of weight percentage, as few careful estimates were made, and for many samples the small number of grains present—in some cases as few as two—gave results that can not be considered fair averages. Most of the grains were from 0.5 to 2 millimeters in size, but a few were as large as 3 millimeters and many were smaller than 0.5 millimeters. Some of the quartz grains are well-rounded, but the pyroxene and perovskite grains are mostly angular. On the pyroxene, spine-like projections parallel to the elongation were common. Usually the pyroxene was surprisingly fresh.

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[1] Table V is a folder at the end of the paper.

The average percentage of minerals found in the residue from the eolianites is given in Table VII.

Fossil soils. The results of thirty-seven petrographic analyses of the fossil soils are compiled in Table V and arranged according to localities. Table VI gives the range in composition of each soil and also the average composition. It also shows that the insoluble residue in the fossil soils ranges from 0.001 per cent to 4.3 per cent. The low percentages are from the light-colored soils; the high values from the deep-red soils.

Significance of the Petrology. From a study of Tables V, VI, and VII it is obvious that the individual analyses vary greatly. Undoubtedly the insoluble residue from any soil or eolianite is not uniform, but the differences actually found are in large part due to the small size of the samples studied. Although each original sample of eolianite weighed from five to a thousand grams, the number of sand grains remaining after treatment with acid was always very small, frequently fewer than five and seldom more than ten. The individual analyses, therefore, do not have any great significance. A study of the totals of several analyses, however, may give some valuable information.

TABLE VI. RANGE AND AVERAGE MINERAL COMPOSITION OF THE SOILS.[1]

	Shore Hills	Harrington	St. George's	Signal Hill	McGall's	Ave.
No. analyses	10	8	11	7	1	
Residue	0.003–4.3 0.650	0.001–1.01 0.004	0.001–0.2 0.073	0.001–0.02 0.003	0.02	
Quartz	7–100 43	0–60 32	3–87 25½	0–58 25	40	32
Pyroxene	0–50 12	0–75 36	0–70 25	0–70 43	6	27
Perovskite	0–67 30	0–30 16	tr–40 19	0–40 21	6	21
Magnetite	0–24 6	0–5 1	0–55 12	0–25 9	4	7
Orthoclase	0–5 ½	0–18 3	0–83 9½	0–10 1½	6	4
Glass	tr	0–40 5	0–45 5	0	38	3½
Titanite	0–7 2	0–30 5	0–5 1	tr		2
Brown Garnet	0–20 5	0	0–2 tr	0		1
Plagioclase	0	0–20 2	0–25 2	tr		1
Pink Garnet	0–2 ½	tr	0–7 1	tr		½
Misc.	Epidote Zircon Tourmaline Rutile Leucoxene Corundum Muscovite Cyanite Staurolite	Epidote Zircon	Zircon Biotite Spinel Chlorite	Zircon Biotite Hornblende		1
						100

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[1] Expressed in percentages.

Table VI shows that there are no systematic and constant differences in the mineralogy of the various soils. Any discrepancies are probably due to the limitations of the study. The relatively large amount of quartz and deficiency of pyroxene in the Shore Hills soil compared with the other soils are the only facts that may have any possible significance. But even these are of no great importance in correlation, for individual analyses vary tremendously from the average. The mineralogy of the fossil soils, as far as the work has gone, therefore, is of no great service in correlating the fossil soils, except in the case of the Shore Hills soil, where quartz is usually high.

In Table VII the mineralogy of the eolianites and fossil soils is compared. Column 1 gives the average composition of all the soil analyses; column 2, that of the eolianites. In general, the insoluble residues are seen to be similar. This is of course exactly what we should expect if the fossil soils were derived from the eolianites.

TABLE VII. COMPARATIVE MINERALOGY OF AVERAGE SOIL AND AVERAGE EOLIANITE.

	Soils (37 analyses)	Eolianites (9 analyses)
Quartz	32%	37%
Pyroxene	27	10
Perovskite	21	15½
Magnetite	7	5
Orthoclase	4	18½
Glass	3½	6
Titanite	2	tr
Brown Garnet	1	3½
Red Garnet	½	1
Hydrargillite?	0	3½
Plagioclase	1	0
Misc.	1	0
Total	100	100

To calculate how many feet of limestone have been weathered to produce one foot of soil involves many difficulties. The average amount of the insoluble residue in nine eolianites is 0.0008 %. The percentage of residue in the soils, however, varies greatly. In the light-colored soils on the hill slopes it ranges from 0.001 % to 0.005 %, but such soils have lost much material due to rain-wash. In the deep-red soils of the pockets the residue is as much as 4.3 %, but a large amount of this material has probably been washed in from the surrounding slopes. A red soil developed on a flat surface, however, represents a strictly residual soil. The Shore Hills (?) soil at Pink Beach was developed under such conditions and the insoluble residue is 0.1 %; the Shore Hills soil from Grape Bay carries 0.05 % insoluble residue; four Red St. George's soil samples at St. George's gives an average of 0.08%. Assuming that the last figure is a fair average and dividing it by the percentage of insoluble residue in the eolianites (0.0008%) we conclude that one hundred cubic feet of limestone are destroyed to give one cubic foot of soil. A. E. Verrill (*1907*, p. 57) obtained a figure twice as great; that is, he calculated that to form one foot of soil two hundred feet of eolianite must be destroyed. Verrill does not state what method he employed, but it is probable that the present estimate is the more accurate. A foot of soil would form twice as fast as he estimated, that is, in 60,000 years instead of 120,000 years. Obviously all these figures have a large percentage of uncertainty and can be regarded only as approximations.

The question of the source of the minerals in the residue of the soils and eolianites introduces a new problem. Many of the more common minerals have obviously been derived from the volcanic platform. The perovskite, pyroxene, magnetite, titanite, biotite, and glass have all been described by L. V. Pirsson (*1914*) as occurring in the lavas. Although brown garnet is not mentioned by him, it is frequently found in such basic rocks. But the presence of such typically continental minerals as quartz, orthoclase, pink garnet, zircon, tourmaline, rutile, muscovite, hornblende, staurolite, and cyanite is quite unexpected in Bermuda. Clearly these minerals are not native to Bermuda and must have been introduced from elsewhere. Once arrived in Bermuda a grain of quartz would long preserve its volume and shape. It would suffer very little wear while drifting in sands composed largely of calcium carbonate. The high percentage of quartz is merely a function of its resistance to corrosion, and does not necessarily indicate an abundant supply. The quartz particles are well rounded, and it is safe to assume that much of the rounding, if not all, was produced elsewhere than in Bermuda.

I have no satisfactory solution of the source of these exotic minerals, but a number of possible explanations may be mentioned. (1) Large numbers of birds stop for a few days in Bermuda while on their seasonal migrations between the West Indies and Nova Scotia.[1] Many birds carry a variety of sand grains in their crops and intestines. In the course of thousands of years a quantity of quartz grains and other continental minerals might well accumulate in this way to the amount found in Bermuda. (2) Dr. Arthur Keith has suggested to me that tropical hurricanes might be competent to transport sand from the Atlantic coastal plain to Bermuda. (3) Raymond and Stetson[2] have recently shown that a jelly-like organic substance may be a very effective agent in transporting sand grains, and I also suggest that the sargasso weed, so common on Bermuda shores, might have transported sand grains. I have found that sand, in which rounded grains are common, adheres firmly to that weed on Florida beaches.

CHEMISTRY.

Two chemical analyses of the St. George's soil from Ferry Road, St. George's, are given below. They were made by Mrs. House. The first is that of a pink variant; the second, that of the deep-red phase taken from a hollow and composed largely of silica, alumina, iron, and water.

TABLE VIII. CHEMISTRY OF THE SOILS.

	Pink soil	Red soil
SiO_2	0.59%	28.68%
Al_2O_3	0.39	26.24
Fe_2O_3	0.36	12.09
MgO	1.00	1.72
CaO	50.81	3.13
Na_2O	1.21	0.77
K_2O	0.19	0.73
CO_2	39.56	2.03
TiO_2	tr	1.58
P_2O_5	0.01	0.50
SO_3	0.56	0.26
Cl	1.63	0.28
MnO	n.d.	0.25
H_2O	4.06	21.80
Organic Matter		
	100.37	100.06

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[1] Bradlee, Thomas S., and Mowbray, Louis L. A List of Birds recorded from the Bermudas. With additional notes compiled by Warren F. Eaton. Proc. Boston Soc. Nat. Hist. vol. 39, no. 8, pp. 279–374. 1931. (In press.) Contrib. Ber. Biol. Sta. for Research. no. 164.

[2] P. E. Raymond and H. C. Stetson, A new factor in the transportation and distribution of marine sediments, Sci., vol. 73, pp. 105–106, 1931.

PHYSIOGRAPHY OF BERMUDA.

Older and Younger Bermuda.

In describing the topography of Bermuda, the lack of streams and river valleys has been mentioned. Throughout most of the islands the depressions are either sink-holes, due to the collapse of the roofs of caves, or are residual depressions left by the construction of encompassing sand dunes. Nevertheless, in the low lying northwest portion of Warwick parish, in the islands between Hamilton Harbor and Great Sound, in the Ireland Island group and in Pembroke parish the topography is of lower relief and more rolling than elsewhere. These relations may be well observed from the Gibbs' Hill lighthouse, which is built in a region of pronounced dune topography. A mile to the north, across Riddle's Bay, is the Riddle's Bay golf course, where the subdued, rolling topography is in striking contrast to the dune topography of the coasts. Two miles to the north are the islands of the Sounds, and they likewise have a low, rolling surface. In the vicinity of the city of Hamilton similar conditions prevail. The north shore portrays a typical dune topography, but to the northwest of Pembroke Marsh there are areas with subdued surfaces.

In general the rocks underlying these areas of low, rolling relief belong to the older stratigraphic horizons, particularly the Walsingham and Pembroke eolianites, the Belmont and Devonshire marine limestones, and the Shore Hills-Harrington soil. These regions are obviously places where the younger dunes did not arise to any great extent. They are part of the old degraded nucleus of Bermuda, about which the younger dunes have accumulated. A glance at the topographic map of Bermuda (Plate XIV) shows this older Bermuda, forming a partially submerged core and flanked on both the north and south by the younger dunes.

The direction in which the younger dunes have migrated can be determined from the dip of the cross-bedding. On the windward side of a dune the cross-bedding dips about five to twelve degrees, on the leeward side, about thirty degrees. On the south shore of Bermuda, in Warwick and Southampton parishes, the steeper cross-bedding dips northeasterly, indicating that the dunes were migrating toward the northeast. The dunes on the north shore of Pembroke parish, on the other hand, have migrated toward the south. In other words, the dunes on opposite sides of the islands were moving inland toward each other. The direction in which dunes migrate is not primarily a function of the prevailing winds, but is in part controlled by the source of supply. Obviously sand must migrate away from its source.

The present Bermuda has therefore evolved from a much smaller, older Bermuda by a process of accretion. The older Bermuda is now best displayed in regions such as that around Hamilton, in the Walsingham Cave district, in the islands of the Sound, and in northern Warwick. To this original nucleus, reduced by a long interval of subaerial erosion, have been added the younger dunes, which have migrated inland from either coast. This younger Bermuda extends the length of the north and south shores.

MARINE BENCHES.

The marine benches are another feature of special interest in the physiography of Bermuda. The most conspicuous, stand between eight and twelve feet above sea-level. They are well displayed at Grape Bay, Hungry Bay, McGall's Bay, Spanish Rock, and many other places. Daly[1] has called attention to this feature on many coasts and regards it as due to a very recent, negative, eustatic shift of sea-level. A bench 25 feet above sea-level is well exposed at Mangrove Bay. A sea-cave on the east side of St. David's Head, to the east of the lighthouse, also indicates this level.

CAVES OF BERMUDA.

The caves are most conspicuously developed in the Walsingham district, between Castle Harbor and Harrington Sound. They have recently been described by Swinnerton.[2] "The accessible parts are not over a few hundred yards in length, although it is undoubtedly true that a network of passages underlies much of the cave area. The presence of sea water in the caves, the variously colored deposits stained with orange cave-earth, and fossil soils, contrasted with the many brilliant white stalactites and stalagmites, make the caves extremely picturesque and memorable spots for the tourist to visit and likewise for the geologist to study. . . . Primarily the caves are zones of solution by downward migrating rain-water along steeply dipping and intersecting joints in the compact Walsingham limestone."

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[1] R. A. Daly, Recent world-wide sinking of ocean level. Bull. Geol. Soc. America, vol. 31, p. 112, 1920.

[2] A. C. Swinnerton, The Caves of Bermuda, Bull. Geol. Soc. America, vol. 40, p. 194, 1929.

The bottom of some of the caves is fifty or sixty feet below sea-level, a fact of great significance, indicating shifts in the water table. Stalactites and stalagmites are abundant below sea-level, but unfortunately no figure is given concerning the exact limits. In 1869, while blasting in Hamilton Harbor, stalactites were found at a depth of 30 feet. Swinnerton has concluded from studies in Kentucky and Bermuda that limestone caves are not cut below the water table and that in general caves form most readily just above that level. The Bermuda caves originated, therefore, when the water table was approximately sixty feet lower than now. On a small island of porous limestone, such as Bermuda, the water table is essentially equivalent to sea-level. Hence these caves show that at some time or times in the past, sea-level was sixty feet or more lower than it is now. No evidence has yet been obtained as to the number of such periods of low water.

The depth of water in the caves and on the outer part of the Bermuda platform is about the same. In the caves the water is fifty to sixty feet deep. The Bermuda platform, extending beyond the actual shores, out to distances ranging from one to three miles, lies at a depth of 65 to 75 feet. This correspondence is suggestive of a prolonged period during which sea-level was depressed to about the amount stated.

EVOLUTION OF BERMUDA.

Recapitulation of Significant Facts.

A brief recapitulation of the significant facts is essential before considering the evolution of Bermuda. What facts are important and what are not, is partly a matter of judgment. What appear to be the significant facts are listed and any deduced sequence of events must satisfy these.

(1) Modern Bermuda is built on a volcanic platform. The top of this platform is elliptical in outline, has an area of 250 square miles, and at the site of the well-boring is about 245 feet below sea-level. (2) Resting on this platform is limestone, several hundred feet thick. The rock is dominantly eolianite, with minor quantities of marine limestone, fossil soils, sand-rock, and calcareous ooze. The thickest part of the limestone is in the form of a great ellipse about thirty miles in its greatest dimension and from one to ten miles broad. The southeast sector of this ring projects above water and is modern Bermuda, which has an area of 19¼ square miles. (3) Interbedded

with the eolianites are five fossil soils. (4) The two lower soils, Shore Hills and Harrington, are at many places separated by a marine limestone, the Devonshire, which implies one period of soil formation interrupted by a marine stage. The lower of these two soils, the Shore Hills, is mostly a residual soil, in places four feet thick. For its accumulation 200,000 or more years were necessary. The upper soil, the Harrington, is partly a soil of accretion and probably accumulated with some rapidity. (5) The third soil, the St. George's, also a residual soil, is about two feet thick, and took something like 150,000 years to form. The upper two soils, Signal Hill and McGall's, are very light-colored and in their upper parts are soils of accretion. No very exact estimate can be made of the length of time essential for them to form, but a few thousands or tens of thousands of years would probably suffice. (7) Associated with the Shore Hills and Harrington soils are two thin beds of marine limestone, the Belmont and Devonshire. (8) Numerous changes of level have taken place. Evidence from the caves alone indicates that sea-level has at some time been at least sixty feet lower than at present. The eolian material in the deep boring, extending as it does to a depth of at least 225 feet below sea-level, would indicate that the ocean level stood much lower than the evidence from the caves alone gives us. The marine limestones show that sea-level has stood at least twenty-five feet higher than now. (9) Eolianite-forming periods and soil-forming periods have alternated. (10) Eolianite is not forming today on any notable scale, but a soil is slowly thickening. A corollary is that the fossil soils formed under conditions largely similar to those now existing in Bermuda, the eolianites under conditions very different. (11) The geological age of the rocks is not strictly demonstrable. The volcanics are Tertiary or earlier, the eolianites and soils presumably all Pleistocene.

INADEQUACY OF SUBSIDENCE HYPOTHESIS.

Earlier workers in Bermuda, impressed by the submerged caves and forests, and realizing that the eolianites must have formed when the island was much larger, have invoked the idea of subsidence—that is, that the Bermuda volcano has gradually sunk beneath the ocean waves. Submerged forests and caves were thus explained. The eolianites might also be explained on this hypothesis, having been formed when Bermuda stood relatively high and the land surface was larger. Such an hypothesis was competent to account for the few facts available thirty to fifty years ago.

But the interbedding of fossil soils with the eolianites was then not properly allowed for in the theory of the case. This paper has its main purpose in stressing a quite different picture of the evolution, soon to be more expressly stated. According to this new hypothesis, the eolianites were developed under low-water conditions; the fossil soils under high-water conditions. It will also appear highly probable that not one of the high-water stages represents sinking of Bermuda. On the other hand, each of these stages is correlated with an eustatic (world-wide) shift of sea-level.

PLEISTOCENE OSCILLATIONS OF SEA-LEVEL.

A feasible explanation of such changes of level is available. All geologists agree that during maximum glaciation sea-level must have stood much lower than at present (Fig. 18). During the Tertiary, when the Greenland and Antarctic ice-caps presumably did not exist, sea-level must have stood higher. Moreover, we know that glaciation was multiple. Hence during the Pleistocene there must have been oscillations of sea-level, high-water stages alternating with low-water stages. The ranges of the fluctuation of sea-level are of the right order for this Bermuda case, and equally important, their geological dates are also appropriate. Where so many correspondences exist, the new theory seems much better supported than is any theory involving vertical displacements (repeated up-and-down movements) of Bermuda.

Daly[1] has recently summarized the present status of our knowledge concerning the changes in level due to glaciation and deglaciation. He has also emphasized certain complexities entering into the problem because of distortion of the solid earth geoid due to plastic flow, elastic effects, and the gravitative attraction of the ice caps. Daly (1929, p. 727) concludes that "when the Wisconsin-Würm ice-caps were of full size, the sea-level over most of the earth was not far from eighty meters (262 feet) lower than at present. In late Tertiary time, if the Greenland and Antarctic ice-caps were non-existent, sea-level was presumably 26 to 52 meters (85 to 170 feet) higher than now.

ORIGIN OF FOSSIL SOILS AND EOLIANITES.

Bermuda is now a land of luxuriant vegetation, and a soil similar to the fossil soils is forming. Modern dunes are rare and in nearly all

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[1] R. A. Daly, Swinging Sea Level of the Ice Age, Bull. Geol. Soc. America, vol. 40, pp 721–734, 1929.

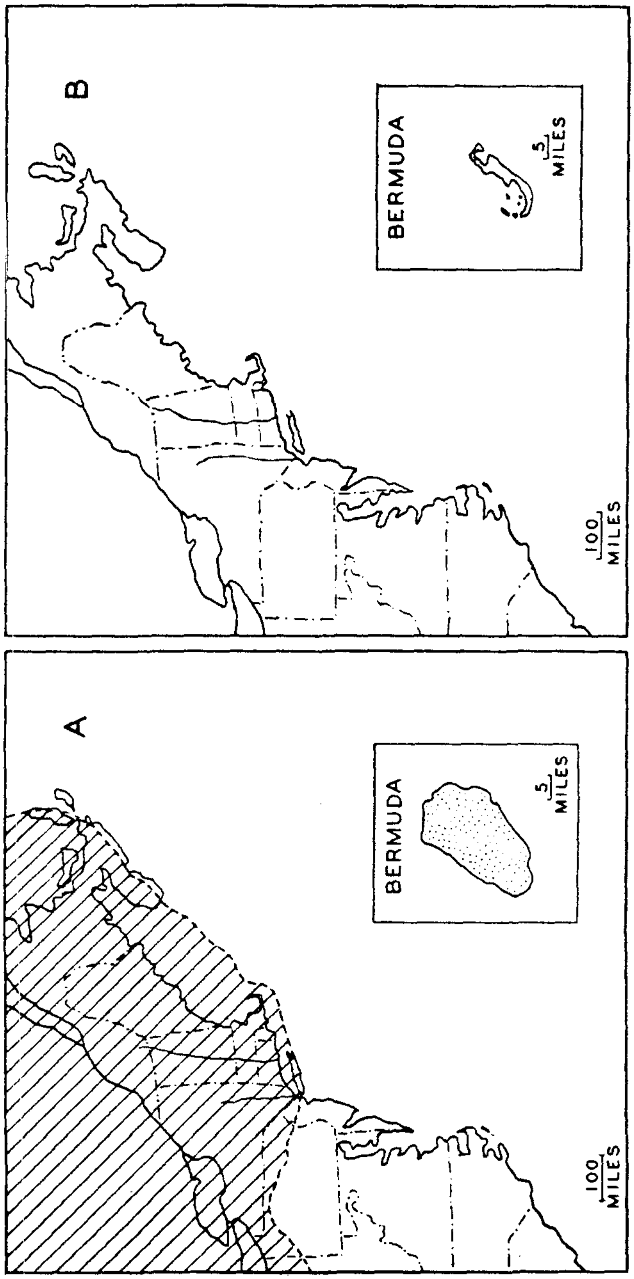
A
BERMUDA
5 MILES
100 MILES
B
BERMUDA
5 MILES
100 MILES

FIGURE 18. A. Bermuda enlarged during glaciations. B. Bermuda reduced during deglaciations. (After Swinnerton.)

cases are the result of deforestation by man. The fossil soils were doubtless formed under conditions not unlike those ruling Bermuda today. The truly residual fossil soils, particularly the Shore Hills, might well be the product of long periods of weathering in the interglacial ages. But the eolianites must have formed under vastly different conditions from those now existing in Bermuda. They are witnesses of a different climate. Can they be correlated with glacial stages of the Pleistocene Ice Age?

The Bermuda *reef* encloses a great lagoon, the floor of which, even in its deepest part, is only about sixty feet deep. The reef is also surrounded by a bench, one to three miles wide and about seventy feet below sea-level. As the water surface fell because of glaciation on the continents, more and more of Bermuda was exposed to the winds. At stages, when sea-level had dropped, great flats covered by marine shells were exposed to the air. Strong winds sweeping across the now dry, but naturally little vegetated flats, would be likely to drift great quantities of broken shells, to pile up as dunes. Even when the continental glaciers had reached a third or half their total size the whole Bermuda platform would stand above the sea. Throughout the tens of thousands of years that the ice-front was moving south, while it was standing at its maximum extent, and again while it was retreating, the Bermuda platform was emergent.

That the Pleistocene vegetative cover hindered this growth of dunes in but slight degree, is a natural assumption. For thousands of years there was no soil on the broad, emerged flat. Hence the establishment of a binding mantle of vegetation on the sands was there practically impossible. Moreover, with the increased cold at Bermuda, many indigenous plants must have been inhibited from vigorous growth. I believe that in certain protected places, like the Walsingham cave district, plant growth went on during eolianite-forming stages.

Furthermore, the Pleistocene winds of Bermuda must have been much stronger than those at present. When the continents were covered with great ice-caps and their accompanying anticyclones, the storm tracks must have been pushed farther south. This meant more wind and also a colder climate.

In summary, the eolianites were certainly formed under climatic and topographic conditions different from those now existing. The eolianite epochs alternated with the soil-forming epochs. The glacial and interglacial stages of the Pleistocene also represent a series of alternating climatic conditions and it seems logical to associate the two. Moreover, if we assume that the eolianites are contemporaneous with the glacial stages, we have a satisfactory explanation of the abundance of available calcareous sand, the high winds, and the dearth of vegetation, all of which are conditions favorable to the formation of sand dunes.

SUBDIVISIONS OF THE PLEISTOCENE.

The Pleistocene deposits of North America have been recently classified by Leighton[1] as shown in the accompanying table. Many attempts have been made to estimate the relative and absolute lengths of the interglacial stages. The most recent, that by Professor George F. Kay, has not been published, but he has very kindly discussed this problem with the writer. His figures for relative lengths of the interglacial stages and sub-stages, are given in a column at the right, post-Wisconsin time being taken as unity.

TABLE IX. CLASSIFICATIONS OF THE PLEISTOCENE.

A. *North America.*

		Relative length
Recent		1
IV. Wisconsin Glacial	Hudsonian sub-stage of advance (late Wisconsin)	
	Unnamed sub-stage of recession	¼
	Quebecan (Early and Middle Wisconsin) sub-stage of advance	
	Peorian sub-stage of recession	¼
	Manitoban (Iowan) sub-stage of advance	
Sangamon interglacial		6
III. Illinoian Glacial		
Yarmouth interglacial		12
II. Kansan Glacial		
Aftonian interglacial		8
I. Nebraskan Glacial		

B. *The Alps.*

Penck and Brückner[2] give the following classification for the Alps:

IV. Würm Glacial (three-fold)
Riss-Würm interglacial
III. Riss Glacial
Mindel-Riss interglacial
II. Mindel Glacial
Günz-Mindel interglacial
I. Günz Glacial

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[1] M. W. Leighton, The Peorian loess and the classification of the glacial drift sheets of the Mississippi Valley, Jour. Geol., vol. 39, pp. 45–53, 1931.

[2] Penck und Brückner: Die Alpen in Eiszeitalter, 1909.

Both classifications agree in recognizing four glacial stages separated by three interglacial stages, the last glacial stage being three-fold, though the American classification is here the more detailed. Kay[1] has suggested calling the last glacial stage the Eldoran, to be subdivided into four sub-stages of advance, the Iowan, Early, Middle and Late Wisconsin, and three sub-stages of recession. Leighton's three-fold classification is the one adopted in this paper. There is no reason to suppose that the continental ice-cap completely melted away at any time during the Wisconsin, but there were obviously great changes in the amount of ice. These oscillations undoubtedly caused shifts in sea-level and were themselves the product of climatic changes which may well have been felt in Bermuda.

In our discussion, Kay stated that the amount of leaching of calcium carbonate during the various interglacial stages has been as follows: Aftonian, 20 feet; Yarmouth, 30 feet; Sangamon, 12 feet; post-Wisconsin, 2½ feet. Assuming that post-glacial time is 25,000 years[2] and that the rapidity of leaching does not vary with depth—which is undoubtedly not strictly correct—we would conclude that the Aftonian lasted 200,000 years, the Yarmouth 300,000, the Sangamon 120,000. For the sub-stages of recession of the Wisconsin (Eldoran) no direct evidence is available, but from indirect evidence they must have been very short, perhaps 5,000 years or less each. The relative length of the various interglacial stages of recession and sub-stages of advance, as thus determined, is entered in Table IX, A.

CORRELATION OF BERMUDA WITH NORTH AMERICA.

The attempt to correlate the Pleistocene deposits of Bermuda with those of North America is exceedingly hazardous. The fossil snails are certainly of no value in this problem, and the marine fossils of the Belmont and Devonshire marine formations likewise apparently give no help. Only one method is available. If the reader accepts the contention of the writer that the eolianites represent glacial stages and the fossil soils interglacial stages, the relative and absolute length of the interglacial stages in Bermuda may be compared with those of North America. The fundamental assumption is made, of course, that whatever major changes took place in the ice-caps had their effect in Bermuda. In the accompanying table (X) the stratigraphic data for Bermuda are placed in one column. In the next column is indicated the nature of the conditions under which the formation was deposited. The approximate length of time represented by each soil—that is, interglacial stage—is indicated and, finally, the relative length of the interglacial stages, taking post-Southampton time as unity.

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[1] Personal communication.

[2] George F. Kay, The relative ages of the Iowa and Wisconsin drift sheets, Am. Jour. Sci., vol. 21, pp. 158–172, 1931.

TABLE X. PLEISTOCENE IN BERMUDA.

Formation Name	*Conditions*	*Length in yrs.*	*Relative length*
Present soil	*Post-glacial*	25,000	1
Southampton eolianite	Glacial		
McGall's soil	*Interglacial*	10,000 ?	⅖?
Somerset eolianite	Glacial		
Signal Hill soil	*Interglacial*	10,000 ?	⅖?
Warwick eolianite	Glacial		
St. George's soil	*Interglacial*	120,000	6
Pembroke eolianite	Glacial		
Harrington soil Devonshire limestone Shore Hills soil Belmont limestone	*Interglacial*	250,000	10
Walsingham eolianite	Glacial		

This table may be compared with that of North America in several ways. One method is to correlate the longest interglacial age of which we have any record in Bermuda with the longest in North America—that is, to correlate the Belmont-Shore Hills-Devonshire-Harrington group with the Yarmouth. This method leads to the correlation shown in the following table. A second method is to compare the relative lengths of the interglacial ages. This leads to a similar result. The correlation of Bermuda with North America would then be as follows:

TABLE XI. CORRELATION OF BERMUDA WITH NORTH AMERICA.

	North America	*Bermuda*
Wisconsin	Hudsonian sub-stage of advance	Southampton eolianite
	Unnamed sub-stage of recession	*McGall's soil*
	Quebecan sub-stage of advance	Somerset eolianite
	Peorian sub-stage of recession	*Signal Hill soil*
	Manitoban glacial	Warwick eolianite
Sangamon interglacial		*St. George's soil*

Illinoian glacial	Pembroke eolianite
Yarmouth interglacial	*Harrington soil* *Devonshire limestone* *Shore Hills soil* *Belmont limestone*
Kansan glacial	Walsingham eolianite
Aftonian interglacial	*Part of Walsingham?*
Nebraskan glacial	Part of Walsingham?

The physiography of Bermuda supplies a very weighty argument supporting the correlation given above. The dune topography of the Warwick, Somerset, and Southampton eolianites is well preserved both on the north and south shores. The dune topography of the Pembroke and Walsingham eolianites, on the other hand, has been completely destroyed, as in the vicinity of Hamilton and Riddle's Bay. It is well known that in continental North America post-Wisconsin time has been so short that the topography of glacial deposits of Wisconsin age is well preserved. The original topography of Illinoian deposits, however, is more or less completely destroyed. If these relations hold in Bermuda, we would conclude that the eolianites with well-preserved dune topography—the Warwick, Somerset, and Southampton—are Wisconsin. Eolianites on which the dune topography is lacking, such as the Pembroke and Walsingham, are by the same criteria pre-Wisconsin.

Not only does the correlation satisfy the requirements imposed by the relative lengths of the various interglacial and sub-glacial stages, but it also indicates that there was unusually high water during the Yarmouth. This is in accord with evidence elsewhere. The fauna and flora found in the Yarmouth interglacial deposits near Toronto[1] indicate a stage warmer than the present. As a corollary we might conclude that the Greenland and Antarctic ice-caps were not as large as now and that therefore sea-level stood higher.

The conclusion has been reached that the Walsingham eolianite represents the Kansan glacial stage and possibly the Aftonian interglacial and the Nebraskan glacial. The problem has not yet been solved, but there are several possible solutions. (1) The Aftonian soil may be present but not exposed. (2) Bermuda may not have been high enough during the Aftonian to project above sea-level. (3) Aftonian soil may have been formed but later destroyed. Of these suggestions, the second seems the most reasonable. The failure to identify definitely Aftonian and Nebraskan deposits by no means invalidates the correlation, any more than the failure to identify Pre-Cambrian rocks in the Mississippi Valley invalidates the identification of the Paleozoic rocks.

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[1] A. P. Coleman, Ice Ages, Recent and Ancient.

CUTTING OF THE BERMUDA PLATFORM.

In the earlier pages it has been assumed that the volcanic platform of Bermuda is elliptical in ground-plan, with an area of about two hundred square miles, and essentially flat, the depth being about 245 feet below sea-level. On this platform repose the eolian limestones of Bermuda. Is the surface of this platform a product of Tertiary marine planation or has it been greatly reduced by the Pleistocene waves? Daly[1] has shown that during Tertiary time, when the Antarctic and Greenland ice-caps were presumably non-existent, the strand-line in the vicinity of such oceanic islands as Bermuda was from 35 to 40 meters (115 to 131 feet) higher than now. At the time of maximum glaciation sea-level was about 80 meters (262 feet) lower than at present.

According to Barrell[2] the profile of equilibrium on a stormy sea shelf is about 300 feet below sea-level. The Bermuda platform, 245 feet below present sea-level, would be from 360 to 375 feet below the surface of the Tertiary ocean and 20 feet above the lowest Pleistocene sea-level. Obviously these figures are only approximations, but they readily accord with the hypothesis that the Bermuda platform was base-leveled during the Tertiary. If the volcano had become extinct in early Tertiary time, by the end of the Tertiary it might readily have been reduced to the base-level of marine erosion, which must be at least three hundred feet below sea-level on an isolated and exposed oceanic island. Marine planation during the maximum extent of the Pleistocene ice sheets would have reduced Bermuda to a much lower platform than that now existing. We are thus forced to conclude that Tertiary planation was fully competent to form the Bermuda platform, but that Pleistocene planation would have formed a lower surface.

There is an even more weighty argument in favor of Tertiary planation. Eolianites and fossil soils from many early glacial and interglacial stages have been preserved in Bermuda. If Pleistocene marine planation were competent to actively erode and reduce the volcanic platform during each time of maximum ice advance, the superficial eolianites and soils would be the first to be destroyed. The very preservation of Kansan and Illinoian eolianites and Yarmouth and Sangamon soils during successive glacial and interglacial stages shows that the Pleistocene waves were not even competent to destroy all the superficial deposits, much less to erode the volcanic platform. Once again we are driven to the conclusion that the Bermuda volcanic platform was reduced to its present depth in pre-Pleistocene time.

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[1] R. A. Daly, Swinging sea level of the Ice Age, Bull. Geol. Soc. America, vol. 40, pp. 721–734, 1929.

[2] Joseph Barrell, The piedmont terraces of the northern Appalachians, Am. Jour. Sci., vol. 49, p. 354, 1920.

Observations on the Location of the Bermuda Islands on the South and Southeast Edge of the Volcanic Platform.

It may be noted that the main mass of the Bermuda Islands is located on the south and southeast sides of the Bermuda platform. The reason for this arrangement has not heretofore been satisfactorily explained. To leave this question without any attempt at an explanation, would not be satisfactory, especially as I believe it is possible to throw some light on it, even though what now seems like light may eventually prove to be inapplicable to the problem in hand.

A glance at the map of Bermuda will show that the islands as a group have a northeast trend. The directions of the dips in the dunes prove that for the greater part of the south and southwest parts of the islands the wind came from the southwest. It is true that on the "north shore" of the main Bermuda island the dunes show that the winds came from the north or northeast, and that on the easternmost part of the island they came from the northeast. Still, it remains true that for the greater part the winds came from the southwest, and not directly on shore from the south. In other words the winds which formed the dunes did not come directly on shore, but at an angle of about 45°. If the winds were from the southwest during glacial stages the greatest amount of land building would be at the southwest, as it is in Southampton and Somerset. The main island has been built onto, somewhat like shingles on a roof in a southerly and southwesterly direction. As one goes northeast from Simmons Beach it is seen that the dune formations are progressively thinner in that direction.

Cressey[1] has shown, in his study of the dunes of Lake Michigan, that the oldest dunes are several miles back from the present lake front and the youngest dunes are at the present shoreline of the lake. As the lake shrank in size the dune building followed the receding waters. It is believed that the same thing happened at Bermuda, and the structure of the islands themselves lends support to this belief.

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[1] George B. Cressey; The Indiana Sand Dunes and Shore Line of Lake Michigan Basin, The Geog. Soc. of Chicago, Bull. No. 8, May 1928.

In other words, I believe that the optimum location for dune building, which would mean the largest dunes, would be at the south and southwest parts of the islands, especially the main island. North and northwest winds might have been too strong for the preservation of dunes made on the northern rim of the platform. That dunes were there is proved by the remnants at North Rock. As the waters receded, after an interglacial stage, the dunes would follow them towards the new shoreline at a lower level. Here dunes would be built, but only to be destroyed as the waters returned with an onset of glacial conditions on the continents. The place best fitted for the preservation of dunes, and also the greatest height of dunes would be just where they are located today, not at the north or northwest or northeast—where strong anticyclonic winds from the continental ice sheets would destroy any dunes—but at the south or southwest, where that which had been built could have an opportunity to persist through the battle between the winds and dunes. That they were preserved at all, is probably due to the very quick cementation which takes place when this eolian shell sand is exposed to the atmosphere. Although not hard, the dunes would be able to offer more resistance to the attack of the waves than would be the case with quartz sand, where cementation could not be affected. By the time the waves reached the centre of the platform the cementation would have reached such a stage that resistance would be considerable and thus the life of the Bermuda group would be extended for a much longer period than with quartz sand.

Turning now to the Bahama Islands, what do we see? Exactly the same conditions. Take, for example, a few of the more northern islands of this group; Long Island, Cat Island, Eleuthera Island, Nassau, Great Abaco Island, Andros Island and Bahama Island, are located distinctly on the south and southeast of their platforms. The northern parts of the platforms are mostly free from islands. Why? The answer has already been given, I believe. The northern winds were too strong for the dunes of eolianite at the Bahamas, as well as at Bermuda, for such relatively soft rocks to endure the strength of the waves. The waves from the south and southwest were strong enough for building good-sized dunes, but not so strong as to destroy them during the glacial stages, or late glacial stage when the waters came back from the ice, released by melting. There is an opportunity here to base a study of the winds of the Pleistocene on the location, size and structure of the dunes.

The evidence that the north winds were severe at Bermuda is indicated by the large dune, Prospect Hill, built parallel to north winds, which separates Pembroke and Devonshire marshes. Here we have a dune of Southampton or Somerset age, built from the north, which cuts across the direction in which were built the other dunes along the north shore. Very strong winds are known to build dunes parallel to their direction, as in this case. In no other part of Bermuda have I found a case similar to this. If there were other cases of the kind, it is to be presumed that such dunes were built on the northern part of the platform and not on the southern part and that they have been destroyed.

It is known that dunes protect the country farther away from the water than themselves. This is evidenced at Provincetown, on Cape Cod. Here, as one enters the town, is a pond about a mile east of the town. The pond has not yet filled but has been protected by the dunes on its northeast border, and has been in its present state as long as anyone can remember. Such must have been the case also at Bermuda. Parts were protected, and during long intervals, especially as the islands grew in height, the protection afforded by the high dunes would insure them from burial. As has already been noted, this has been especially true in the Walsingham cave district and about Hamilton and among the inner islands.

Not only in Bermuda, therefore, but in the Bahama group as well, the islands are located where the winds were most favorable for their growth and later protection. That this location was at the south and southwest, and not at the north and northwest, is indicated by the actual locations of the islands of both groups, the Bermudas and the Bahamas.

SUMMARY OF GEOLOGIC HISTORY OF BERMUDA.

The Bermuda volcanoes were erupted during the Tertiary, or earlier. By the end of the Tertiary they had been reduced to submarine platforms. The most northeasterly and largest of these was elliptical in plan, essentially flat, and several hundred feet below sea-level. With the coming of the Pleistocene great quantities of water were subtracted from the oceans to form the continental ice-sheets.

When the ice was at its maximum extent the strand-line fell as much as 260 feet below modern sea-level. While the ice-cap grew, large parts of the Bermuda banks, covered by mollusc shells and unprotected by vegetation, were exposed to the sweep of the winds and the dried sands were piled up in great dunes. The higher winds in the vicinity of the Bermuda of the Ice Age and the inevitable changes in flora were important factors favorable for the construction of dunes. When the sea rose at the close of each glacial stage, the source of supply for the dunes was buried beneath the ocean waves, the winds became less violent, and a permanent flora anchored the dunes. A long period of slow decay began, during which red and brown soils accumulated. Such conditions lasted for tens of thousands of years. But with the advent of a new glacial stage, conditions were once more favorable to the formation of dunes. Thus dune- and soil-forming conditions alternated during the Pleistocene.

The early Pleistocene history, as elsewhere in the world, is obscure. Eolianites formed during the Nebraskan glacial stage and a soil of Aftonian age have not yet been recognized in Bermuda. They may be present beneath the zone of observation, or Aftonian Bermuda may not have projected above the sea. During the Kansan stage the Walsingham eolianite was deposited as great dunes, which even today extend scores of feet above the level of the sea. With the advent of the long Yarmouth interglacial age, sea-level rose even higher than it is today and the Belmont marine limestone was deposited. Meantime on the emerged part of Bermuda the Shore Hills soil began to accumulate, and, with a slight fall of sea-level, weathering of the Belmont marine limestone began. Sea-level again rose and the Devonshire marine limestone was laid down. During the transition between the Yarmouth and the Illinoian, a new soil, the Harrington, was formed, but it was largely a soil of accretion, due to the increased activity of the wind. When the Illinoian glacial stage was in full swing the Pembroke eolianite was deposited. The Older Bermuda, the nucleus of the Younger Bermuda, had now formed. During the Sangamon interglacial stage, which was considerably shorter than the preceding Yarmouth interval of warmth, that part of Bermuda already formed was reduced to a low rolling topography. The St. George's soil formed during this stage is by no means as thick as that formed during the Yarmouth. The Sangamon was succeeded by the three-fold Wisconsin stage, which opened with the Manitoban (Iowan) glaciation, during which the Warwick eolianite was deposited. In the brief Peorian sub-stage of recession, in the course of which the continental ice-sheets probably did not completely melt away, the Signal Hill soil was formed. Although a red soil washed into local pockets by the rain accumulated to some extent, the deposit of this time was dominantly a fine-grained, wind-blown material, most of which was presumably deposited when conditions were changing between glacial and interglacial. With the next great forward push of the ice, the Quebecan (Early and Middle Wisconsin), dune-forming conditions once more held sway on Bermuda and the Somerset eolianite was deposited. It was largely confined to the north and south shores, as in the case of the Warwick eolianite. Again the ice-caps retreated to the north, and another short unnamed sub-stage of recession ensued. The McGall's soil, dominantly a product of accretion, was rapidly deposited at this time. While the ice-caps were making the last great drive toward the south during the Hudsonian (Late Wisconsin), the Southampton eolianite accumulated on the coasts of Bermuda. With the final melting of the ice-caps, the water returned to the ocean, caused the sea-level to rise, and the Bermuda plateau, now provided with much new dry land, completed by the dunes of the Wisconsin stage, assumed its present appearance. The shorelines have been driven back somewhat by erosion. A twelve-foot drop in sea-level is recorded by an emerged bench, cut in geologically very recent time, perhaps during the time of written history.

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KEY TO SOIL LOCALITIES ON MAP

Soil No.	*Location*	*Soil No.*	*Location*	*Soil No.*	*Location*	*Soil No.*	*Location*	*Soil No.*	*Location*
1.	S–2	29.	H–4	56.	H–5	84.	K–L–5	112.	G–6
2.	S–2	30.	H–4	57.	I–5–6	85.	K–L–5	113.	G–6
3.	S–2	31.	H–5	58.	I–6	86.	K–L–5	114.	G–7
4.	S–2	32.	H–I–5	59.	H–5	87.	P–5	115.	G–H–7
5.	S–2	33.	F–7	60.	T–5	88.	D–6	116.	G–7
6.	S–2	34.	E–6	61.	M–4	89.	Q–R–2	117.	H–7
7.	S–2	35.	E–6	62.	L–5	90.	G–1	118.	M–5
8.	S–2	36.	F–6	63.	L–5	91.	L–6	119.	M–5
9.	R–2	37.	G–6	64.	L–6	92.	B–C–2	120.	M–4
10.	R–2	38.	F–7	65.	L–6	93.	I–5	121.	F–1
11.	R–2	39.	E–7	66.	L–6	94.	I–5	122.	P–4–5
12.	R–3	40.	D–E–6	67.	L–6	95.	I–5	123.	P–5
13.	M–6	41.	I–6	68.	L–6	96.	I–5	124.	G–6
14.	M–6	42.	C–D–6	69.	L–6	97.	D–1	125.	G–6
15.	N–6	43.	C–6	70.	L–6	98.	I–5	126.	G–5
16.	N–6	44.	C–6	71.	L–6	99.	D–1	127.	C–4
17.	N–6	45.	H–6	72.	L–6	100.	E–7	128.	E–5
18.	M–N–4	46.	D–2	73.	L–6	101.	E–6	129.	C–6
19.	N–3	47.	C–D–2	74.	L–6	102.	G–6	130.	I–7
20.	O–3	48.	C–D–2	75.	L–6	103.	G–7	131.	P–Q–6
21.	Q–6	49.	D–6	76.	L–M–6	104.	F–G–5	132.	P–4–5
22.	Q–6	50a.	H–7	77.	L–M–6	105.	G–5–6	133.	J–7
23.	T–3	50b.	H–7	78.	L–M–6	106.	G–7		
24.	R–3	51.	D–7	79.	K–6	107.	O–6		
25.	I–6	52.	H–5	80.	K–6	108.	G–7		
26.	J–6	53.	H–5	81.	K–6	109.	G–7		
27.	G–H–4	54.	H–5	82.	P–5	110.	S–2		
28.	H–4	55.	H–5	83.	K–L–5	111.	G–6		

Parishes

DE. = Devonshire
HA. = Hamilton
PA. = Paget
PE. = Pembroke
SM. = Smith's
SO. = Southampton
STG. = St. George's
—— = Sandy's
WA. = Warwick

PLATE 1.

1. A view of Bermuda looking east from the top of Gibbs' Hill lighthouse, about 370 feet above sea-level. Gibbs' Hill is about 246 feet above sea-level. The older Bermuda is seen on the left in the lowland about Riddles' Bay golf course and farther east. On the right are the much younger hills of eolianite deposited on the eroded Pembroke surface, and according to the theory advanced in this paper, dating from the Warwick stage of dune formation, and this in turn is correlated with the Iowan stage of the Pleistocene. At the low-water glacial stages, dunes to the north and south are thought to have formed and protected this region from these later dunes. These later southern dunes never extended farther north than the northern base of Gibbs' Hill.

2. View looking north from the top of Gibbs' Hill lighthouse, showing old lands of the Pembroke-St. George's stages, and eroded to its present form since that time. The islands in the foreground owe their shapes to erosion. The erosion must have taken place as much during the glacial stages of lowered ocean level as during the interglacial high water phases. The finding, by divers, of a cave with stalactites and red earth, in the ship channel in Hamilton harbour, at a depth of 6 fathoms, would necessitate a lowered sea-level or an uplifted Bermuda.

1

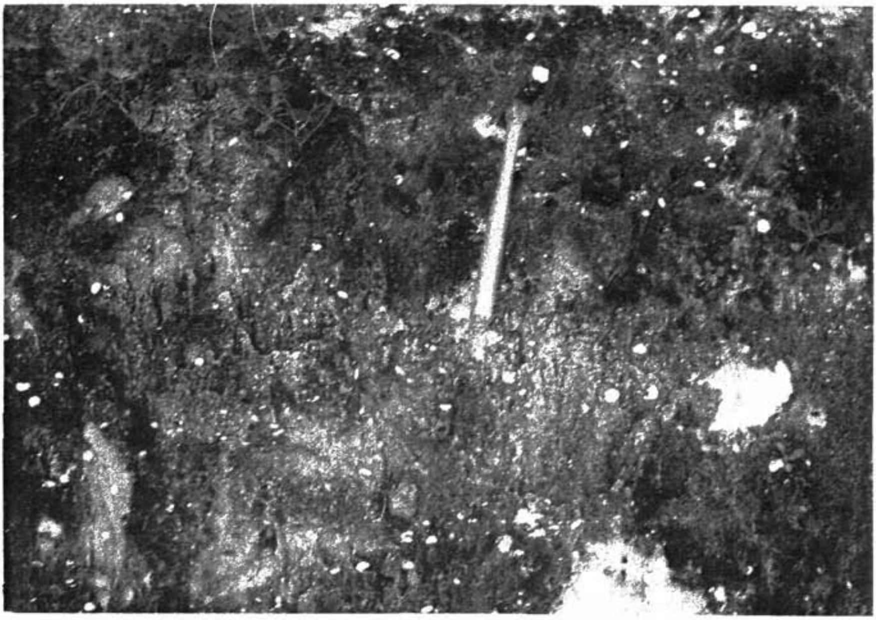
2

PLATE 2.

1. A typical close-up view of a fossil soil. The white objects are mainly *Poecilozonites bermudensis*, the most common extinct land snail in Bermuda. This photograph was taken in 1924 at Ruth's point, on St. David's island. The soil is probably of Harrington age. Map locality 60.

2. The deeply weathered fossil soil on Elliott Street, Hamilton. It is thought to be the St. George's soil resting on Pembroke eolianite. The solution pocket close to the writer is 5 feet deep below the soil proper. This soil is of a deep reddish-brown color. Fossil land snails were not found in this soil. A small piece of carbonized wood was found. In the most thoroughly weathered fossil soils the shells are usually absent or so much decomposed as to yield only fragments. Note the manner in which this soil merges with the more recent soil on the left.

Locality 98.



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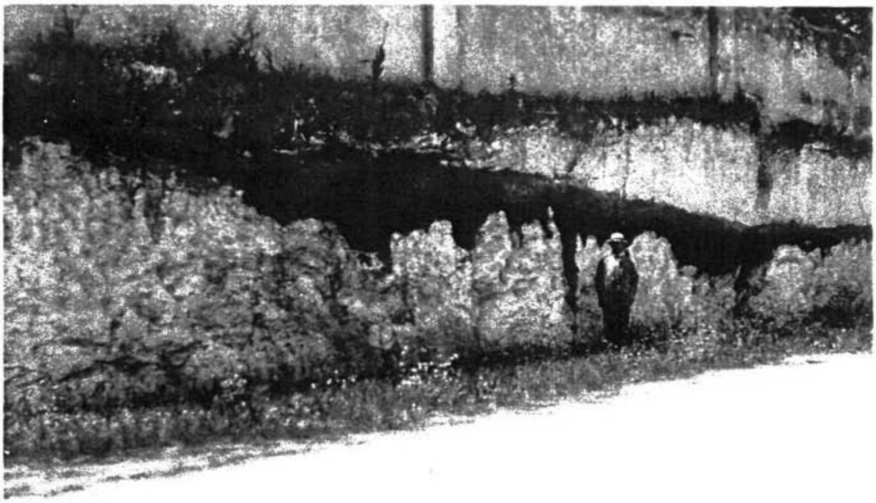


2

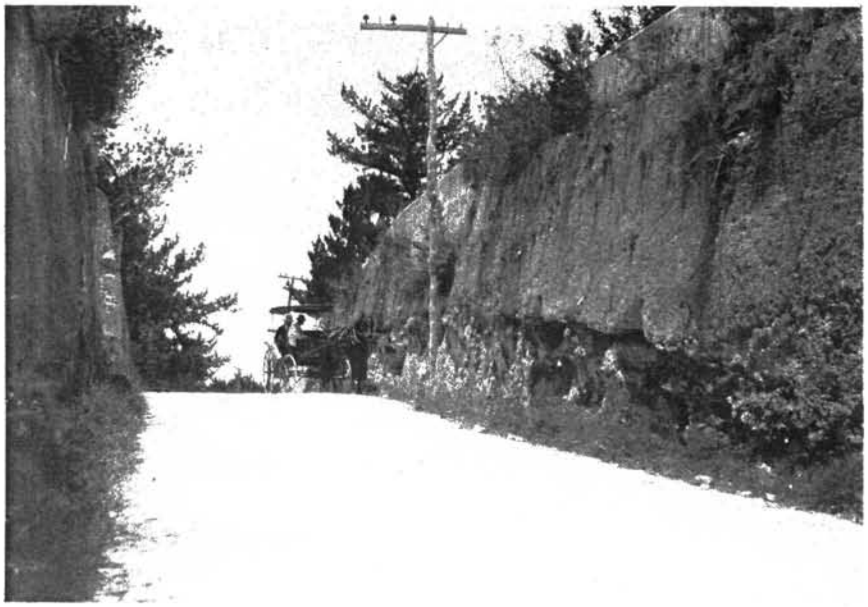
PLATE 3.

Professor T. H. Clark shown close to the pocket mentioned in the description of Pl. 2, No. 2.

Locality 98.

PLATE 4.

1. and 2. A fossil soil seen in the new cut on Elliott Street, Hamilton. These two views are taken looking north. On the south side of the street this soil is similarly well exposed. The age of this soil is problematical, but is probably St. George's. If it is of later date, it is the only case of so young a soil intercalated between eolianites in the Hamilton region. It is evident that considerable erosion had taken place before the overlying eolianite was deposited. Note the soil stains from above, resembling pockets, at the top of both views. By placing the upper view to the right of the lower a complete section of the overlying dune can be had.



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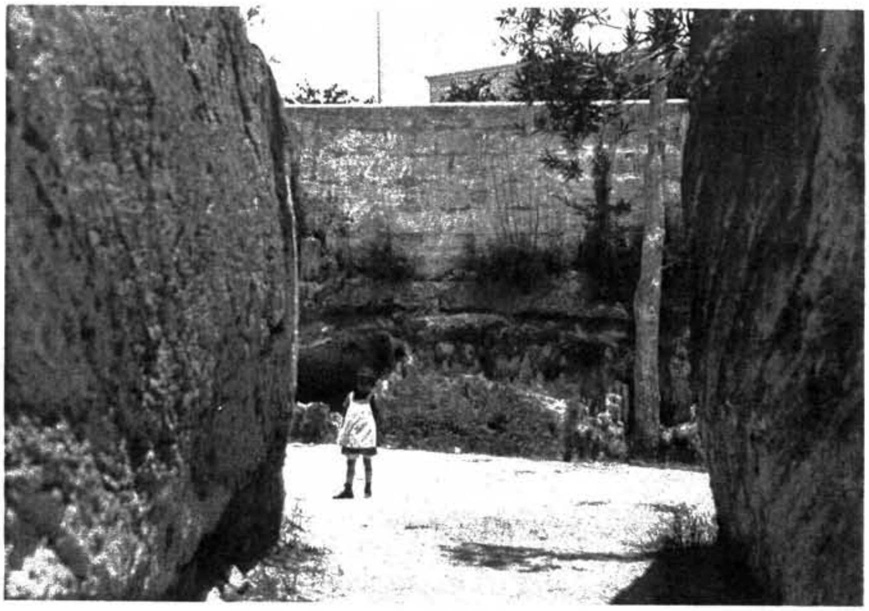


2

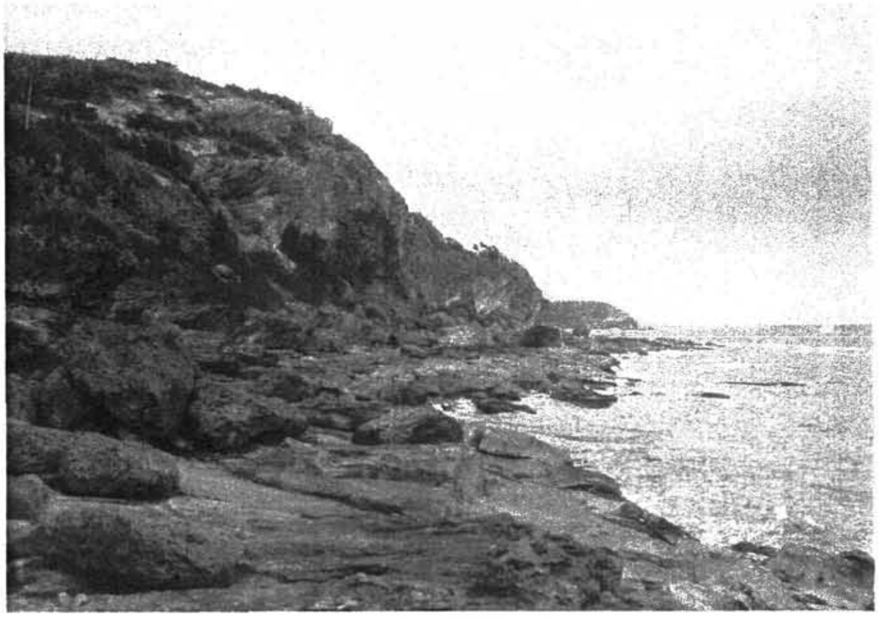
PLATE 7.

1. View looking east along the marine bench at McGall's bay. In the cliff appear 4 fossil soils, noted first by Clark in 1926. On the bench itself the writer found patches of the oldest soil, the Shore Hills, thus making 5 soils in all for this locality. With the exception of the uppermost soil, the McGall's soil, all these soils show evidence of considerable weathering, and the hill as it now stands is the resultant of eolianite and soil formations, covering the interval between the Shore Hills soil stage and the present time, or in terms of the Pleistocene, and according to the conclusions reached in this paper, the time between the Yarmouth and the present.

Localities 66–72.

2. View taken under the cliff at McGall's bay to show Harrington soil overlain by the windward beds of a dune of Pembroke eolianite, dipping about 12° to the east. Wind scour has eroded the soil beneath as can be noted on the right. The texture and fresh condition of this soil in many places would indicate that it is a soil of accretion to some extent, and that the lowering of the water level at the end of the Devonshire had exposed beds of fine material off shore to wind action. The indurated deep red soil, supposed to be the Shore Hills soil, was found under this Harrington soil in small patches not far from this spot. Another indurated red soil was found about 300 yds. west of this locality with the usual "palmetto holes" and this is presumably the same indurated red soil found at this locality. The limestone on which the red soil is found is without much doubt the Belmont marine limestone.

Locality 66.



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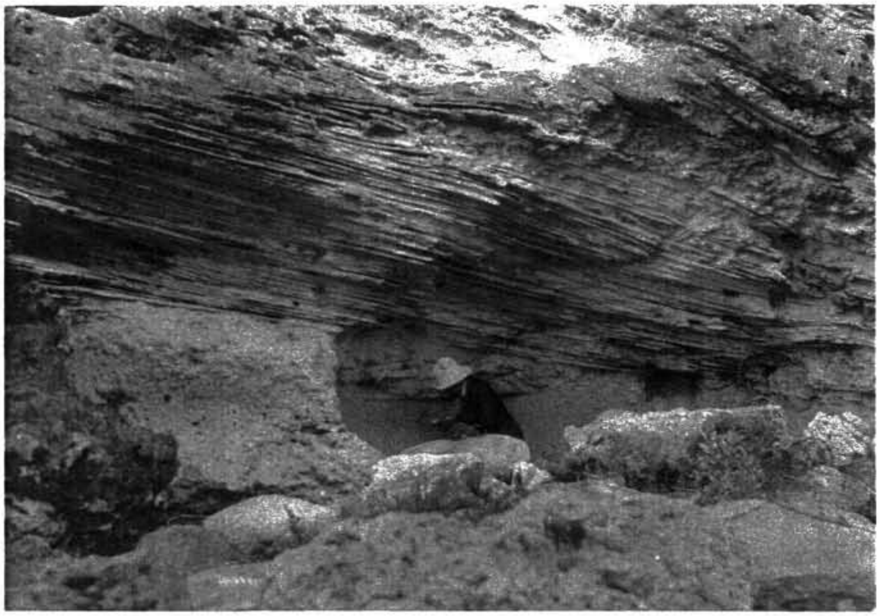


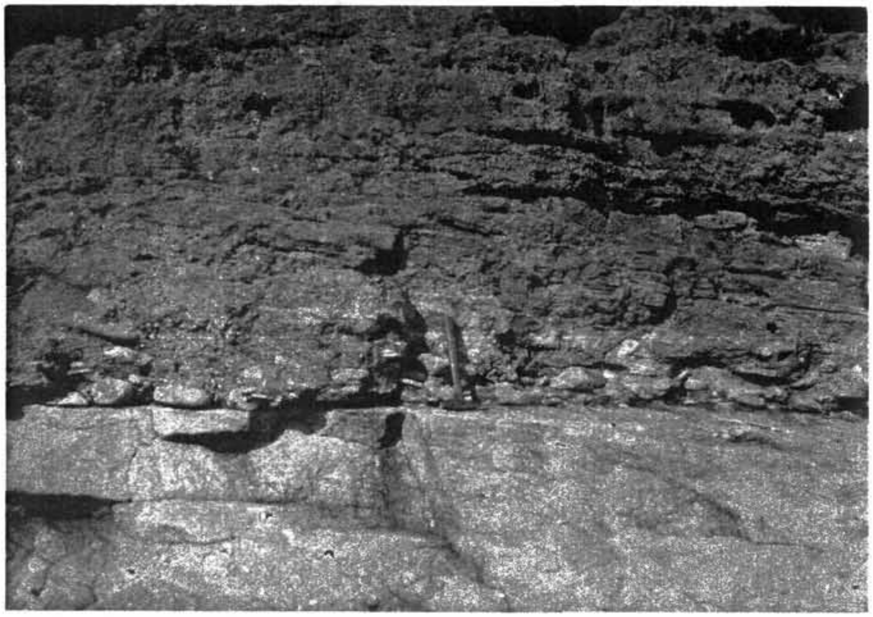
2

PLATE 8.

1. This view, taken at Devonshire bay, shows what the writer believes to be the Belmont marine formation, overlain by the Devonshire beach conglomerate and sandstone. The Shore Hills soil is absent and was probably washed away when the conglomerate was formed. At many places along the south shore, what Verrill calls "palmetto stumps" can be seen penetrating the Belmont formation. In the opinion of the writer and also of Professor Schuchert these "palmetto stumps" are nothing but solution holes caused in the development of soils, like those found in plate 2. At the Grape bay locality 146 this condition can be seen better than anywhere else. No one who has seen these solution holes in the Shore Hills soil at Grape bay can doubt the similar origin of all the "palmetto stumps." The Grape bay locality was not discovered until the plates were finished.

Locality 80.

2. A view of the Gurnet rocks, taken from a motor boat south of Castle Harbor. The Gurnet rocks, like north rock, are remnants of a former dune. The Gurnet rocks lie close to the southern edge of the lagoon, just as the north rock lies at the northern edge, and have a depth of water about them of from 2 to 3 fathoms. Suddenly the water deepens to 10 or more fathoms southward. At the close of the Wisconsin stage of the Pleistocene, it is evident that dunes lay close to the edge of the present outer reefs and irregularly over the present lagoon floor. With the return of the waters from the melting icecaps these dunes were partly submerged and almost entirely destroyed by wave action. The Gurnet rocks and the North rock remain as witnesses of this Wisconsin dune stage.



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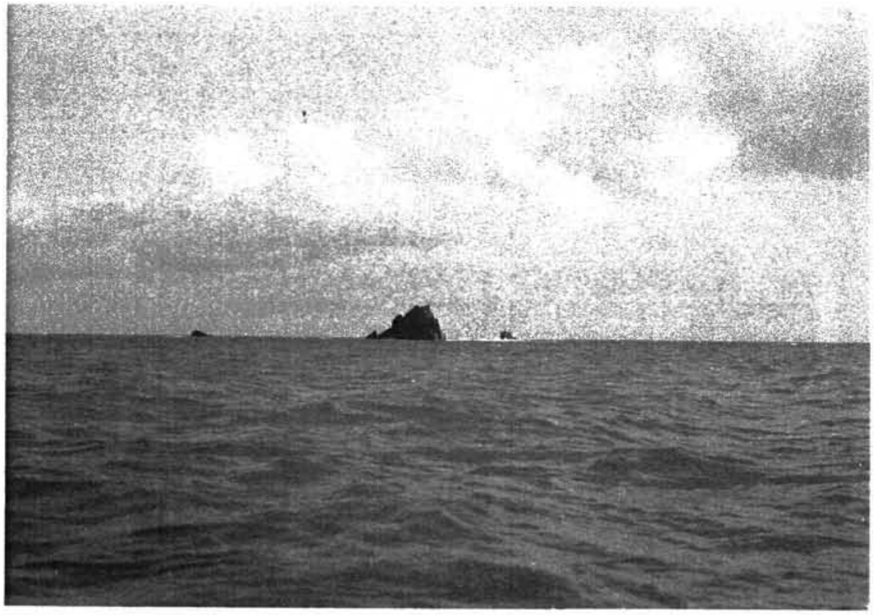


2

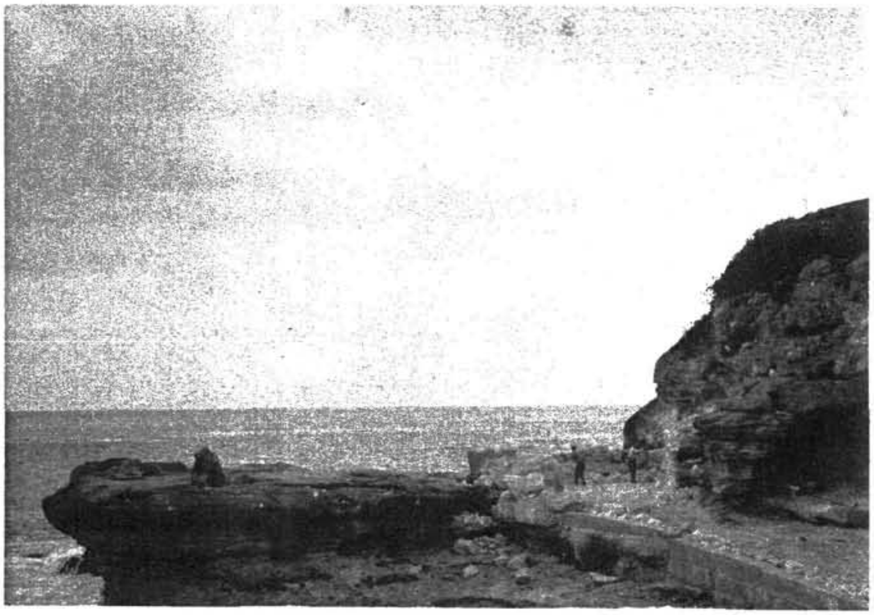
PLATE 9.

1. View taken at Hungry bay. The marine bench at the left is about 12 feet above sea-level. At the right under the cliff is an excavation made by laborers.

Locality 143.

2. A view of what might be called a fossil sea-cave just above horizontal marine sandstone. The cave is just above the excavation shown in No. 1 on this plate, but is not visible in No. 1. Loose blocks cemented together may be seen in the upper left part of this picture. At the lower left corner a pail may be seen which gives an idea of the scale. The floor of this cave has an altitude above sea-level of about 220 feet. Several feet above the cave is a fossil soil, the Harrington, which can be seen in the cliff as it rises from a point almost at sea-level.

Locality 133.



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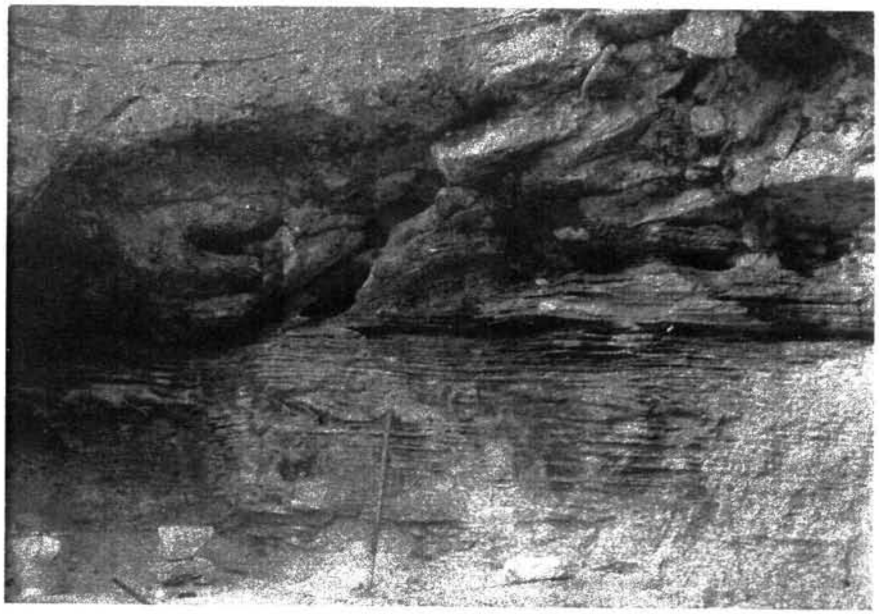

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PLATE 10.

1. A view of the typical marine corrosion in the Belmont marine sandstone at Pink Beach. At the upper left is a corner of a stack of eolianite, the remnant of a former dune, which formed, and then later when the sea-level attained a height of about 15 feet higher than the present sea-level, the dune of eolianite was almost obliterated. Fossil soils are found about 100 feet north of the stack and the lowest rests on the Belmont formation. This lowest soil is absent between the stack and the Belmont, nor do the upper ones appear in the stack. It is evident that the eolianite of the stack is younger than the fossil soils, perhaps of Wisconsin age.

Locality 107.

2. An example of deep corrosion at the shore of Castle Harbour back of Walsingham House. The limestone seen is not in loose blocks but is part of the solid ledge. The much deeper corrosion in No. 2, as compared with that shown in No. 1, of this plate, would indicate that the uncovering of the Belmont marine sandstone in No. 1, is an event of no great antiquity geologically. The 15-foot recession of sea-level, advocated by Daly, may throw light on the history of the stack at Pink Beach and also on the deep weathering at Walsingham House, and may give a means of determining comparative time estimates.

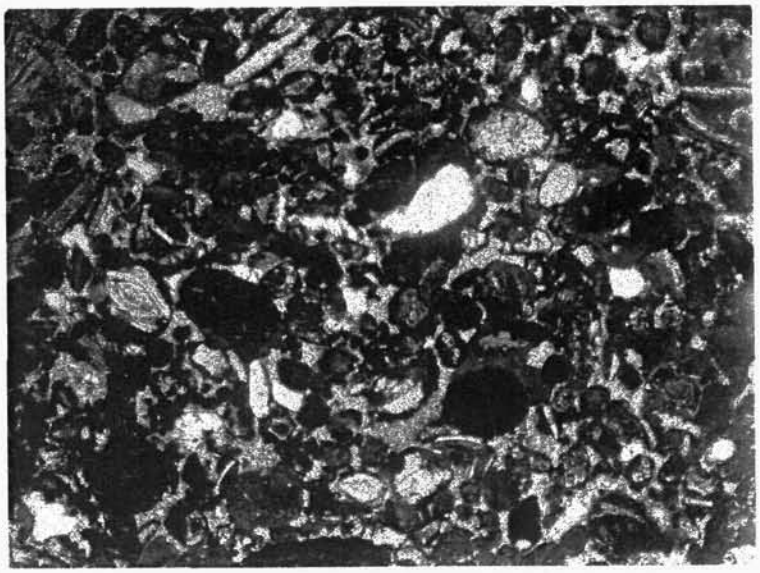
1

2

PLATE 11.

1. A microphotograph of the reddish rock underlying the Shore Hills soil at Stokes point St .George's. This rock is full of much weathered eolianite grains and foraminifera. The Shore Hills soil lies on this rock and in solution pockets. The soil is overlain by the Devonshire marine formation. The reddish rock may be a phase of the Belmont formation, but if so it must have been exposed to the air to account for the much weathered grains of which it is composed. Marine forms and brecciated fragments of the red rock are mixed together at the bottom of the soil, while at the top, nothing but land mollusca are found. The problem which Stoke's Point presents has not been conclusively solved at this writing. Magnification in microphotograph about 20 diam. Microphotograph by E. C. Jeffrey. Locality 24.

2. A microphotograph of the Shore Hills soil at Hinson island, Hamilton harbor. The conditions in this soil are not so very different from those in the red rock of No. 1, on this plate. Weathering of the foraminifera and eolianite grains is marked. Magnification about 20 diam. Microphotograph by E. C. Jeffrey. Locality 105.



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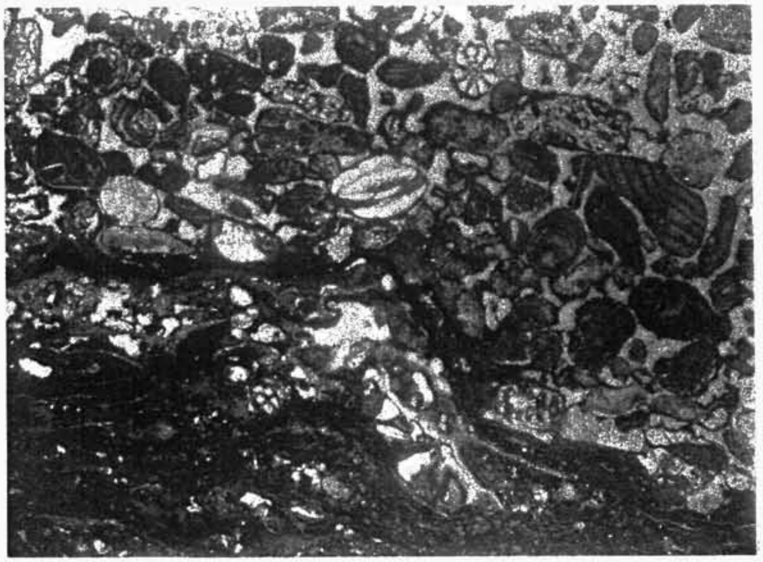
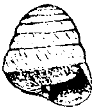


2

PLATE 12.

Some of the more common land mollusca found in the fossil soils of Bermuda. After Verrill.

1. Poecilozonites dalli, × 1½.
2. Zonitoides bristoli, × 20.
3. Poecilozonites cupula, × 1.
4–5. Strobilops hubbardi, × 4.
6–7. Carychium bermudensis, Gul. × 13. Profile and front views.
8. Vertigo numellata, Gul. × 13.
9. Vertigo marki, Gul. × 13.
10–11. Euconulus turbinatus, Gul. × 8.
12–13. Thysanophora hypolepta, Pilsbury, much enlarged.
14–15. Poecilozonites circumfirmatus, Verrill, much enlarged.
16–17. Zonitoides minusculus, Binney enlarged.
18. Poecilozonites nelsoni, × 1.
19. Poecilozonites callosus, × 1.



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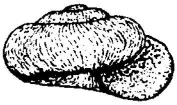

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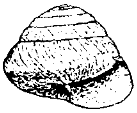

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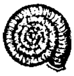

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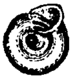

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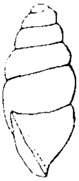

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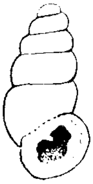

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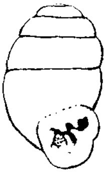

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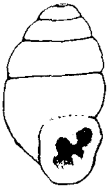

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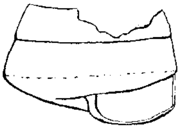

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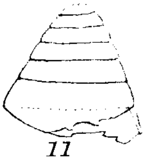

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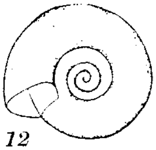

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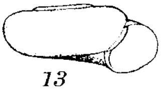

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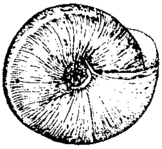

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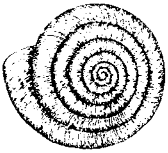

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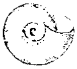

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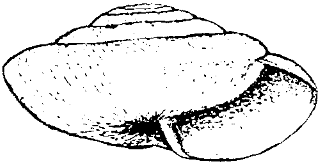

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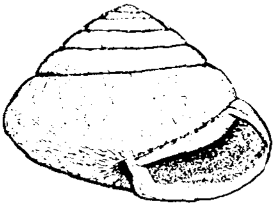

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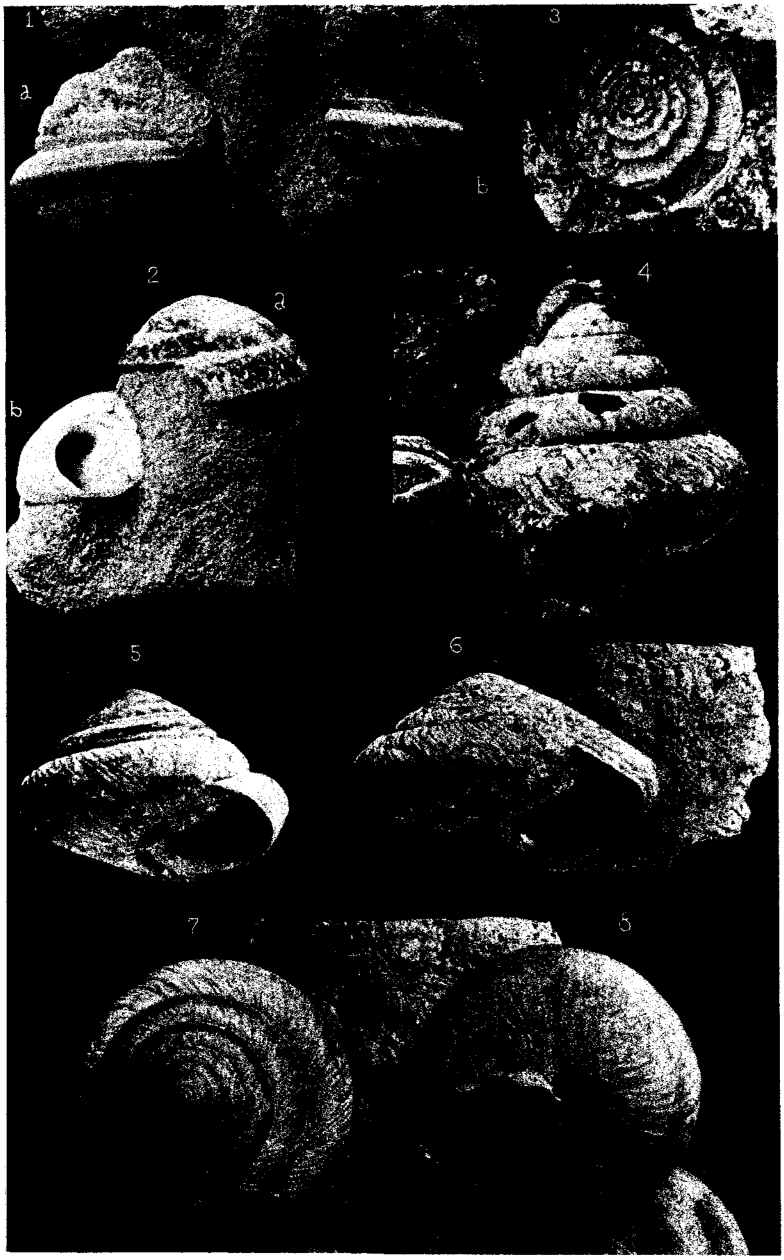
PLATE 13.

Some of the more important fossil land snails found in the fossil soils of Bermuda. *Poecilozonites nelsoni* is the most important fossil as an horizon marker so far discovered. It has never been found above the Harrington-Pembroke formations. It has been found by the writer in the Walsingham formation at the entrance of Crystal Cave, just inside the gate, and has been mentioned by Verrill as occurring in the Walsingham formation. Small varieties of *Poecilozonites bermudensis* are found more commonly in the Shore Hills and Harrington soils and the larger varieties, especially var. *zonatus*, are found in large quantities in the St. Georges, Signal Hill and McGall's soils. As the paleontology of the soils is only partly done, little more can be said at present.

1–2. Poecilozonites bermudensis, var. zonatus, Nat. size.
3. Poecilozonites reinianus var. antiquus, × 2½
4. Poecilozonites nelsoni, var. concoides, Nat. size.
5–6. Poecilozonites nelsoni, var. callosus. Nat. size.
7–8. Poecilozonites nelsoni, var. nelsoni, Nat. size.
Photographed from Verrill.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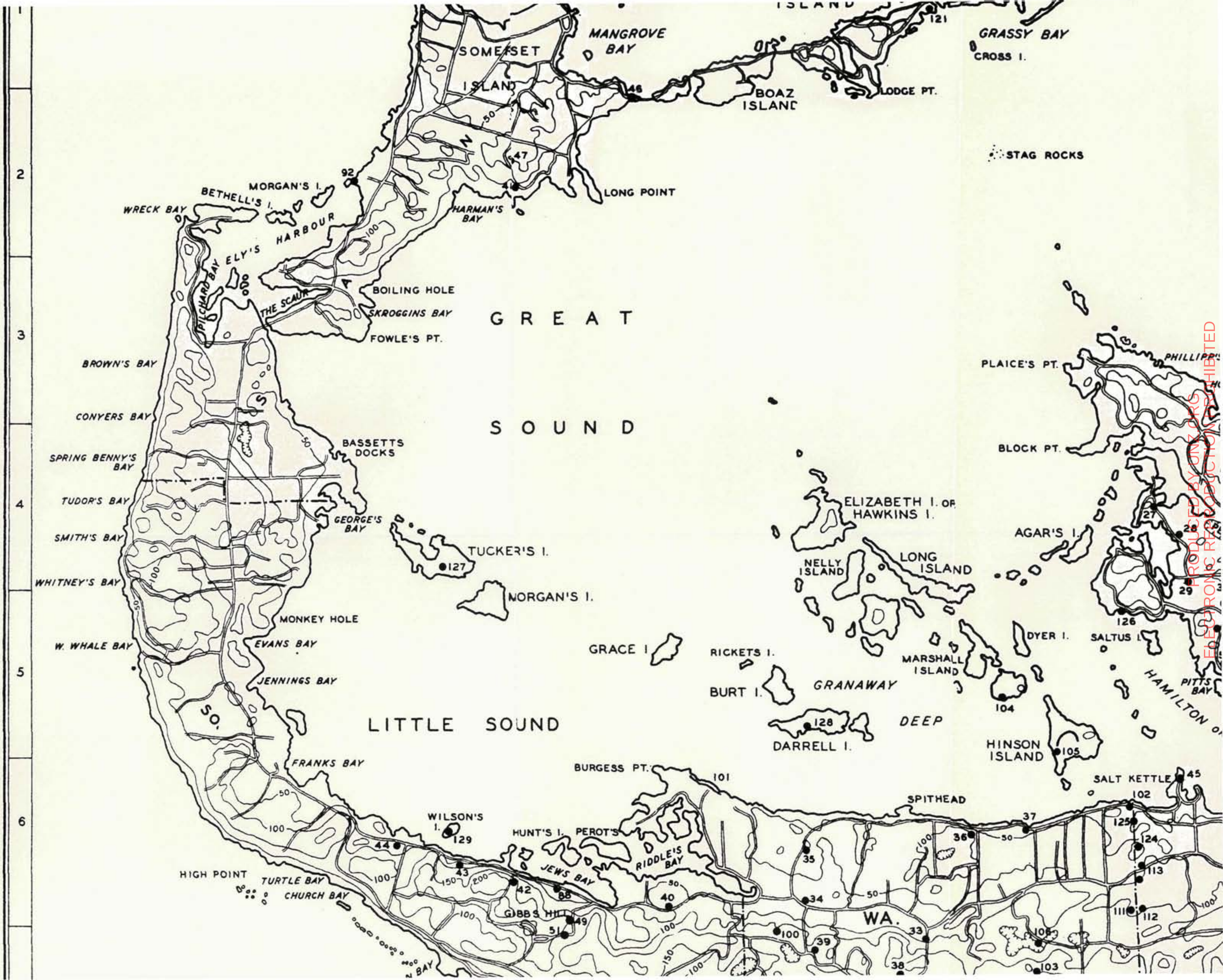
1
2
3
4
5
6
ISLAND
MANGROVE BAY
GRASSY BAY
CROSS I.
SOMERSET ISLAND
BOAZ ISLAND
LODGE PT.
121
46
STAG ROCKS
47
92
41
MORGAN'S I.
BETHELL'S I.
WRECK BAY
HARBOUR
ELY'S
PILCHARD BAY
LONG POINT
HARMAN'S BAY
BOILING HOLE
SKROGGINS BAY
THE SCAUR
FOWLE'S PT.
GREAT SOUND
BROWN'S BAY
CONYERS BAY
SPRING BENNY'S BAY
TUDOR'S BAY
SMITH'S BAY
WHITNEY'S BAY
W. WHALE BAY
BASSETTS DOCKS
GEORGE'S BAY
TUCKER'S I.
127
MORGAN'S I.
MONKEY HOLE
EVANS BAY
JENNINGS BAY
FRANKS BAY
LITTLE SOUND
SANDYS
SO.
GRACE I.
RICKETS I.
BURT I.
DARRELL I.
128
GRANAWAY DEEP
ELIZABETH I. OF HAWKINS I.
NELLY ISLAND
LONG ISLAND
MARSHALL ISLAND
104
AGAR'S I.
PLAICE'S PT.
PHILLIP'S
BLOCK PT.
DYER I.
SALTUS I.
126
27
28
29
HAMILTON
PITTS BAY
HINSON ISLAND
105
SALT KETTLE
45
102
BURGESS PT.
101
SPITHEAD
37
36
125
124
113
111
112
WILSON'S I.
129
HUNT'S I.
PEROT'S
RIDDLE'S BAY
JEWS BAY
44
43
42
88
49
51
GIBBS HILL
40
35
34
33
100
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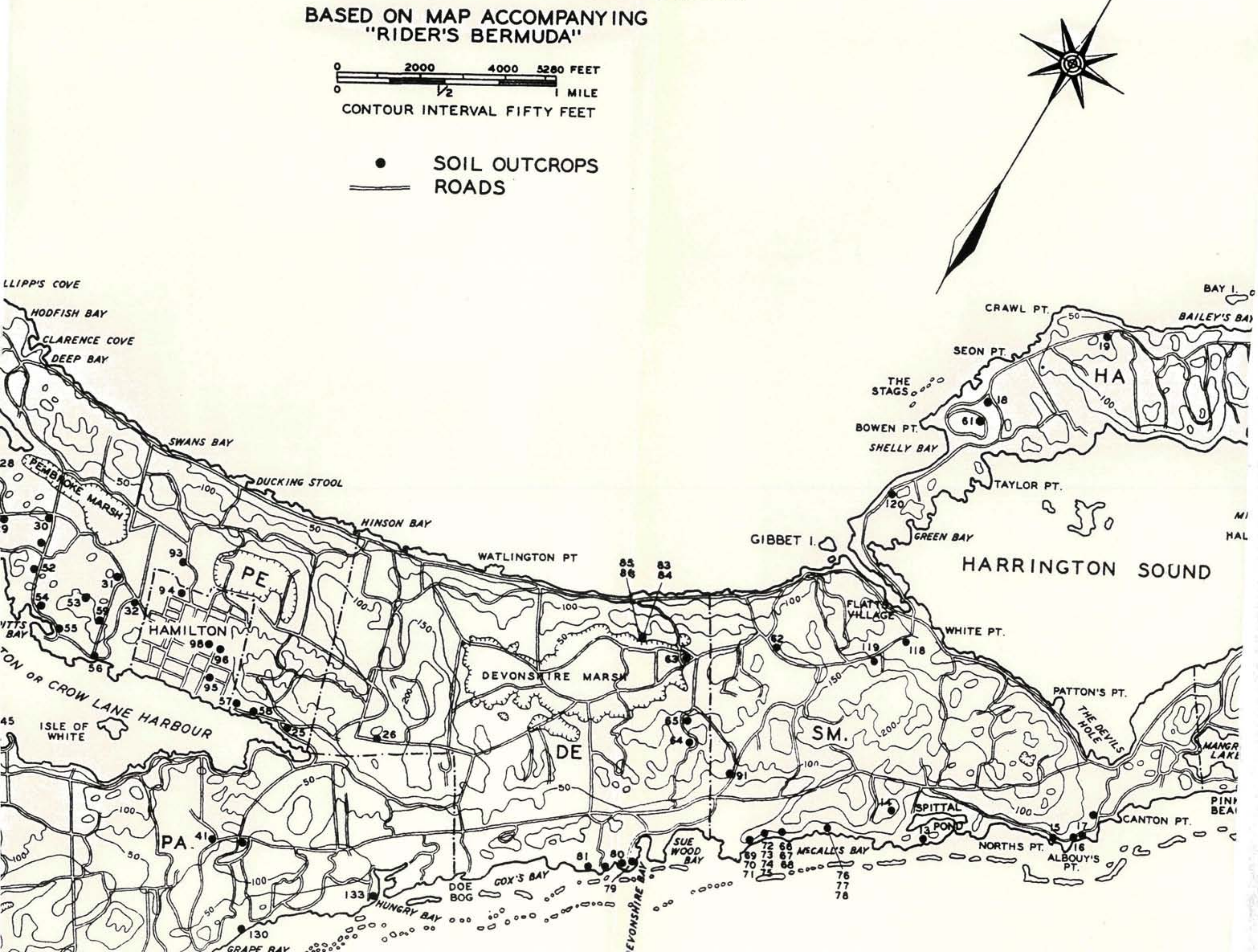
38
106
103
WA.
HIGH POINT
TURTLE BAY
CHURCH BAY
BAY
50
100
150
200

FOSSIL SOIL LOCALITIES
OF THE
THE BERMUDA ISLANDS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ROADS
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HODFISH BAY
CLARENCE COVE
DEEP BAY
SWANS BAY
PEMBROKE MARSH
DUCKING STOOL
HINSON BAY
WATLINGTON PT
HAMILTON
PE
DEVONSHIRE MARSH
DE
GIBBET I.
FLATTS VILLAGE
WHITE PT.
HARRINGTON SOUND
SM.
PATTON'S PT.
THE DEVILS HOLE
SPITTAL POND
NORTHS PT.
ALBOUY'S PT.
CANTON PT.
PINK BEA
MANGR LAKE
CRAWL PT.
BAY I.
BAILEY'S BAY
SEON PT.
THE STAGS
BOWEN PT.
SHELLY BAY
TAYLOR PT.
GREEN BAY
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MI
WTTS BAY
ON OR CROW LANE HARBOUR
ISLE OF WHITE
PA.
HUNGRY BAY
GRAPE BAY
DOE BOG
COX'S BAY
DEVONSHIRE BAY
SUE WOOD BAY
MCALL'S BAY
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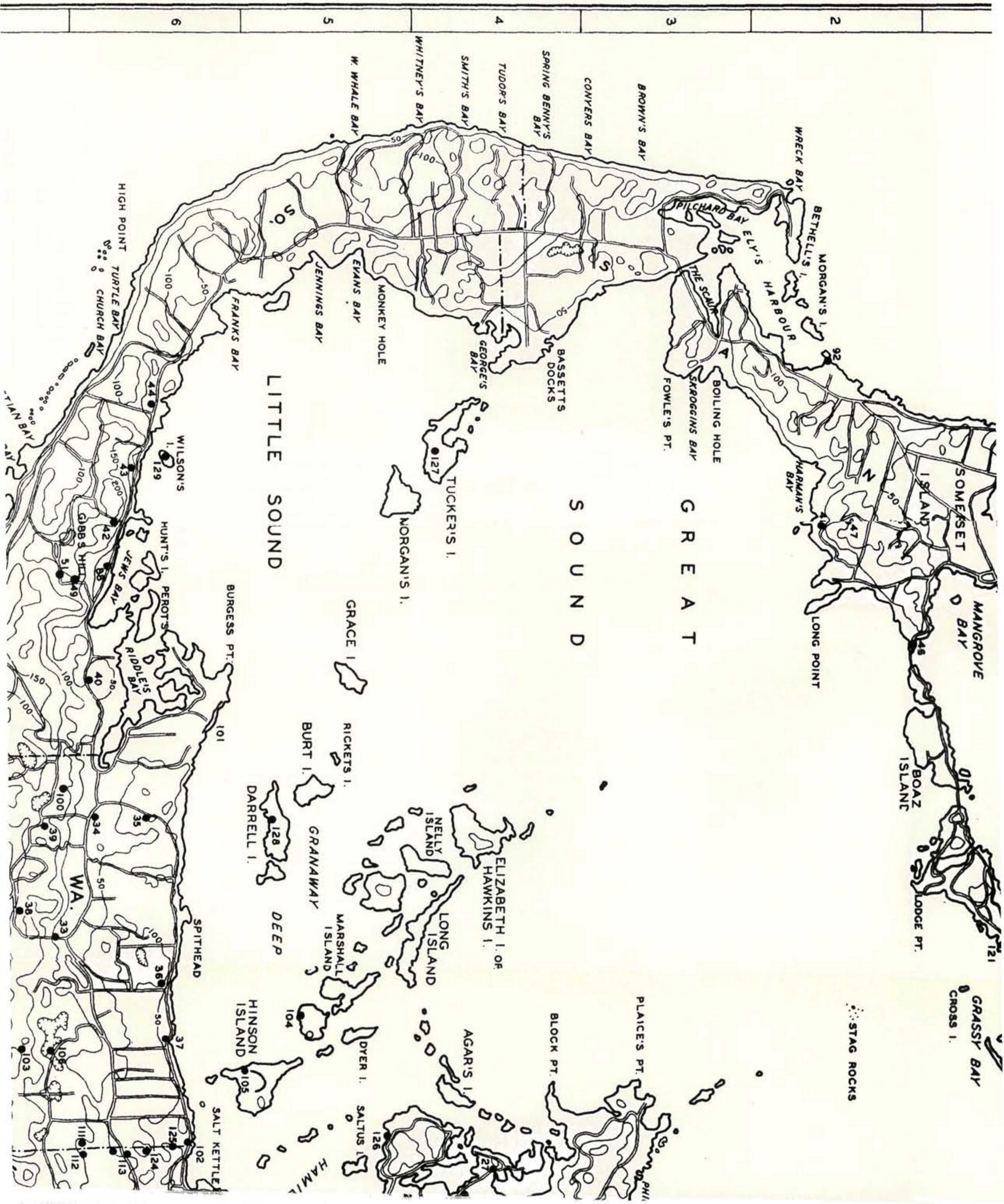
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W. WHALE BAY
WHITNEY'S BAY
SMITH'S BAY
TUDOR'S BAY
SPRING BENNY'S BAY
CONYERS BAY
BROWN'S BAY
WRECK BAY
BETHELL'S I.
MORGAN'S I.
PILCHARD BAY
ELY'S HARBOUR
THE SCAUR
SOMERSET ISLAND
MANGROVE BAY
BOAZ ISLAND
LODGE PT.
GRASSY BAY
CROSS I.
STAG ROCKS
HIGH POINT
TURTLE BAY
CHURCH BAY
FRANKS BAY
JENNINGS BAY
EVANS BAY
MONKEY HOLE
GEORGE'S BAY
BASSETTS DOCKS
FOWLE'S PT.
SKROGGINS BAY
BOILING HOLE
HARMAN'S BAY
LONG POINT
GREAT SOUND
LITTLE SOUND
TUCKER'S I.
MORGAN'S I.
GRACE I.
WILSON'S I.
HUNT'S I.
PEROT'S
JEWS BAY
GIBBS HILL
RIDDLE'S BAY
BURGESS PT.
RICKETS I.
BURT I.
DARRELL I.
GRANAWAY DEEP
NELLY ISLAND
ELIZABETH I. OF HAWKINS I.
LONG ISLAND
MARSHALL ISLAND
HINSON ISLAND
DYER I.
SALTUS I.
AGAR'S I.
BLOCK PT.
PLAICE'S PT.
SPITHEAD
SALT KETTLE
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TABLE V. PETROLOGY OF EOLIANITES AND SOILS.

	Color	Percentage of insoluble coarser than silt	Quartz	Perovskite	Pyroxene	Magnetite	Orthoclase	Volcanic glass	Titanite	Zircon	Rutile	Brown garnet	Red garnet	Biotite	Hornblende	Hydrargillite (?)	Miscellaneous
uth of oyal	White	0.0004	36	18	34		6	tr				6					
	White	0.002	48			21										31	
	White	0.0006	13	18	9		10	50?									
	White	0.0004	45	20	15	20											
Bay	White	0.001	90	tr		5		5									
Bay	Flesh	0.001	30	35			30	?			?	5					
Bay	White	0.0005	20				60					20					
Bay	Gray	0.001	20	5	5		60		tr				10				Chlorite
Bay	White	0.0005	30	40	30												
Haycock Valsingham	Red	1.25	40	51	tr	2	tr		2	tr	1	1	tr				Tourmaline 1% Epidote 2%
s Vale	Red	0.5	19	38	2	24	tr		7	tr	tr	6			tr		Muscovite tr Epidote 4%
Rock	Gray	0.0003	10	80												10	
ire Bay	White	0.001	50		50												
ire Bay	Pink	0.01	9	25	40	5	18		2						1		
ire Bay	Red	0.005	20	25	10	45											
ire Bay	Pink	0.001		33	67												
ach	Red	0.1	20	20	20	20		tr	tr			20					
Bay	White	0.005	50	25	25												
Bay	White	0.01	30	tr	40				30	tr							
Bay	White	0.002	30	30	30				10				tr				
ay	Red	0.05	20	20	50	9	tr		1				tr				
s Beach	Gray	0.001	50		50												
rge's	Pink	0.001		25	50	25											
rge's	Pink	0.005	20	40	40												
rge's	Red	0.13	10	25	5	55			5								
rge's	Pink	0.05	10	40	50			tr									Chlorite tr
rge's	Deep-red	0.2	10	28	46	10	1		5								
rge's	Light Red	0.03	3	10	70	14	3										
rge's	White	0.003		25	75												Epidote tr
rge's	Red	4.3	17	67	3	3	tr		4	tr			2				Epidote 2% Corundum (?) t Leucoxene 2%
rge's	Pink	0.01	25	25	25	25											
Street, on	Red		87	2	4			5		1			1				Spinel tr
on	Red	0.08	20	36	31		5	4	2			2					
on	Mottled Pink	0.001	55	tr		tr		45					?				
ie	Pink	0.1	77	18		3				1							Cyanite ½% Staurolite ½
t	Red	0.1	100														
yal	Pink	0.001	58	22		10	10			tr							
Island	Pink	0.003	90		10												
's Bay	Pink	0.02	40	6	6	4	6	38									
's Bay	Pale Pink	0.001	20	3	70		tr		tr	tr		?	tr	3	tr		Plagioclase tr
's Bay	Red	0.2	3	30	42	10	12		3								
's Bay	Pink	0.01	50	10	15												Plagioclase 25%
's Bay	Pink	0.02	12	2	2		83						1	tr			
's Bay	White	0.002	25	25	25		5										Plagioclase 20%
's Bay	White	0.001	60					40									
's Bay	Pink	0.02	7	46	23				1			20	2				Epidote 1%
's Bay	Deep-red	0.05	40	40	10		5					5					

Proceedings of the American Academy of Arts and Sciences.

Vol. 66. No. 12.—November, 1931.

ON THE APSIDAL MOTION OF A FAINT ECLIPSING BINARY.

By Harlow Shapley.

ON THE APSIDAL MOTION OF A FAINT ECLIPSING BINARY.

BY HARLOW SHAPLEY.

(Read by title October 14, 1931)

THE speed of revolution of the line of apsides in the relative orbit of a close pair of stars that are flattened and elongated by rotation and tidal distortion is significant in the study of the internal structure of the stars. Every effort should be made, therefore, to increase our meager body of observational material on apsidal motion. Until recently, Y Cygni, studied of late by Dugan, Waterfield, Redman, and Russell,[1] was the only system for which we had clear evidence of an advancing periastron; but new photographic work at Harvard on the eclipsing binary RU Monocerotis, in verification of the visual observations of Dubiago and Martinoff at Kasan,[2] show not only that this close double star moves in an orbit of abnormally high eccentricity, but that the line of apsides is slowly revolving.

If t_1 and t_2 are the times of primary and secondary minima, P the orbital period, and e the eccentricity of the orbit, the longitude of periastron, ω, measured from the ascending node, is given by

$$\cos \omega = \frac{\pi[2(t_2-t_1)-P]}{2Pe},$$

to a sufficient approximation for binaries whose orbits are so little inclined to the line of sight that deep eclipses occur. The period of revolution of periastron is measured by the periodicity of the difference $d = t_2 - t_1$, that is, by the cyclic shift of the secondary minimum with respect to the primary.

The accurate determination of $e \sin \omega$, and the consequent evaluation of e and ω separately, from the width and shape of the light curve at the two minima, is a difficult matter for which observations rarely suffice; and certainly the variations in ω cannot be measured satisfactorily from the character of the light curves during eclipses. The apsidal motion can best be determined from the relatively simple study of the times of eclipse for those systems whose light curves show two distinct minima in each revolution; but the determination of the dimensions and ellipticities of the component stars from analyses of the light curves also becomes necessary before the apsidal motion can be used to estimate the distribution of matter inside the stars.[3] For RU Monocerotis the data are as yet insufficient either to yield complete orbital elements, or to determine the period of revolution of the line of apsides. All that we can do at present is to show that the highly elongated orbit is revolving and to suggest that close and prolonged observation of this object is important in an attractive problem of celestial mechanics as well as in the astrophysics of stellar interiors.

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[1] For bibliography see Russell and Dugan, M. N., **91,** 212, 1930.

[2] A. N., **236,** 225, 1929.

RU Monocerotis is a tenth magnitude eclipsing variable in the position $6^h\ 50^m\ 56^s$, $-7°\ 25'.6$ (1932), spectral class A0, and period 3.58 days. It was discovered as an Algol variable many years ago by L. Ceraski. It has been observed by Blažko, Lehnert, Graff, Wendell, and Dubiago and Martinoff.[4] Wendell's early and rather fragmentary observations at Harvard show the astonishing and quite unparalleled situation that

$$d = t_2 - t_1 = 2.9(t_3 - t_2),$$

but it remained for Dubiago and Martinoff to call attention to the abnormality after finding a similarly high eccentricity indicated by their own visual observations. For nearly all other eclipsing binaries the ratio of intervals lies between 0.8 and 1.2.

Since $$e \geqq \frac{\pi}{2P}\left|\left(d - \frac{P}{2}\right)\right|,$$

the large displacement of the secondary minimum from the midpoint between successive primaries means that the orbit of RU Monocerotis has an eccentricity not less than 0.375, which is three times that of the orbit of the average binary of short period, and two and a half times that of the orbit of Y Cygni—perhaps the most eccentric among known eclipsing stars.

The observations on the Harvard plates, a total of 543 in number, extend through the interval from 1898 to 1931. Two estimates of magnitude were made on all plates during maximum light; four estimates on all plates during minima. The comparison stars are as follows:

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[3] Russell, M. N., **88,** 641, 1928.

[4] For bibliography see A. N., **236,** 225, 1929; cf. also Nizhny-Novgorod, Veränd. St., **1,** No. 8–9, 1929.

	B.D.	Spectrum	Magnitude
a	−7°1616	A3	10.39
b	−7°1611	A0	10.92
c	. . .	A	11.75

The magnitudes of the comparison stars were estimated with reference to the astrographic sequence[5] at 7^h, − 2°.5, on two plates of the AC series, one of the AM series, and one of the RH series. The magnitudes of the variable were derived almost wholly from plates of AC and AM series.

The observations on RU Monocerotis are given in the table where magnitudes for individual plates are listed for the minima, but for the maximum light the observations are grouped in means of four or five. Four successive intervals of time are treated separately in the table and in the accompanying figures which are based upon the tabulated results.

INTERVAL 1

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.050	10.55	5	2.176	10.59	5
0.232	10.57	5	2.275	10.55	4
0.506	10.56	5	2.358	10.55	4
0.816	10.54	5	2.436	10.60	1
1.022	10.54	5	2.503	11.17	1
1.092	10.55	5	2.523	11.25	1
1.206	10.55	5	2.569	11.21	1
1.278	10.55	4	2.585	11.25	1
1.450	10.56	4	2.610	10.81	1
1.624	11.17	1	2.702	10.58	5
1.659	11.25	1	2.774	10.54	5
1.676	11.17	1	2.852	10.53	4
1.740	10.66	1	2.962	10.57	4
1.766	11.09	1	3.102	10.60	4
1.782	10.53	5	3.372	10.57	5
1.862	10.57	5			
2.008	10.57	5			

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[5] Harvard Annals, **86**, 145, 1924.

INTERVAL 2

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.150	10.54	5	2.390	10.52	4
0.398	10.56	5	2.478	10.68	1
0.674	10.55	5	2.575	10.80	1
0.948	10.56	5	2.598	10.89	1
1.308	10.54	5	2.612	10.87	1
1.420	10.74	1	2.652	10.66	1
1.487	10.68	1	2.668	10.54	4
1.524	10.76	1	2.820	10.54	4
1.529	10.81	1	2.970	10.53	4
1.543	11.09	1	3.048	10.60	4
1.547	10.89	1	3.158	10.58	4
1.579	11.00	1	3.514	10.59	5
1.617	11.21	1			
1.696	10.83	1			
1.711	10.71	1			
1.796	10.59	5			
1.956	10.57	5			
2.100	10.54	5			
2.272	10.58	4			

INTERVAL 3

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.024	10.55	5	1.591	11.25	1
0.102	10.57	5	1.596	11.17	1
0.198	10.58	5	1.623	11.04	1
0.280	10.53	5	1.666	10.75	1
0.382	10.55	5	1.677	10.87	1
0.432	10.56	5	1.701	10.72	1
0.626	10.56	5	1.770	10.55	5
0.754	10.53	5	1.880	10.57	5
0.790	10.55	5	1.988	10.56	5
0.914	10.54	5	2.064	10.55	5
1.038	10.52	5	2.116	10.56	5
1.178	10.56	4	2.168	10.54	5
1.275	10.52	4	2.270	10.60	4
1.370	10.57	4	2.325	10.60	4
1.424	10.50	1	2.370	10.58	4
1.495	10.87	1	2.422	10.58	1
1.509	10.89	1	2.464	10.81	1

INTERVAL 3—*Continued*

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
1.520	11.09	1	2.482	11.00	1
1.567	11.21	1	2.497	10.96	1
2.560	10.92	1			
2.569	11.25	1			
2.605	10.81	1			
2.696	10.54	5			
2.784	10.58	5			
2.898	10.54	5			
2.972	10.58	5			
3.100	10.59	5			
3.206	10.58	5			
3.278	10.54	5			
3.342	10.57	5			
3.478	10.60	4			
3.555	10.59	4			

INTERVAL 4

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.036	10.56	5	1.463	10.78	1
0.120	10.64	5	1.472	10.84	1
0.178	10.60	5	1.475	10.60	1
0.288	10.58	5	1.504	11.11	1
0.354	10.63	5	1.528	11.17	1
0.452	10.58	5	1.530	11.21	1
0.542	10.63	5	1.539	11.17	1
0.726	10.56	5	1.558	10.92	1
0.850	10.56	5	1.565	11.13	1
1.016	10.53	5	1.587	11.09	1
1.160	10.53	5	1.604	10.76	1
1.245	10.54	4	1.648	10.60	1
1.300	10.61	4	1.660	10.66	1
1.375	10.56	4	1.734	10.57	5
1.401	10.74	1	1.814	10.57	5
1.416	10.55	1	1.902	10.61	5
1.417	10.77	1	1.996	10.58	5
1.450	10.84	1	2.088	10.58	5
2.190	10.58	4	2.644	10.66	1
2.268	10.54	4	2.818	10.55	5
2.358	10.60	4	2.960	10.57	5

INTERVAL 4—*Continued*

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
2.412	10.62	4	3.108	10.54	5
2.462	10.58	1	3.310	10.57	5
2.473	10.68	1	3.358	10.64	4
2.496	10.74	1	3.448	10.58	4
2.497	10.76	1	3.530	10.57	4
2.501	10.87	1			
2.537	11.25	1			
2.547	11.17	1			
2.553	11.17	1			
2.554	11.34	1			
2.571	11.17	1			
2.596	11.11	1			
2.609	10.59	1			
2.632	10.83	1			

Phases were computed with the elements

$$\text{Minimum} = \text{J.D. } 2416016.845 + 3^d.5847\ E$$

The duration of minimum is short compared with the length of period (the radii of the stars, an approximate solution shows, are scarcely an eighth of the average distance between centers); accordingly, though maximum light is very satisfactorily measured for all four intervals, too few observations are available to define the light curve during eclipses, and hence the width of the minima cannot be estimated with sufficient precision to give a value of $e \sin \omega$. The times of the minima are not very accurately determined, but the following results are derivable from the four light curves:

Median date	$t_2 - t_1$	$\frac{t_2 - t_1}{t_1 - t_2}$
1902.0	2.69	3.02
1907.0	2.65:	2.85:
1914.0	2.60	2.65
1925.1	2.56	2.51

They indicate clearly the advance of periastron. If we take $e = 0.4$, then

$$\cos \omega = 0.547\ (2d - 3.585)$$

and for the foregoing median dates we have

$$\omega = 0^\circ.9,\ 2^\circ.8,\ 6^\circ.4,\ 9^\circ.0$$

If we take $e = 0.5$, then

$$\omega = 12^\circ.2,\ 14^\circ.4,\ 16^\circ.8,\ 19^\circ.0$$

The arc covered by the advancing periastron is too small and the observations too uncertain to make possible a good estimate of the period of apsidal revolution. It may be of the order of a thousand years.

With the Harvard polarizing photometer Wendell found the visual range of RU Monocerotis at primary eclipse to be

$$11^{\rm m}.20 - 10^{\rm m}.48 = 0^{\rm m}.72$$

and the amplitude at the secondary perhaps a little less. Dubiago and Martinoff from observations made by the Argelander method find the two minima practically equal with a range of $0^{\rm m}.60$. The mean photographic range derived from the Harvard observations is, within the uncertainties, the same for both eclipses:

$$11^{\rm m}.24 - 10^{\rm m}.56 = 0^{\rm m}.68$$

The differences between the Harvard visual and photographic magnitudes at maximum and minimum are in agreement with the spectral class, A0. The amplitudes at the two eclipses show that we observe the nearly central eclipse of bodies essentially equal in size, luminosity, mass, and surface brightness. These similarities will simplify the discussion of the central concentration of matter within the stars when further observations have given us the period of apsidal revolution and accurate values of the radii of the component stars.

For the four groups of photographic observations the mean values of maximum light are, respectively:

Median date	Photographic magnitude	Mean error	Number of observations
1902.0	10.557	0.004	98
1907.0	10.559	0.006	73
1914.0	10.560	0.004	157
1925.1	10.578	0.006	140
All	10.565	0.003	468

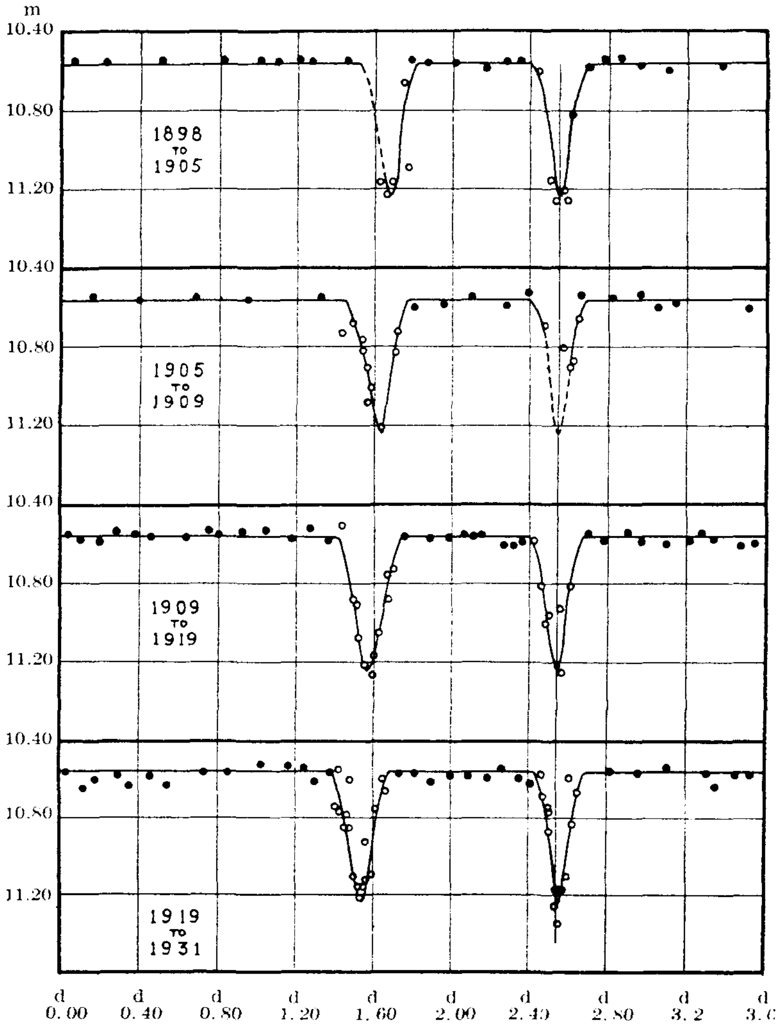
Both the maximum and minimum are fainter by approximately two hundredths of a magnitude during the fourth interval. The difference probably reflects merely the introduction of series of plates with color equations different from the average for the earlier intervals. About sixty plates of the series AX, AY, RB, and RH were used after 1920.

Of ellipticity or reflection effect there is no evidence, which is in keeping with our past experience with stars that are small relative to their separation. At periastron, however, the bodies for several hours are separated by less than two diameters; and yet during that interval of time, there appears to be no visible effect on the total light of the system, for we compute from the data in the foregoing table (134 observations) that the maximum light is then essentially constant at 10.563. The absence of a periastron brightening, such as is frequently observed for binaries with much lower eccentricities, can be taken, in connection with the very slow revolution at periastron, to indicate that the components of RU Monocerotis have high central concentration.

I am indebted to Mrs. Esther Robinson for assistance in the investigations reported in this paper and to the Permanent Science Fund of the American Academy of Arts and Sciences for a grant to cover a part of the expenses.

August 12, 1931.

Photographic light curve of RU Monocerotis, illustrating the anomalous separation of the minima and its change with time. Ordinates are photographic magnitudes; abscissae are days. Open circles in the minima represent single observations; the dots represent groups of four and five observations.



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Proceedings of the American Academy of Arts and Sciences.

Vol. 66. No. 13.—January, 1932.

RECORDS OF MEETINGS, 1930-1931.

OFFICERS AND COMMITTEES FOR 1931-1932.

LIST OF THE FELLOWS, ASSOCIATES, AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

INDEX.

(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

One thousand one hundred and eighty-third meeting.

October 8, 1930.—Stated Meeting.

The Academy met at its House at 8.33 P. M.

The President in the Chair.

There were present forty-six Fellows and Associates and eight guests.

The records of the annual meeting of May 14 were read and approved.

The Corresponding Secretary reported the receipt of letters accepting Fellowship from LeRoy Abrams, David Cheever, John Erskine, Henry C. Fall Philip Franklin, Thomas H. Gage, James Geddes, Jr., William H. P. Hatch, Frank B. Jewett, Lafayette B. Mendel, Lyman C. Newell, Greenleaf W. Pickard, Harry H. Plaskett, George E. Russell, James H. Ryan, George A. Sanderson, Karl Sax, Torald H. Sollmann, Dirk J. Struik, Fritz B. Talbot, Ernest H. Wilkins; and of letters accepting Foreign Honorary Membership from Alfredo Casella, Louis E. Lapicque, Archibald B. Macallum, and Sir Aurel Stein.

He also announced on behalf of the Council that the following grants had been made from the Permanent Science Fund:

$250 to Charles B. Lipman toward the support of studies on the biology of unusually heat-resisting organisms;

$300 to Ralph H. Cheney for apparatus and animals necessary for the completion of researches on the relation of caffeine to striated muscle and motor fatigue;

$500 to Columbus Iselin to be applied toward the cost of publication of a paper entitled "A Report on the Coastal Waters of Labrador, based on the Explorations of the 'Chance' during the Summer of 1926";

$50 to George H. Shull to be used in the collection of materials for the study of the genetics and cytology of Oenothera Lamarckiana hybrids;

$1000 to G. W. Kenrick for construction of apparatus for the study of radio transmission in the high frequency or short wave length region.

It was announced that the President had appointed the following delegates: Arthur E. Kennelly, to represent the Academy at the inauguration of Karl T. Compton as president of the Massachusetts Institute of Technology on June 6, 1930; Merritt L. Fernald, at the International Botanical Congress at Cambridge, England, August 16–23, 1930; Arthur A. Noyes (with William B. Munro as alternate) at the inauguration of Robert G. Sproul as president of the University of California.

The President announced the death of seven Fellows:—Florian Cajori (Class I, Section 1), Jesse Walter Fewkes (Class III, Section 2), Kuno Francke (Class III, Section 4), William Suddards Franklin (Class I, Section 2), Ira Nelson Hollis (Class I, Section 4), William Caleb Loring (Class III, Section 1), Horatio C. Wood (Class II, Section 4); and of one Foreign Honorary Member:—Adolf von Harnack (Class III, Section 3).

The Academy passed a vote of thanks to Ambassador W. Cameron Forbes for his gift of a Philippine narra wood table and three chairs.

The Fellows elected in May were then presented to the Academy.

The following communication was presented:

Mr. C. Iselin: "Physical Oceanography," with lantern slide illustrations.

Seven papers were read by title:

"An Equation of State for Gas Mixtures. II. A Study of the Methods of Combination of the Constants of the Beattie-Bridgeman Equation of State," by James A. Beattie and Shikao Ikehara;

"A Study of the Absolute Temperature Scale. I. The Construction of Several Types of Platinum Resistance Thermometers," by James A. Beattie, David D. Jacobus, and John M. Gaines;

"Equations for Vapor Pressures and Latent Heats, including Approximate Equations for Solid Compounds containing a Gaseous Component," by Louis J. Gillespie;

"A Report on the Coastal Waters of Labrador, Based on Explorations of the 'Chance' during the Summer of 1926," by C. Iselin, presented by Thomas Barbour;

"The Green's Matrix and Expansion Problem for Systems of Integro-Differential Equations," by F. C. Jonah, presented by Jacob Tamarkin;

"The Joule-Thomson Effect in Air. Second Paper," by J. R. Roebuck, presented by P. W. Bridgman;

"The Volume of Eighteen Liquids as a Function of Pressure and Temperature," by P. W. Bridgman.

The meeting was dissolved at 9.58 P. M.

One thousand one hundred and eighty-fourth meeting.

November 12, 1930.—Stated Meeting.

The Academy met at its House at 8.35 P. M.

The President in the Chair.

There were present forty-two Fellows and Associates and ten guests.

The records of the meeting of October 8 were read and approved.

The President announced the death of two Fellows:—Frederick Perry Fish (Class III, Section 1) and Ernest Henry Wilson (Class II, Section 2); of one Associate:—Henry Winchester Cunningham; and of one Foreign Honorary Member:—Adolf Engler (Class II, Section 2).

The following communication was presented:

Mr. James F. Norris: "Interatomic Forces in Molecules containing Carbon."

One paper was presented by title: "Contribution towards a Monograph of the Laboulbeniaceæ. Part V," by Roland Thaxter.

The meeting was dissolved at 9.43 P. M.

One thousand one hundred and eighty-fifth meeting.

December 10, 1930.—Stated Meeting.

The Academy met at its House at 8.30 P. M.

The President in the Chair.

There were present twenty-seven Fellows and five guests.

The records of the meeting of November 12 were read and approved.

The following communication was presented:

Mr. Truman L. Kelley: "Some Problems in the Analysis of Mental Life."

Two papers were presented by title: "Necessary Conditions in the Problem of Lagrange and Mayer under General End Conditions," by Marston Morse and Sumner Myers.

"Compressibility and Pressure Coefficient of Resistance, including Single Crystal Magnesium," by P. W. Bridgman.

The meeting was dissolved at 9.35 P. M.

One thousand one hundred and eighty-sixth meeting.

JANUARY 14, 1931.—STATED MEETING.

The Academy met at its House at 8.25 P. M.

The PRESIDENT in the Chair.

There were present thirty-five Fellows and Associates and one guest.

The records of the meeting of December 10 were read and approved.

The Corresponding Secretary reported the receipt of a letter from Dr. Kenneth D. Blackfan, resigning Fellowship.

He announced that the Council had appointed Mr. Alfred L. Ripley a member of the Committee of Finance to fill the unexpired term of Mr. Frederick P. Fish, deceased.

He also announced that Messrs. C. H. Grandgent, of Cambridge, Henry Guy, of Grenoble, and Paul Hazard, of Paris, had been appointed delegates to represent the Academy at the 400th Anniversary of the Collège de France, June 18–20, 1931.

The President announced the death of two Fellows:—Edward Channing (Class III, Section 3), Morris Gray (Class III, Section 4); and of two Foreign Honorary Members:—Joseph Jacques Césaire Joffre (Class I, Section 4), Sir George Otto Trevelyan (Class III, Section 3).

The following communication was presented:

Dr. Francis G. Benedict: "New Studies in Human and Reptile Metabolism," with lantern slide illustrations.

One paper was presented by title: "Researches on the Rotation of Permalloy and Soft Iron by Magnetization and the Nature of the Elementary Magnet," by S. J. Barnett.

The meeting was dissolved at 9.43 P. M.

One thousand one hundred and eighty-seventh meeting.

FEBRUARY 11, 1931.—STATED MEETING.

The Academy met at its House at 8.35 P. M.

The PRESIDENT in the Chair.

There were present twenty-nine Fellows and Associates and four guests.

The records of the meeting of January 14 were read and approved.

The Corresponding Secretary presented the summary of a report to the Council by a special committee appointed by that body to consider the matter of increasing the membership of the Academy. On motion of Vice-President Ford, it was

Voted, To refer the matter to a committee which should make a report to be considered at the next meeting.

The President then reappointed the same committee: A. E. Kennelly, Chairman, W. B. Cannon, and J. H. Beale.

The Corresponding Secretary announced that the Council had made the following grants from the Permanent Science Fund:

To Prof. Walter C. Schumb, Massachusetts Institute of Technology, $500. for the purpose of procuring apparatus to be used in the study of the effect of the electrodeless discharge upon chemical reactions occurring in the gaseous state;

To the American Association of Variable Star Observers (Dr. Alice H. Farnsworth, President, Mount Holyoke College), $200. for replenishing the stock of variable star charts for distribution to its members;

To Dr. Zenon M. Bacq, Faculty of Medicine, University of Brussels, $300. for apparatus and monkeys necessary for researches in the physiology of the sympathico-adrenal system.

The President announced the death of two Foreign Honorary Members:—Alfred Percival Maudslay (Class III, Section 2) and Ignatz Urban (Class II, Section 2); and of one Associate:—Charles Ernest Fay.

The following communication was presented:

Mr. William J. Crozier: "The Quantitative Treatment of Animal Behavior," with lantern slide illustrations.

The meeting was dissolved at ten o'clock.

One thousand one hundred and eighty-eighth meeting.

MARCH 11, 1931.—STATED MEETING.

The Academy met at its House at 8.30 P. M.

The PRESIDENT in the chair.

There were present thirty-five Fellows and Associates.

The records of the meeting of February 11 were read and approved.

The President announced the death of Fred Dayton Lambert, Fellow in Class II, Section 2.

The President appointed the Nominating Committee as follows:

Albert Sauveur, of Class I
William H. Weston, Jr., of Class II
Chandler R. Post, of Class III

On recommendation of the Council the following appropriations were made for the ensuing year:

From the income of the General Fund, $8,050, to be used as follows:

for General and Meeting expenses	$ 900
for Library expenses	2,000
for Books, Periodicals, and Binding	1,400
for House expenses	2,800
for Treasurer's expenses	950
	$8,050

From the income of the Publication Fund, $3,936.80, to be used for publication.

From the income of the Rumford Fund, $3,822.42, to be used as follows:

for Research	$2,250.00
for Books, Periodicals, and Binding	350.00
for Publication	600.00
for use at the discretion of the Committee	622.42
	$3,822.42

From the income of the C. M. Warren Fund, $1,192.25, to be used at the discretion of the Committee.

The report of the special committee appointed to consider the matter of increasing the membership of the Academy was presented in printed form by the Chairman, Mr. A. E. Kennelly, as follows:

REPORT OF SPECIAL COMMITTEE ON MEMBERSHIP.

Gentlemen:

Your Special Committee on Membership of the Academy, appointed 11th February, 1931, desires to report as follows:

The Committee considers that although the title of our Academy implies equal recognition of the "Arts and Sciences," yet the emphasis at the present time, in regard to membership, papers, communications, discussions and library representation, is principally on the "Sciences." The Committee believe that this inequality in representation has not come about through design, but probably quite fortuitously.

We think that it would be desirable to alter this policy, without delay, for the purpose of building up an Academy in which both the "Arts" and the "Sciences" should be equally represented. At the present time, there are two Classes (I and II) representing the mathematical, physical, natural and physiological sciences, comprising in all 411 Fellows; while there is only one Class (III) for all the arts and humanities, comprising 179 Fellows. Judging from our conversations with individual Fellows in Class III, it would seem that if the Academy should openly decide upon a policy of balancing its activities between the "Arts" and "Sciences," there would be no difficulty in nominating new Fellows of suitably high grade to the "Arts" group, so as to terminate the existing disparity.

To bring about approximate numerical balance between the Fellows representing "Arts" and "Sciences," using these terms in a purely classification sense, it would be necessary to have two Classes in "Arts" (III and IV) to correspond to the existing two Classes (I and II) in "Sciences." It is the opinion of the Committee that the new Class IV should be created forthwith by action of the Academy, in amending its statutes, under the rights therein contained. The existing Class III would have to be rearranged and split into two Classes (III and IV) each with Vice Presidential and Councillor representation, and each with four sections. After consulting with the officers of Class

III, the plan of Section reorganization hereinafter set forth is suggested.

We therefore recommend the passage of the following votes:

(1) That existing Class III be divided into two Classes, III and IV.
(2) That the sections of the new Classes III and IV be designated as follows:

CLASS III. THE SOCIAL ARTS.

Section I. Jurisprudence.
II. Government, International Law, and Diplomacy.
III. Economics and Sociology.
IV. Administration and Affairs.

CLASS IV. THE LITERARY AND THE FINE ARTS.

Section I. History and Literary Criticism.
II. Philosophy, Psychology, and Homiletics.
III. Philology and Archæology.
IV. Creative Arts.

For the purpose of carrying out the above votes, the following changes in the Statutes are recommended:

(a) Chapter II, Art. 1, Line 3, substitute "four" for "three"; also for "Class III, The Moral and Political Sciences" in the same article, substitute Classes III and IV as presented above in recommended Vote 2.

(b) Chapter II, Art. 2, Line 1, substitute "Eight hundred" for "Six hundred," and in line 2, substitute "Six hundred" for "Four hundred."

(c) Chapter II, Art. 3, Line 2, substitute "One hundred and thirty" for "One hundred."

(d) Chapter IV, Art. 1, Line 2, substitute "Four" for "Three." Second Paragraph, Line 1, substitute "sixteen" for "twelve"; Line 2, substitute "four" for "three."

(e) Chapter IV, Art. 3, Line 2, substitute "Four" for "Three."

(f) Chapter X, Art. 2, Line 1, substitute "Nine" for "Seven."

(g) Chapter XI, Art. 2, (I) Line 1, substitute "four" for "three."
Chapter XI, Art. 2, (IV) Line 2, substitute "four" for "three."

Chapter XI, Art. 2, (IV) Line 14, substitute "four" for "three."
Chapter XI, Art. 2, (V) Line 2, substitute "four" for "three."
Chapter XI, Art. 2, (VI) Line 1, substitute "four" for "three."
Chapter XI, Art. 2, (VII) Line 2, substitute "four" for "three."

(h) Chapter XII, Art. 2, Line 1, substitute "Twenty-five" for "Twenty."

Chapter XII, Art. 2, Line 3, substitute "Eighteen" for "Fifteen."

(i) Chapter XII, Art. 7, Line 4, substitute "twenty-five" for "twenty."

We suggest that any action taken in regard to the Class of Associates shall be postponed at least until after the foregoing recommendations shall have been disposed of.

The recommended changes in the Statutes will require an amendment by the General Court of the Act of Incorporation, Chapter forty-six of the Acts of the year seventeen hundred and seventy-nine. We therefore recommend further that the Academy at once petition the General Court of Massachusetts for the passage of the following Act, which we believe may be obtained before the April Meeting of the Academy:—

An Act relative to the American Academy of Arts and Sciences.

Be it enacted, etc., as follows:

Section 1. Section four of Chapter forty-six of the acts of the year seventeen hundred and seventy-nine, as amended by Section 1 of Chapter forty-seven of the acts of the year nineteen hundred and eleven, is hereby further amended by striking out the word "four" in the last line, and inserting in place thereof the word "six," so as to read as follows:—

> *Section 4.* That the fellows of the said academy may from time to time elect such persons to be fellows thereof as they shall judge proper; and that they shall have full power and authority from time to time to suspend, expel or disfranchise any fellow of the said academy who shall by his conduct render himself unworthy of a place in that body, in the judgment of the academy; and also to settle and establish the rules, forms and conditions of election, suspension, expulsion and disfranchisement; *provided,* that the number of the

said academy, who are inhabitants of this state, shall not at any one time be more than six hundred nor less than forty.

Section 2. This act shall take effect upon its passage.

Respectfully submitted,

Special Committee on Membership
A. E. Kennelly, Class I, Chairman,
W. B. Cannon, Class II,
J. H. Beale, Class III.

The report was discussed by Vice-President Ford, and Messrs. Joshua Whatmough, W. H. P. Hatch, J. H. Beale, W. B. Cannon, and B. L. Robinson.

On the motion of Vice-President Ford, it was

Voted, That the Academy approves the general principle of increasing its membership as outlined in the report of the Special Committee on Membership.

On the motion of Mr. J. H. Beale, it was

Voted, To petition the General Court of Massachusetts to amend Section four of Chapter forty-six of the acts of the year seventeen hundred and seventy-nine, as amended by Section 1 of Chapter forty-seven of the acts of the year nineteen hundred and eleven by striking out the word "four" in the last line, and inserting in place thereof the word "six."

The following communication was presented:

Mr. George LaPiana: "The Romance of Early Christianity in Byzantine Miniatures and the Sources of the Christian Religious Theater," with lantern slide illustrations.

The meeting was dissolved at 10.23 P. M.

One thousand one hundred and eighty-ninth meeting.

April 8, 1931.—Stated Meeting.

The Academy met at its House at 8.25 P. M.

The President in the Chair.

There were present thirty-eight Fellows and Associates and one guest.

In the absence of the Recording Secretary, Vice-President Ford was asked to assume his duties.

The records of the meeting of March 11 were read and approved.

In the absence of the Corresponding Secretary, the President made several announcements on behalf of the Council:

A grant of $500. from the Permanent Science Fund has been made to Professor Tenney L. Davis, of the Massachusetts Institute of Technology, for the employment of two Chinese students for the purpose of continuing translations of Chinese alchemy;

Mr. Arthur E. Kennelly has been reappointed for a new term of three years as the Academy's representative on the Division of Foreign Relations of the National Research Council, dating from June 30, 1931;

Mr. Tenney L. Davis has been appointed the delegate to represent the Academy at the Faraday Centennial in London, September 21, 1931.

The President announced the death of William Coolidge Lane, Fellow in Class III, Section 4, and of Carl Hansen Ostenfeld, Foreign Honorary Member in Class II, Section 2.

A second report of the Special Committee on Membership was presented by the Chairman, Mr. A. E. Kennelly, as follows:

The Committee held a meeting on March 17th, at which Vice-President Ford of Class III, Recording Secretary Clark, Treasurer Bowditch, and Councillor Knudson of Class III were present.

After considerable discussion, the following resectionalisations of Classes III and IV were adopted for recommendation to the Academy:

CLASS III. THE SOCIAL ARTS.

Section 1. Jurisprudence.
2. Government, International Law, and Diplomacy.
3. Economics and Sociology.
4. Administration and Affairs.

CLASS IV. THE FINE ARTS.

Section 1. Theology, Philosophy, and Psychology.
2. History and Archæology.
3. Philology.
4. Creative Arts and Criticism.

On motion of Mr. Kennelly, it was

Voted, That the old Class III be divided into two classes, III and IV, with eight sections as indicated in his reports.

On motion of Mr. Beale, it was

Voted, That the changes in the Statutes indicated in the report be adopted.

On motion of Mr. E. W. Forbes, it was

Voted, That in Class IV, the general designation, The Humanities, be substituted for The Fine Arts.

On motion of Mr. Hopkinson, it was

Voted, That Section 4 of Class IV be termed The Fine Arts and Belles Lettres instead of The Creative Arts and Criticism, as proposed in the report.

The following communication was presented:

Mr. Charles Hopkinson: "Portrait Painting from the Painter's Point of View."

One paper was presented by title: "Radioactive Disintegration Applied to the Measurement of Geologic Time Illustrated by Application to the Wilberforce Uraninite," by Gerhard Kirsch and Alfred C. Lane.

The meeting was dissolved at 9.58 P. M.

One thousand one hundred and ninetieth meeting.

MAY 13, 1931.—ANNUAL MEETING.

The Academy met at its House at 8.23 P. M.

The PRESIDENT in the Chair.

There were present forty-four Fellows and Associates and three guests.

The records of the meeting of April 8 were read and approved.

The Corresponding Secretary reported that several letters and manuscripts of Joseph Hale Abbot, a former Fellow of the Academy, had been received as a gift from his granddaughter, Miss Miriam Abbot.

He announced that the Council had made the following grants from the Permanent Science Fund:

1. To Professor Earnest A. Hooton, of Harvard University, one

thousand dollars toward a budget of twelve thousand dollars required to complete studies of data already collected in an investigation of "The Physical Characteristics of the Criminals in the United States in their Relation to Offense, Nativity, Nationality, Race, and to Certain Conditions, Sociological, Psychological, and Pathological," the grant however, contingent upon the collection by Professor Hooton of the remainder of the required budget of twelve thousand dollars, and the grant to be available only until May first, 1932;

2. To Professor J. LeRoy Conel, of Boston University, two hundred and fifty dollars toward expenses in the collecting of embryos of the Atlantic eel, *Myxine glutinosa,* to be used in the study of the development of its brain;

3. To Professor James H. Woods, of Harvard University, one thousand dollars, toward the expense of publishing, in six volumes, the works of the late Professor Charles S. Peirce;

4. To Professor H. S. Jennings, Director of the Zoological Laboratory of Johns Hopkins University, five hundred dollars, to be used in aiding Dr. Alexander Weinstein in carrying out an investigation of the effects of radiation in producing mutations and on the nature and genetic behavior of the mutations so produced.

The President announced the death of two Fellows:—Albert Abraham Michelson (Class I, Section 2) and Edward Robinson (Class III, Section 2).

The following report of the Council was presented:

REPORT OF THE COUNCIL.

Since the last report of the Council there have been reported the deaths of fifteen Fellows: Florian Cajori, Edward Channing, Jesse Walter Fewkes, Frederick Perry Fish, Kuno Francke, William Suddards Franklin, Morris Gray, Ira Nelson Hollis, Fred Dayton Lambert, William Coolidge Lane, William Caleb Loring, Albert Abraham Michelson, Edward Robinson, Ernest Henry Wilson, Horatio C. Wood; seven Foreign Honorary Members: Adolf Engler, Adolf von Harnack, Joseph Jacques Césaire Joffre, Alfred Percival Maudslay, Carl Hansen Ostenfeld, Sir George Otto Trevelyan, Ignatz Urban; and two Associates: Henry Winchester Cunningham, Charles Ernest Fay.

Twenty-one Fellows and four Foreign Honorary Members were elected by the Council and announced to the Academy in May 1930.

The roll now includes 586 Fellows, 93 Foreign Honorary Members, and 30 Associates (not including those elected in May 1931).

The annual report of the Treasurer, Ingersoll Bowditch, was read, of which the following is an abstract:

GENERAL FUND.

Receipts.

Income on hand April 1, 1930	$1,534.85	
From Investments	2,852.48	
From Assessments	5,385.00	
From Admissions	140.00	
From Interest on Deposits	404.90	
From Rumford Fund Income	200.00	$10,517.23

Expenditures.

Expenses of Library	$2,078.00		
Treasurer's Expenses	970.19		
Books and Binding	1,697.84		
General Expenses	1,309.96		
House Expenses	2,673.44		
President's Expenses	1.75		
Printing Biographical Notices	32.75	$8,763.93	
Income transferred to Principal		366.50	$ 9,130.43

RUMFORD FUND.

Receipts.

Income on hand April 1, 1930	$7,037.40	
From Investments	4,120.10	$11,157.50

Expenditures.

Publications	$1,422.16
Purchase and Binding of Books	342.44
Research	3,036.00

Engraving Medals	15.00		
Printing	7.70		
Transferred to General Fund Income for care of books	200.00		
Books and Periodicals—bal. 1929 transferred to Capital	21.69		
Discretion of Committee—bal. 1929 transferred to Capital	1,105.26	$6,150.25	
Interest on bonds bought	17.50		
Income transferred to Principal	211.10	228.60	$6,378.85

C. M. Warren Fund.

Receipts.

Income on hand April 1, 1930	$ 76.06	
From Investments	1,255.00	$1,331.06

Expenditures.

Research	$908.70		
Vault Rent, part.	3.00	$ 911.70	
Interest on bonds bought	12.13		
Income transferred to Principal.	62.75	74.88	$ 986.58

Publication Account.

Receipts.

Income on hand April 1, 1930		$9,625.48	
From Income Appleton Fund	$1,709.02		
From Income Centennial Fund.	2,632.50		
From Authors' Reprints	181.32		
From Sale of Publications	620.04		
From Amer. Council of Learned Societies	1,500.00		
From Income Permanent Science Fund	500.00	$7,142.88	$16,768.36

Expenditures.

Publications	$3,703.45		
Vault Rent, part	10.00	$3,713.45	
Interest on Bonds bought	24.95		
Income transferred to Principal	222.93	247.88	$ 3,961.33

PERMANENT SCIENCE FUND.

Receipts.

Income on hand April 1, 1930	$ 639.42	
Received for above fund.	4,310.58	$ 4,950.00

Expenditures.

Grants from above fund	$4,450.00	
Transferred to Publication Account	500.00	$ 4,950.00

FRANCIS AMORY FUND.

Receipts.

From Investments		$ 2,275.50

Expenditures.

Publishing Statement	$ 46.75	
Income transferred to Principal	2,162.37	
Interest on bonds bought	30.00	
Interest on bonds bought	36.38	$ 2,275.50

The following reports were also presented:

REPORT OF THE LIBRARY COMMITTEE.

During the year 161 volumes and 30 unbound numbers of periodicals have been borrowed by 20 different Fellows and 15 different libraries, and many more have been consulted at the Academy. All books taken out have been returned or satisfactorily accounted for.

The number of bound volumes on the shelves at the time of the last report was 42,026. During the year 552 volumes were added, making the number now 42,578. This includes 44 purchased from the General Fund, 28 from the Rumford Fund, and 480 received by gift or exchange. The number of pamphlets received was 305.

One addition has been made to the periodical subscription list, the new Quarterly Journal of Mathematics (Oxford Series).

The following appropriations were placed at the disposal of the Committee during the past year:

Balance from General Fund	$ 621.99
Appropriation from General Fund	3,400.00
Balance from Rumford Fund	65.05
Appropriation from Rumford Fund	300.00
Total	$4,387.04

The expenses charged to the Library during the financial year ending April 1, 1931, were:

Salaries		$2,000.00
Binding:		
General Fund		$ 722.25
Rumford Fund		75.25
		$ 797.50
Purchase of Books and Periodicals:		
General Fund	$975.59	
Rumford Fund	267.19	$1,242.78
Miscellaneous		$ 78.00
Total		$4,118.28

There remained on April 1, 1931, an unexpended balance of $268.76, as follows:

General Fund	$246.15
Rumford Fund	22.61
Total	$268.76

There have been applications to permit photostats to be made from our old and rare volumes. Unless the Academy otherwise directs, it will be the policy to be very liberal in such permissions, but unless there is especially good reason and adequate insurance of volumes unreplaceable or not easily replaceable, this should not involve shipping the volumes away.

Respectfully submitted,
ALFRED C. LANE, *Librarian.*

May 13, 1931.

REPORT OF THE RUMFORD COMMITTEE.

The Rumford Committee held five meetings during the academic year 1930–31 (May 14th, Nov. 12th, Dec. 10th, Jan. 14th, and March 11th).

The Committee made the following grants from the Rumford Fund, during the year, in aid of researches on heat and light:

1930		
No. 284.	April 9th—Prof. T. C. Poulter, Iowa Wesleyan College, Mt. Pleasant, Iowa, for purchase of equipment in researches on effects of pressure on the behaviour of light	$ 300
No. 285.	April 9th—Messrs. E. C. Kemble and F. H. Crawford, Harvard University, for purchase of equipment in spectrophotographic researches.	375
1931		
No. 286.	Jan. 14th—Prof. H. Shapley and W. J. Fisher for equipment on meteor research	450
No. 287.	Jan. 14th—Prof. J. W. McBain, Stanford University, for equipment of research on light-scattering power of sols, gels, and liquid crystals	475
No. 288.	Jan. 14th—Dr. J. C. Stearns, University of Denver, for equipment in research on cosmic rays	300
No. 289.	Jan. 14th—Prof. A. T. Evans, Miami University, Oxford, Ohio, for equipment in research on influence upon certain lower plants	200

No. 290.	Mar. 11th—Prof. Geo. R. Harrison, Mass. Inst. Technology for equipment in research on intensities of spectral lines from multiply-ionised atoms	500
No. 291.	Mar. 11th—Prof. C. E. Mendenhall, University of Wisconsin, for equipment in research on the photo-electric properties of metals	500
No. 292.	Mar. 11th—Prof. W. R. Frederickson, Syracuse University, for equipment in studying the Faraday effect spectra in diatomic molecules	300
	Total	$3,400

The committee voted unanimously, first on January 14th and again on March 11th, to recommend the award of the Rumford Medals to Dr. Karl Taylor Compton "for researches in Thermionics and Spectroscopy."

Reports of progress in researches aided by grants from the Rumford Fund have been received from Messrs. P. W. Bridgman, Annie J. Cannon, F. H. Crawford, G. H. Dieke, N. Feather, Geo. R. Harrison, Norton A. Kent, G. W. Pierce, H. H. Plaskett, T. C. Poulter, J. R. Roebuck, Harlow Shapley, and Harlan T. Stetson.

The following recent publications concern researches that have been aided by grants from the Rumford Fund:

(1). By Prof. P. W. Bridgman, Harvard University, "The Volume of Eighteen Liquids as a Function of Pressure and Temperature," *Proc. Am. Academy of Arts and Sciences,* Vol. 66, No. 5, February 1931, pp. 185–233.

(2). By Prof. P. W. Bridgman, Harvard University, "Compressibility and Pressure Coefficient of Resistance, including Single Crystal Magnesium," *Proc. Am. Academy of Arts and Sciences,* Vol. 66, No. 7, March 1931, pp. 255–270.

(3). By Prof. Harlan T. Stetson, Director Perkins Observatory, Ohio Wesleyan University, "The Influence of Sun Spots on Radio Reception," *Miscellaneous Scientific Papers, Franklin Institute* Reprint No. 3, *Journal Fr. Inst.* Vol. 210, No. 4, Oct. 1930, pp, 403–419.

(4). By Prof. Harlan T. Stetson, Director Perkins Observatory, Ohio Wesleyan University, "On the Correlation of Radio Reception with the Moon's Position in the Observer's Sky," *Terrestrial Magnetism and Atmospheric Electricity*, Vol. 36, No. 1, March 1931, pp. 1–5.

(5). By Dr. N. Feather, "Concerning the Absorption Method of Investigating β-Particles of High Energy. The Maximum Energy of the Primary β-Particles of Mesothorium 2," *Physical Review*, Vol. 35, No. 12, June 15, 1930, pp. 1559–1567.

(6). By Dr. N. Feather, "An unsuccessful attempt to influence the Normal Decay of a weak Source of Polonium," *Proc. Cambridge Philosophical Society*, Vol. 26, Part 4, Oct. 1930, pp. 538–541.

Respectfully submitted,
A. E. KENNELLY, *Chairman.*

REPORT OF THE C. M. WARREN COMMITTEE.

The Committee had at its disposal at the beginning of the fiscal year 1930–1931, $1,242.56, of which $908.70 has been appropriated during the past year.

Since the last annual report grants have been made as follows:

June 5, 1930: to Professors Conant and Kistiakowsky, $350. for the purchase of a quartz spectrograph to be used in the study of the absorption spectra of organic compounds in solution at liquid air temperatures.

June 5, 1930: to Dr. Nelson K. Richtmyer, Bryn Mawr College, $260, for the purchase of a micro balance and 30 grams of platinum to be used in the investigation of certain methylated sugars, and the hydrogenation of certain glycosides.

June 5, 1930: to Professor Henry Gilman, Iowa State College, $200, for the purchase of rare chemicals to be used in the study of chemiluminescence.

June 5, 1930: to Professor H. W. Underwood, Jr., $100, for the employment of an assistant in connection with the study of the action of anhydrous zinc chloride on ethers.

Four applications for grants amounting to $1,450 are now before the Committee for action.

Reports of progress have been received from Dr. Charles F. H. Allen, Professor Ralph H. Bullard, Professor Henry Gilman, Professor Carl L. A. Schmidt, Dr. Nelson K. Richtmyer, Professor H. W. Underwood, Jr.

The papers listed below, which have been published since the last report of the Committee, describe the results of investigations aided by the Warren Fund. In each case there is an acknowledgment by the author of the assistance received:

H. Gilman, R. E. Fothergill and J. McGlumphy, "On the sensitivity of the luminescence of some arylmagnesium halides," *Rec. Trav. Chim.* T. 49, No. 6 (June 1930).

L. M. Chapman Rawlius and Carl L. A. Schmidt, "The Mode of Combination Between Certain Dyes and Gelatin Granules," *The Journal of Biological Chemistry*, Vol. LXXXVIII, No. 1, August 1930.

W. M. Hoskins, Merle Randall, and Carl L. A. Schmidt, "The Conductance and Activity Coefficients of Glutamic and Aspartic Acids and Their Monosodium Salts," *The Journal of Biological Chemistry*, Vol. LXXXVIII, No. 1, August 1930.

C. V. Smythe and Carl L. A. Schmidt, "Studies on the Mode of Combination of Iron with Certain Proteins, Amino Acids, and Related Compounds," *The Journal of Biological Chemistry*, Vol. LXXXVIII, No. 1, August 1930.

Carl L. A. Schmidt, Paul L. Kirk, and W. K. Appleman, "The Apparent Dissociation Constants of Arginine and of Lysine and the Apparent Heats of Ionization of Certain Amino Acids," *The Journal of Biological Chemistry*, Vol. LXXXVIII, No. 1, August 1930.

John B. Dalton, Paul L. Kirk, and Carl L. A. Schmidt, "The Apparent Dissociation Constants of Diiodotyrosine, Its Heat of Solution, and Its Apparent Heat of Ionization," *The Journal of Biological Chemistry*, Vol. LXXXVIII, No. 2, September 1930.

J. B. Conant and F. H. Crawford, "The Study of Absorption Spectra of Organic Compounds at Liquid Air Temperatures," *Proc. N. Acad. Sci.*, Vol. 16, No. 8, pp. 552–554, August 1930.

Charles F. H. Allen, "Cyclohexanone-Benzil," *Canadian Journal of Research*, 4: 264–274, 1931.

Respectfully submitted,

JAMES F. NORRIS, *Chairman*.

REPORT OF THE COMMITTEE OF PUBLICATION.

1. The Committee of Publication has held but one meeting during the year, the rest of the work of the Committee being carried on by

correspondence. The Chairman of the Committee by invitation attended one meeting of the Rumford Committee. Since the present Committee took charge eleven papers for the Proceedings and one Memoir have been accepted for publication. Owing to limited funds and obligations assumed by the previous Committee, it has been necessary to decline for publication several papers presented by non-members. The papers accepted are as follows:

A. Records number for 1928–29 and 1929–30.

1. Iselin, C.—A Report on the Coastal Waters of Labrador. 37 pages.
2. Creighton, W. S.—The New World Species of the Genus Salenopsis. 112 pages.
3. Gillespie, L. J.—Equations for Vapor Pressures and Latent Heats. 12 pages.
4. Beattie, Jacobs and Gaines—An Experimental Study of the Absolute Temp. Scale. 17 pages.
5. Bridgman, P. W.—The Volume of Eighteen Liquids as a Function of Pressure and Temperature. 48 pages.
6. Morse, Marston, and Myers, Byron—The Problems of La Grange and Mayer with Variable End Points. 18 pages.
7. Bridgman, P. W.—Compressibility and Pressure Coefficient of Resistance, including Single Crystal Magnesium. 16 pages.
8. Barnett, S. J.—Researches on the Rotation of Permalloy and Soft Iron by Magnetization and the Nature of the Elementary Magnet.
9. Keyes, F. G.—Note on the Kelvin Scale Temperature of Freezing Water.
10. Kirsch, G. and Lane, A. C.—Radio-active Disintegration Applied to the Measurement of Geological Time Illustrated by Application to the Wilberforce Uraninite.
11. Thaxter, R.—Contribution to a Monograph on the Laboulbeniaceæ. Part V.

Eight of these have already appeared in print. The remaining four are in the printer's hands.

2. The financial accounts of the Committee may be briefly summarized as follows:

Total Appropriation for the year 1930–31	$13,860.77
Total Expenditures for the year 1930–31	3,703.45
Balance	$10,157.32
Additional Receipts	
From Sales	$ 620.04
From Author's Reprints	181.32
From Am. Council Learned Societies	1,500.00
From Permanent Science Fund	500.00
Total Amount Available April 1, 1931	$12,958.68

In addition to these funds there have been placed at the disposal of the Committee

Rumford Funds available Apr. 1, 1930	$3,252.58
Expenditures for Publication	1,422.16
Rumford Funds Balance carried over	$1,830.42
Farlow Herbarium Funds for Thaxter Memoir	$3,000.00

3. Some apologies from a committee which has thirteen thousand dollars at its disposal at the end of a fiscal year may seem in order. The explanation for such a large balance is found in the fact that the expensive Sapir volume and the Thaxter Memoir have not yet been paid for. The two together will probably cost not less than eight thousand dollars.

4. The delay in the appearance of the Sapir volume on the Paiute Language has been explained by Professor Sapir as the result of the necessity of ordering a considerable number of matrixes for special characters. The proofreading he says has been slow and difficult because of the technical nature of the material. Parts one and two are already in print and part three—the last—is now in galley proof. Quoting from his last letter, "the volume should be out before the end of the summer at latest, I should think."

5. The Committee wishes to express its appreciation of the support received from President E. B. Wilson, from the Rumford Committee, and from the Committee on the Permanent Science Fund.

Respectfully submitted,
H. V. Neal, *Chairman.*

Report of the House Committee.

The House Committee has had funds at its disposal amounting to $3,597.77, made up as follows:

Balance from previous year	$ 837.77
Appropriation for 1930–31	2,500.00
Received for use of rooms	260.00
	$3,597.77

Of this amount, the sum of $2,648.27 has been spent for the routine expenses, janitor, light, power, heat, telephone, etc., and $630.07 has been spent for upkeep and equipment, making a total of $3,278.34, and leaving an unexpended balance of $319.43. The chief items of upkeep this year have been the painting of the lecture hall, the staircase hall, and the kitchen, and rather extensive repairs on the roof.

Meetings have been held as follows:

The Academy	8
American Antiquarian Society	1
American Chemical Society, Northeastern Section	8
Archæological Institute of America	2
Copley Society	1
Harvard-Technology Chemical Club	7
New England Botanical Club	9
	36

The Council Chamber has been used for Academy Council and Committee meetings, and also by the Trustees of the Children's Muesum, the New England Farm and Garden Association, the Thursday Evening Club, etc.

A detailed list of expenditures follows:

Janitor		$ 977.00
Electricity	Light	203.10
	Power	96.65
Coal		988.83
Elevator		85.59
Gas		72.50

Telephone	103.88
Water	36.80
Ash tickets	40.20
Upkeep	580.57
Furnishing and equipment	49.50
Janitor's supplies and sundries	43.72
Total expenditure	$3,278.34

Respectfully submitted,
S. Burt Wolbach, *Chairman.*

May 13, 1931.

It was *Voted*, To award the Rumford Premium to Dr. Karl Taylor Compton for researches in thermionics and spectroscopy.

On the recommendation of the Treasurer, it was *Voted*, That the annual assessment for 1931–1932 be $15.

The annual election resulted in the choice of the following officers and committees:

Jeremiah D. M. Ford, *President*
Harry M. Goodwin, *Vice-President for Class I*
Walter B. Cannon, *Vice-President for Class II*
Edwin F. Gay, *Vice-President for Class III*
Charles B. Gulick, *Vice-President for Class IV*
Tenney L. Davis, *Corresponding Secretary*
Walter E. Clark, *Recording Secretary*
Ingersoll Bowditch, *Treasurer*
Alfred C. Lane, *Librarian*
Herbert V. Neal, *Editor*

Councillors for Four Years.

Harry E. Clifford, of Class I
Oakes Ames, of Class II
George G. Wilson, of Class III
Charles Hopkinson, of Class IV

Councillor for Three Years.

Roscoe Pound, of Class III

Councillor for Two Years.

Arthur N. Holcombe, of Class III

Councillor for One Year.

George H. Chase, of Class IV

Finance Committee.

Thomas Barbour
Paul J. Sachs
Alfred L. Ripley

Rumford Committee.

Arthur E. Kennelly
Elihu Thomson
Harry M. Goodwin
Harlow Shapley
Percy W. Bridgman
Charles L. Norton
Norton A. Kent

C. M. Warren Committee.

James F. Norris
Gregory P. Baxter
Arthur D. Little
Reid Hunt
Walter L. Jennings
Frederick G. Keyes
Charles A. Kraus

Publication Committee.

(Editor) Herbert V. Neal, *ex officio Chairman*
Edwin C. Kemble, of Class I
Joseph H. Beale, of Class III
Frederic T. Lewis, of Class II
George F. Moore, of Class IV

Library Committee.

(Librarian) Alfred C. Lane
Raymond C. Archibald, of Class I
Robert P. Blake, of Class III
Thomas Barbour, of Class II
Henry B. Washburn, of Class IV

House Committee.

S. Burt Wolbach, *Chairman*
Robert P. Bigelow
William H. Lawrence

Committee on Meetings.

The President
The Recording Secretary
George H. Parker
Daniel F. Comstock
Joshua Whatmough

Auditing Committee.

George R. Agassiz
John E. Thayer

The Corresponding Secretary reported that the following gentlemen had been elected members of the Academy by the Council:

FELLOWS.

CLASS I.

Section 1. George Rutledge, Belmont.
Section 2. Karl Taylor Compton, Cambridge.
George Russell Harrison, Cambridge.
Robert Bruce Lindsay, Providence, R. I.
Otto Oldenberg, Cambridge.
Section 3. William Albert Noyes, Jr., Providence, R. I.
Section 4. Charles Harold Berry, Belmont.
George Leonard Hosmer, Woburn.

CLASS II.

Section 1. George Francis McEwen, La Jolla, Cal.
Section 2. Charles Alfred Weatherby, Cambridge.
Section 3. William King Gregory, New York, N. Y.
Section 4. George Blumer, New Haven, Conn.
William Bosworth Castle, Boston.
Eugene Floyd Du Bois, New York, N. Y.

CLASS III.

Section 1. George Weston Anderson, Wellesley Hills.
Roland William Boyden, Beverly.
Henry Addington Bruce, Cambridge.
Richard Clarke Cabot, Cambridge.
Zechariah Chafee, Jr., Cambridge.
Calvin Coolidge, Northampton.
Fred Tarbell Field, Newton.
James Ford, Cambridge.
Manley Ottmer Hudson, Cambridge.
Nathan Isaacs, Cambridge.
Truman Lee Kelley, Cambridge.
William DeWitt Mitchell, Washington, D. C.
Edmund Morris Morgan, Arlington.

Herbert Parker, South Lancaster.
Thomas Nelson Perkins, Milton.
Thomas Reed Powell, Cambridge.
Henry Knox Sherrill, Boston.
Jeremiah Smith, Jr., Cambridge.
George Woodward Wickersham, New York, N. Y.
Section 2. Edward Cooke Armstrong, Princeton, N. J.
Samuel Hazzard Cross, Cambridge.
Francis Peabody Magoun, Jr., Cambridge.
Section 3. John Donald Black, Belmont.
Melvin Thomas Copeland, Cambridge.
William Leonard Crum, Cambridge.
Sidney Bradshaw Fay, Cambridge.
Wesley Clair Mitchell, New York, N. Y.
Pitirim Alexandrovich Sorokin, Cambridge.
Oliver Mitchell Wentworth Sprague, Cambridge.
Donald Skeele Tucker, Cambridge.
Section 4. Stephen Vincent Benét, New York, N. Y.
Robert Frost, South Shaftsbury, Vt.
Raymond Dexter Havens, Baltimore, Md.
Robert Silliman Hillyer, Cambridge.
Henry James, New York, N. Y.
Paul Manship, New York, N. Y.
Daniel Gregory Mason, New York, N. Y.
Frank Jewett Mather, Princeton, N. J.
Paul Elmer More, Princeton, N. J.
Kenneth Ballard Murdock, Cambridge.
Robert Kilburn Root, Princeton, N. J.

FOREIGN HONORARY MEMBERS.

Class I.

Section 4. Karl Willy Wagner, Berlin.

Class II.

Section 2. Ludwig Diels, Berlin.
Section 3. August Krogh, Copenhagen.

CLASS III.

Section 2. Robert Seymour Conway, St. Albans.
Sylvain Lévi, Paris.
Antoine Meillet, Paris.
Section 3. Herbert Albert Laurens Fisher, Oxford.
George Macaulay Trevelyan, Cambridge.
Section 4. John Galsworthy, London.
Edgar Allison Peers, Liverpool.
Charles Sanford Terry, Aberdeen.

The following communications were presented:

Dr. John L. Bremer: "Motion Pictures of the Living Heart of the Embryonic Chick."

Mr. Joseph H. Beale: "Promising Lines of Legal Research."

The meeting was dissolved at 10.05 P. M.

American Academy of Arts and Sciences.

OFFICERS AND COMMITTEES FOR 1931–1932.

PRESIDENT.

Jeremiah D. M. Ford.

VICE-PRESIDENTS.

Class I.	Class II.
Harry M. Goodwin,	Walter B. Cannon,
Class III.	**Class IV.**
Edwin F. Gay,	Charles B. Gulick.

CORRESPONDING SECRETARY.

Tenney L. Davis.

RECORDING SECRETARY.

Walter E. Clark.

TREASURER.

Ingersoll Bowditch.

LIBRARIAN.

Alfred C. Lane.

EDITOR.

Herbert V. Neal.

COUNCILLORS.

Class I.		Class II.
Charles A. Kraus,		George R. Minot.
	Terms expire 1932.	
Frederick A. Saunders,		Charles T. Brues.
	Terms expire 1933.	
Clarence L. E. Moore,[1]		Hervey W. Shimer.
	Terms expire 1934.	
Harry E. Clifford,		Oakes Ames.
	Terms expire 1935.	

Class III.		Class IV.
W. Cameron Forbes,		George H. Chase.
	Terms expire 1932.	
Arthur N. Holcombe,		Irving Babbitt.
	Terms expire 1933.	
Roscoe Pound,		Albert C. Knudson.
	Terms expire 1934.	
George G. Wilson,		Charles Hopkinson.
	Terms expire 1935.	

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[1] Died December 5, 1931.

COMMITTEE OF FINANCE.

Thomas Barbour, Paul J. Sachs, Alfred L. Ripley.

RUMFORD COMMITTEE.

Arthur E. Kennelly, *Chairman*,
Elihu Thomson, Harry M. Goodwin, Harlow Shapley,
Percy W. Bridgman, Charles L. Norton, Norton A. Kent.

C. M. WARREN COMMITTEE.

James F. Norris, *Chairman*,
Gregory P. Baxter, Arthur D. Little, Reid Hunt,
Walter L. Jennings, Frederick G. Keyes, Charles A. Kraus.

COMMITTEE OF PUBLICATION.

Herbert V. Neal, *Chairman*,
Edwin C. Kemble, of Class I, Joseph H. Beale, of Class III,
Frederic T. Lewis, of Class II, Robert P. Blake, of Class IV.

COMMITTEE ON THE LIBRARY.

Alfred C. Lane, *Chairman*,
Raymond C. Archibald, of Class I, William Y. Elliott, of Class III,
Thomas Barbour, of Class II, Henry B. Washburn, of Class IV.

AUDITING COMMITTEE.

George R. Agassiz, John E. Thayer.

HOUSE COMMITTEE.

S. Burt Wolbach, *Chairman*,
William H. Lawrence, Robert P. Bigelow, Davis R. Dewey.

COMMITTEE ON MEETINGS.

The President,
The Recording Secretary,
Daniel F. Comstock, George H. Parker, Charles J. Bullock,
Joshua Whatmough.

LIST

OF THE

FELLOWS, ASSOCIATES, AND FOREIGN HONORARY MEMBERS.

(Corrected to December 7, 1931.)

FELLOWS.—628.

(Number limited to eight hundred.)

CLASS I.—*Mathematical and Physical Sciences.*—204.

SECTION I.—*Mathematics and Astronomy.*—51.

Charles Greeley Abbot	Washington, D. C.
Walter Sydney Adams	Pasadena, Cal.
George Russell Agassiz	Boston
Raymond Clare Archibald	Providence, R. I.
George David Birkhoff	Cambridge
Ernest William Brown	New Haven, Conn.
William Elwood Byerly	Cambridge
William Wallace Campbell	Mount Hamilton, Cal.
Julian Lowell Coolidge	Cambridge
George Cary Comstock	Beloit, Wis.
Leonard Eugene Dickson	Chicago, Ill.
Philip Fox	Evanston, Ill.
Fabian Franklin	New York, N. Y.
Philip Franklin	Belmont
Edwin Brant Frost	Williams Bay, Wis.
William Caspar Graustein	Cambridge
Frank Lauren Hitchcock	Belmont
Edward Vermilye Huntington	Cambridge
Dunham Jackson	Minneapolis, Minn.
Oliver Dimon Kellogg	Cambridge
Carl Otto Lampland	Flagstaff, Ariz.
Willem Jacob Luyten	Minneapolis, Minn.

George Abram Miller	Urbana, Ill.
John Anthony Miller	Swarthmore, Pa.
Samuel Alfred Mitchell	University, Va.
Eliakim Hastings Moore	Chicago, Ill.
Frank Morley	Baltimore, Md.
Marston Morse	Cambridge
Forest Ray Moulton	Chicago, Ill.
Henry Bayard Phillips	Boston
William Henry Pickering	Mandeville, Jamaica, B. W. I.
Harry Hemley Plaskett	Cambridge
Charles Lane Poor	New York, N. Y.
Roland George Dwight Richardson	Providence, R. I.
Henry Norris Russell	Princeton, N. J.
George Rutledge	Belmont
Frank Schlesinger	New Haven, Conn.
Harlow Shapley	Cambridge
Vesto Melvin Slipher	Flagstaff, Ariz.
Frederick Slocum	Middletown, Conn.
Virgil Snyder	Ithaca, N. Y.
Joel Stebbins	Madison, Wis.
Harlan True Stetson	Delaware. Ohio
Dirk Jan Struik	Cambridge
Henry Taber	Worcester
Jacob David Tamarkin	Providence, R. I.
Harry Walter Tyler	Washington, D. C.
Oswald Veblen	Princeton, N. J.
Joseph Leonard Walsh	Cambridge
Alfred North Whitehead	Cambridge
Frederick Shenstone Woods	Newton

Class I., Section II.—*Physics*.—59.

Adelbert Ames, Jr.	Hanover, N. H.
Joseph Sweetman Ames	Baltimore, Md.
Samuel Jackson Barnett	Los Angeles, Cal.
Carl Barus	Providence, R. I.
Louis Agricola Bauer	Washington, D. C.
Percy William Bridgman	Cambridge

Walter Guyton Cady	Middletown, Conn.
George Ashley Campbell	New York, N. Y.
Leslie Lyle Campbell	Boston
Emory Leon Chaffee	Cambridge
Arthur Holly Compton	Chicago, Ill.
Karl Taylor Compton	Cambridge
Daniel Frost Comstock	Boston
William David Coolidge	Schenectady, N. Y.
Henry Crew	Evanston, Ill.
Harvey Nathaniel Davis	Hoboken, N. J.
Clinton Joseph Davisson	New York, N. Y.
Arthur Louis Day	Washington, D. C.
William Johnson Drisko	Winchester
William Duane	Boston
Alexander Wilmer Duff	Worcester
Arthur Woolsey Ewell	Worcester
Harry Manley Goodwin	Brookline
George Ellery Hale	Pasadena, Cal.
Edwin Herbert Hall	Cambridge
Arthur Cobb Hardy	Wellesley
George Russell Harrison	Cambridge
Hammond Vinton Hayes	Boston
John Charles Hubbard	Baltimore, Md.
Gordon Ferrie Hull	Hanover, N. H.
Charles Clifford Hutchins	Brunswick, Me.
Frederick Eugene Ives	Philadelphia, Pa.
James Edmund Ives	Washington, D. C.
William White Jacques	Boston
Edwin Crawford Kemble	Cambridge
Norton Adams Kent	Newton Center
Frank Arthur Laws	Salem
Henry Lefavour	Boston
Robert Bruce Lindsay	Providence, R. I.
Theodore Lyman	Brookline
Ernest George Merritt	Ithaca, N. Y.
Dayton Clarence Miller	Cleveland, O.
Robert Andrews Millikan	Pasadena, Cal.

Harry Wheeler Morse	Stanford University, Cal.
Edward Leamington Nichols	Ithaca, N. Y.
Charles Ladd Norton	Boston
Otto Oldenberg	Cambridge
George Washington Pierce	Cambridge
Michael Idvorsky Pupin	New York, N. Y.
Frederick Albert Saunders	Cambridge
John Clarke Slater	Cambridge
John Stone Stone	San Diego, Cal.
Maurice deKay Thompson	Brookline
Elihu Thomson	Swampscott
Manuel Sandoval Vallarta	Brookline
David Locke Webster	Palo Alto, Cal.
Edwin Bidwell Wilson	Brookline
Robert Williams Wood	Baltimore, Md.
John Zeleny	New Haven, Conn.

Class I., Section III.—*Chemistry*.—50.

Roger Adams	Urbana, Ill.
Wilder Dwight Bancroft	Ithaca, N. Y.
Gregory Paul Baxter	Cambridge
James Alexander Beattie	Brookline
Arthur Alphonzo Blanchard	Cambridge
Marston Taylor Bogert	New York, N. Y.
William Crowell Bray	Berkeley, Cal.
Russell Henry Chittenden	New Haven, Conn.
Arthur Messinger Comey	Cambridge
James Bryant Conant	Cambridge
Tenney Lombard Davis	Norwell
Henry Fay	Boston
George Shannon Forbes	Cambridge
Edward Curtis Franklin	Palo Alto, Cal.
Louis John Gillespie	Cambridge
Lawrence Joseph Henderson	Cambridge
Charles Loring Jackson	Boston
Walter Louis Jennings	Worcester
Grinnell Jones	Cambridge

Frederick George Keyes	Cambridge
Elmer Peter Kohler	Cambridge
Charles August Kraus	Providence, R. I.
Arthur Becket Lamb	Cambridge
Irving Langmuir	Schenectady, N. Y.
Gilbert Newton Lewis	Berkeley, Cal.
Warren Kendall Lewis	Newton
Arthur Dehon Little	Brookline
Duncan Arthur MacInnes	New York, N. Y.
Lafayette Benedict Mendel	New Haven, Conn.
Edward Mueller	Cambridge
Samuel Parsons Mulliken	Cambridge
Charles Edward Munroe	Forest Glen, Md.
Lyman Churchill Newell	Brookline
James Flack Norris	Boston
Arthur Amos Noyes	Pasadena, Cal.
William Albert Noyes	Urbana, Ill.
William Albert Noyes, Jr.	Providence, R. I.
Samuel Cate Prescott	Brookline
Robert Hallowell Richards	Jamaica Plain
Martin André Rosanoff	Valencia, Pa.
Allan Winter Rowe	Boston
George Scatchard	Cambridge
Miles Standish Sherrill	Winchester
Harry Monmouth Smith	Brookline
Julius Oscar Stieglitz	Chicago, Ill.
Richard Chace Tolman	Pasadena, Cal.
William Hultz Walker	Pasadena, Cal.
Willis Rodney Whitney	Schenectady, N. Y.
Robert Seaton Williams	Cambridge
Alpheus Grant Woodman	Watertown

CLASS I., SECTION IV.—*Technology and Engineering.*—44.

Comfort Avery Adams	Belmont
Bernard Arthur Behrend	Wellesley Hills
Charles Harold Berry	Belmont
William Hubert Burr	New Canaan, Conn.

Vannevar Bush	Belmont
John Joseph Carty	Winter Park, Fla.
Harry Ellsworth Clifford	Newton
Arthur Powell Davis	Oakland, Cal.
Theodore Harwood Dillon	Cambridge
Philip Drinker	Brookline
Gano Dunn	New York, N. Y.
William Frederick Durand	Palo Alto, Cal.
Charles Leavitt Edgar	Brookline
Gordon Maskew Fair	Cambridge
Frederic Harold Fay	Boston
John Ripley Freeman	Providence, R. I.
John Hays Hammond	New York, N. Y.
George Leonard Hosmer	Woburn
William Hovgaard	Brookline
James Robertson Jack	Watertown
Dugald Caleb Jackson	Cambridge
Frank Baldwin Jewett	New York, N. Y.
Lewis Jerome Johnson	Cambridge
Arthur Edwin Kennelly	Cambridge
William Henry Lawrence	Jamaica Plain
Charles Thomas Main	Winchester
Lionel Simeon Marks	Cambridge
Edward Furber Miller	Newton
Frederick Law Olmsted	Brookline
Charles Francis Park	Taunton
William Barclay Parsons	New York, N. Y.
Langdon Pearse	Chicago, Ill.
Harold Pender	Philadelphia, Pa.
Greenleaf Whittier Pickard	Newton Center
George Edmond Russell	Arlington
Albert Sauveur	Cambridge
William Sowden Sims	Newport, R. I.
Henry Lloyd Smyth	Watertown
Charles Milton Spofford	Brookline
John F. Stevens	Baltimore, Md.
Edward Pearson Warner	New York, N. Y.

Arthur Edward Wells	New York, N. Y.
Robert Spurr Weston	Boston
Joseph Ruggles Worcester	Boston

Class II.—*Natural and Physiological Sciences.*—209.

Section I.—*Geology, Mineralogy, and Physics of the Globe.*—50.

Wallace Walter Atwood	Worcester
Norman Levi Bowen	Washington, D. C.
Isaiah Bowman	New York, N. Y.
Kirk Bryan	Cambridge
Henry Helm Clayton	Canton
Herdman Fitzgerald Cleland	Williamstown
Reginald Aldworth Daly	Cambridge
Edward Salisbury Dana	New Haven, Conn.
William Morris Davis	Palo Alto, Cal.
Benjamin Kendall Emerson	Amherst
William Ebenezer Ford	New Haven, Conn.
James Walter Goldthwait	Hanover, N. H.
Louis Caryl Graton	Cambridge
Herbert Ernest Gregory	Honolulu, T. H.
William Jackson Humphreys	Washington, D. C.
Ellsworth Huntington	New Haven, Conn.
Robert Tracy Jackson	Peterborough, N. H.
Thomas Augustus Jaggar	Honolulu, T. H.
Douglas Wilson Johnson	New York, N. Y.
Arthur Keith	Washington, D. C.
Alfred Church Lane	Cambridge
Esper Signius Larsen, Jr.	Cambridge
Andrew Cowper Lawson	Berkeley, Cal.
Charles Kenneth Leith	Madison, Wis.
Waldemar Lindgren	Brookline
Frederic Brewster Loomis	Amherst
Alexander George McAdie	Hampton, Va.
George Francis McEwen	La Jolla, Cal.
Donald Hamilton McLaughlin	Cambridge
Kirtley Fletcher Mather	Cambridge
Edward Bennett Mathews	Baltimore, Md.

John Campbell Merriam	Washington, D. C.
William John Miller	Los Angeles, Cal.
Charles Palache	Cambridge
Percy Edward Raymond	Lexington
Austin Flint Rogers	Palo Alto, Cal.
Robert Wilcox Sayles	Chestnut Hill
Waldemar Theodore Schaller	Washington, D. C.
Charles Schuchert	New Haven, Conn.
William Berryman Scott	Princeton, N. J.
Hervey Woodburn Shimer	Hingham
Frank Bursley Taylor	Fort Wayne, Ind.
Thomas Wayland Vaughan	La Jolla, Cal.
Charles Hyde Warren	New Haven, Conn.
David White	Washington, D. C.
Herbert Percy Whitlock	New York, N. Y.
Bailey Willis	Palo Alto, Cal.
Arthur Winslow	Boston
John Eliot Wolff	Pasadena, Cal.
Frederick Eugene Wright	Washington, D. C.

CLASS II, SECTION II.—*Botany*.—39.

LeRoy Abrams	Stanford University, Cal.
Oakes Ames	North Easton
Joseph Charles Arthur	Lafayette, Ind.
Irving Widmer Bailey	Cambridge
Liberty Hyde Bailey	Ithaca, N. Y.
Edward Wilber Berry	Baltimore, Md.
Nathaniel Lord Britton	New York, N. Y.
Douglas Houghton Campbell	Palo Alto, Cal.
George Perkins Clinton	New Haven, Conn.
Bradley Moore Davis	Ann Arbor, Mich.
Edward Murray East	Jamaica Plain
Rollins Adams Emerson	Ithaca, N. Y.
Alexander William Evans	New Haven, Conn.
Joseph Horace Faull	Cambridge
Merritt Lyndon Fernald	Cambridge
Richard Thornton Fisher	Petersham

Harvey Monroe Hall	Stanford University, Cal.
Robert Almer Harper	New York, N. Y.
Albert Spear Hitchcock	Washington, D. C.
John George Jack	East Walpole
Edward Charles Jeffrey	Cambridge
Willis Linn Jepson	Berkeley, Cal.
Ivan Murray Johnston	Jamaica Plain
Jacob Goodale Lipman	New Brunswick, N. J.
Burton Edward Livingston	Baltimore, Md.
Elmer Drew Merrill	Berkeley, Cal.
Winthrop John Vanleuven Osterhout	New York, N. Y.
George James Peirce	Palo Alto, Cal.
Alfred Rehder	Jamaica Plain
Benjamin Lincoln Robinson	Cambridge
Karl Sax	Jamaica Plain
William Albert Setchell	Berkeley, Cal.
Arthur Bliss Seymour	Cambridge
Elvin Charles Stakman	St. Paul, Minn.
William Codman Sturgis	New York, N. Y.
Roland Thaxter	Cambridge
William Trelease	Urbana, Ill.
Charles Alfred Weatherby	Cambridge
William Henry Weston, Jr.	Cambridge

Class II., Section III.—*Zoology and Physiology.*—67.

Nathan Banks	Cambridge
Thomas Barbour	Boston
Francis Gano Benedict	Boston
Henry Bryant Bigelow	Concord
Robert Payne Bigelow	Brookline
William T. Bovie	Chicago, Ill.
Edward Allen Boyden	Minneapolis, Minn.
John Louis Bremer	Boston
Charles Thomas Brues	Jamaica Plain
John Wymond Miller Bunker	Belmont
Walter Bradford Cannon	Cambridge
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Gilman Arthur Drew	Eagle Lake, Fla.
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Herbert Spencer Jennings	Baltimore, Md.
Charles Willison Johnson	Brookline
Charles Atwood Kofoid	Berkeley, Cal.
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Ralph Stayner Lillie	Chicago, Ill.
Richard Swann Lull	New Haven, Conn.
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Ernest Gale Martin	Palo Alto, Cal.
Albert Davis Mead	Providence, R. I.
Axel Leonard Melander	New York, N. Y.
Gerrit Smith Miller	Washington, D. C.
Thomas Hunt Morgan	Pasadena, Cal.

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William Patten	Hanover, N. H.
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Henry Augustus Pilsbry	Philadelphia, Pa.
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Class II., Section IV.—*Medicine and Surgery*.—53.

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George Blumer	New Haven, Conn.
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George Augustus Sanderson	Littleton
Austin Wakeman Scott	Cambridge
William Cushing Wait	Medford
Eugene Wambaugh	Cambridge
George Woodward Wickersham	New York, N. Y.

CLASS III., SECTION II.—*Government, International Law, and Diplomacy.*—13.

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Sidney Bradshaw Fay	Cambridge
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Albert Bushnell Hart	Cambridge
Arthur Norman Holcombe	Cambridge
Manley Ottmer Hudson	Cambridge
Abbott Lawrence Lowell	Cambridge
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John Donald Black	Belmont
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Oliver Mitchell Wentworth Sprague	Cambridge
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Jeremiah Smith, Jr.	Cambridge

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Edwin Garrigues Boring	Cambridge
Edgar Sheffield Brightman	Newton
Henry Addington Bruce	Cambridge
Walter Fenno Dearborn	Cambridge
Edmund Burke Delabarre	Providence, R. I.

Edward Staples Drown	Cambridge
William Wallace Fenn	Cambridge
William Henry Paine Hatch	Cambridge
William Arthur Heidel	Middletown, Conn.
William Ernest Hocking	Cambridge
Frederick John Foakes Jackson	Englewood, N. J.
Truman Lee Kelley	Cambridge
Albert Cornelius Knudson	Cambridge
William Lawrence	Boston
Clarence Irving Lewis	Cambridge
William McDougall	Durham, N. C.
Edward Caldwell Moore	Cambridge
Paul Elmer More	Princeton, N. J.
Johnson O'Connor	Boston
George Herbert Palmer	Cambridge
Charles Edwards Park	Boston
Leighton Parks	London, England
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Ralph Barton Perry	Cambridge
James Hardy Ropes	Cambridge
James Hugh Ryan	Washington, D. C.
Henry Knox Sherrill	Boston
Willard Learoyd Sperry	Cambridge
Russell Henry Stafford	Brookline
Henry Bradford Washburn	Cambridge
John Broadus Watson	New York, N. Y.
James Haughton Woods	Cambridge
Robert Mearns Yerkes	New Haven, Conn.

CLASS IV., SECTION II.—*History, Archæology, and Anthropology*.—35.

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Clark Wissler	New York, N. Y.

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James Richard Jewett	Cambridge
George Lyman Kittredge	Cambridge
Henry Roseman Lang	New Haven, Conn.
Charles Rockwell Lanman	Cambridge
David Gordon Lyon	Cambridge
Francis Peabody Magoun, Jr.	Cambridge
Albert Matthews	Boston
William Albert Nitze	Chicago, Ill.
Edward Kennard Rand	Cambridge
Fred Norris Robinson	Cambridge
Robert Kilburn Root	Princeton, N. J.
Rudolph Schevill	Berkeley, Cal.
Herbert Weir Smyth	Cambridge
Franklin Bache Stephenson	Washington, D. C.
Joshua Whatmough	Cambridge
Ernest Hatch Wilkins	Oberlin, Ohio

Class IV., Section IV.—*The Fine Arts and Belles Lettres.*—49.

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Denman Waldo Ross	Cambridge
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Edmund C. Tarbell	Boston
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Charles Henry Conrad Wright	Cambridge

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Charles Foster Batchelder	Cambridge
Frederick Pickering Cabot	Boston
William Brooks Cabot	Boston
John Candler Cobb	Milton
Howard Coonley	Milton
John Albert Cousens	Tufts College
William Lusk Webster Field	Milton
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Edward Jackson Holmes	Boston
William James	Cambridge
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Charles Henry Taylor	Boston
Eliot Wadsworth	Boston
Edwin Sibley Webster	Brookline
Edmund Allen Whitman	Cambridge
Benjamin Loring Young	Weston

FOREIGN HONORARY MEMBERS.—101.

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CLASS I.—*Mathematical and Physical Sciences.*—33.

SECTION I.—*Mathematics and Astronomy.*—9.

Arthur Stanley Eddington	Cambridge, England
Jacques Salomon Hadamard	Paris
Godfrey Harold Hardy	Cambridge, England
Ejnar Hertzsprung	Leyden
Tullio Levi-Civita	Rome
Charles Emile Picard	Paris
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Hermann Weyl	Zürich

CLASS I., SECTION II.—*Physics.*—9.

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Albert Einstein	Berlin
James Franck	Berlin
Abram F. Joffé	Leningrad
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Friedrich Paschen	Charlottenburg
Max Planck	Berlin
Sir Ernest Rutherford	Cambridge, England
Sir Joseph John Thomson	Cambridge, England

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Wilhelm Ostwald	Leipzig
Sören Peter Lauritz Sörensen	Copenhagen
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CLASS I., SECTION IV.—*Technology and Engineering.*—7.

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Aurel Stodola	Zürich
Vsevolod Evgenievich Timonoff	Leningrad
William Cawthorne Unwin	London
Karl Willy Wagner	Berlin

CLASS II.—*Natural and Physiological Sciences.*—31.

SECTION I.—*Geology, Mineralogy, and Physics of the Globe.*—10.

Frank Dawson Adams	Montreal
Charles Barrois	Lille
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Léon William Collet	Geneva
Viktor Goldschmidt	Heidelberg
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Emmanuel de Margerie	Paris
Gustaf Adolf Frederik Molengraaff	Delft
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CLASS III.—*The Social Arts.*—7.

SECTION I.—*Jurisprudence.*—3.

Léon Duguit	Bordeaux
Rt. Hon. Sir Frederick Pollock, Bart.	London
Joseph Redlich	Vienna

SECTION II.—*Government, International Law, and Diplomacy.*—1.

Jean Adrien Antoine Jules Jusserand	Paris

SECTION III.—*Economics and Sociology.*—1.

Arthur Cecil Pigou	Cambridge, England

SECTION IV.—*Administration and Affairs.*—2.

Raymond Poincaré	Paris
Rufus Daniel Isaacs, Marquess of Reading	London

CLASS IV.—*The Humanities.*—30.

SECTION I.—*Theology, Philosophy, and Psychology.*—5

Henri Louis Bergson	Paris
Francis Crawford Burkitt	Cambridge, England
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Étienne Gilson	Melun, France
Edmund Husserl	Freiburg i. B.

SECTION II.—*History, Archæology, and Anthropology.*—8.

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Wilhelm Dörpfeld	Berlin-Lichterfelde

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Karl Pearson	London
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Charles Sanford Terry	Aberdeen
George Macaulay Trevelyan	Cambridge, England

SECTION III.—*Philology*.—8.

Joseph Bédier	Paris
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Hermann Georg Jacobi	Bonn
Sylvain Lévi	Paris
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Antoine Meillet	Paris
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SECTION IV.—*The Fine Arts and Belles Lettres*.—9.

Alfredo Casella	Rome
John Galsworthy	London
Henry Guy	Grenoble
Paul Hazard	Paris
Rudyard Kipling	Burwash, Sussex
Victor Laloux	Paris
Gilbert Murray	Oxford
Edgar Allison Peers	Liverpool
Henri Rabaud	Paris

INDEX OF MEMBERS.

The letters A, F, and FHM refer to the lists of Associates, Fellows, and Foreign Honorary Members, respectively. The class and section are indicated by the numerals following.

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Emerson, B. K. F, II: 1
Emerson, R. A. F, II: 2
Emerson, W. F, IV: 4
Emerton, E. F, IV: 2
Erskine, J. F, IV: 4
Evans, A. W. F, II: 2
Evans, H. M. F, II: 3
Ewell, A. W. F, I: 2
Fair, G. M. F, I: 4
Fairbanks, A. F, IV: 4
Fall, H. C. F, II: 3
Farley, F. E. F, IV: 3
Farnam, H. W. F, III: 3
Farrand, M. F, IV: 2
Faull, J. H. F, II: 2
Fay, F. H. F, I: 4
Fay, H. F, I: 3
Fay, S. B. F, III: 2
Fenn, W. W. F, IV: 1
Ferguson, W. S. F, IV: 2
Fernald, M. L. F, II: 2
Field, F. T. F, III: 1
Field, W. L. W. A
Fisher, H. A. L. FHM, IV: 2
Fisher, I. F, III: 3
Fisher, R. T. F, II: 2
Flexner, S. F, II: 4
Forbes, A. F, II: 3
Forbes, E. W. F, IV: 4
Forbes, G. S. F, I: 3
Forbes, W. C. F, III: 2
Ford, J. F, III: 3
Ford, J. D. M. F, IV: 3
Ford, W. C. F, IV: 2
Ford, W. E. F, II: 1
Fox, P. F, I: 1
Franck, J. FHM, I: 2
Franklin, E. C. F, I: 3
Franklin, F. F, I: 1
Franklin, P. F, I: 1
Freeman, J. R. F, I: 4
Frost, E. B. F, I: 1
Frost, R. F, IV: 4
Gage, T. H. F, III: 1
Galsworthy, J. FHM, IV: 4
Gamble, J. L. F, II: 4
Gay, E. F. F, III: 3
Geddes, J., Jr. F, IV: 3
Gillespie, L. J. F, I: 3
Gilson, E. FHM, IV: 1
Goldschmidt, V. F, II: 1
Goldthwait, J. W. F, II: 1
Goodale, J. L. F, II: 4
Goodnow, F. J. F, III: 4
Goodrich, W. F, IV: 4
Goodwin, H. M. F, I: 2
Grandgent, C. H. F, IV: 3
Grant, R F, III: 4

Graton, L. C. F, II: 1
Graustein, W. C. F, I: 1
Gray, L. H. F, IV: 3
Greene, E. B. F, IV: 2
Greene, W. C. F, IV: 3
Greenough, C. N. F, IV: 4
Greenough, R. B. F, II: 4
Gregory, H. E. F, II: 1
Gregory, W. K. F, II: 3
Grinnell, J. F, II: 3
Gulick, C. B. F, IV: 3
Guy, H. FHM, IV: 4
Haber, F. FHM, I: 3
Hack, R. K. F, IV: 3
Hadamard, J. S. FHM, I: 1
Hale, G. E. F, I: 2
Hall, E. H. F, I: 2
Hall, H. M. F, II: 2
Hammond, J. H. F, I: 4
Hardy, A. C. F, I: 2
Hardy, G. H. FHM, I: 1
Harper, R. A. F, II: 2
Harrison, G. R. F, I: 2
Harrison, R. G. F, II: 4
Hart, A. B. F, III: 2
Hart, F. R. A
Haskins, C. H. F, IV: 2
Hatch, W. H. P. F, IV: 1
Havens, R. D. F, IV: 3
Hayes, H. V. F, I: 2
Hazard, P. FHM, IV: 4
Hazen, C. D. F, IV: 2
Heidel, W. A. F, IV: 1
Heim, A. FHM, II: 1
Henderson, L. J. F, I: 3
Hendrickson, G. L. F, IV: 3
Henshaw, S. F, II: 3
Hertzsprung, E. FHM, I: 1
Hill, B. H. F, IV: 2
Hill, E. B. F, IV: 4
Hills, E. C. F, IV: 3
Hillyer, R. S. F, IV: 4
Hitchcock, A. S. F, II: 2
Hitchcock, F. L. F, I: 1
Hoadley, L. F, II: 3
Hocking, W. E. F, IV: 1
Holcombe, A. N. F, III: 2
Holmes, E. J. A
Holmes, S. J. F, II: 3
Holmes, W. H. F, IV: 4
Hooton, E. A. F, IV: 2
Hopkins, E. W. F, IV: 3
Hopkinson, C. F, IV: 4
Hoskins, R. G. F, II: 3
Hosmer, G. L. F, I: 4
Hovgaard, W. F, I: 4
Howard, L. O. F, II: 3
Howard, W. G. F, IV: 3
Howe, M. A. DeW. F, IV: 4
Howe, P. R. F, II: 4
Howell, W. H. F, II: 4
Hrdlicka, A. F. IV: 2
Hubbard, J. C. F, I: 2
Hudson, M. O. F, III: 2
Hughes, C. E. F, III: 1
Hull, G. F. F, I: 2
Humphreys, W. J. F, II: 1
Hunt, R. F, II: 4
Huntington, A. M. F, IV: 4
Huntington, E. F, II: 1
Huntington, E. V. F, I: 1
Husserl, E. FHM, IV: 1
Hutchins, C. C. F, I: 2
Hyvernat, E. X. L. H. F, IV: 3
Isaacs, N. F, III: 1
Ives, F. E. F, I: 2
Ives, J. E. F, I: 2
Jack, J. G. F, II: 2
Jack, J. R. F, I: 4
Jackson, C. L. F, I: 3
Jackson, C. N. F, IV: 3
Jackson, D. F, I: 1
Jackson, D. C. F, I: 4
Jackson, F. J. F. F, IV: 1
Jackson, H. F, II: 4
Jackson, R. T. F, II: 1
Jacobi, H. G. FHM, IV: 3
Jacques, W. W. F, I: 2

Jaggar, T. A. F, II: 1
James, H. F, IV: 4
James, W. A
Jeffrey, E. C. F, II: 2
Jennings, H. S. F, II: 3
Jennings, W. L. F, I: 3
Jepson, W. L. F, II: 2
Jewett, F. B. F, I: 4
Jewett, J. R. F, IV: 3
Joffé, A. F. FHM, I: 2
Johnson, C. W. F, II: 3
Johnson, D. W. F, II: 1
Johnson, L. J. F, I: 4
Johnston, I. M. F, II: 2
Jones, G. F, I: 3
Jones, Sir R. FHM, II: 4
Joslin, E. P. F, II: 4
Jusserand, J. A. A. J. FHM, III: 2
Keen, W. W. F, II: 4
Keith, A. F, II: 1
Kelley, T. L. F, IV: 1
Kellogg, O. D. F, I: 1
Kemble, E. C. F, I: 2
Kennelly, A E. F, I: 4
Kent, N. A. F, I: 2
Keyes, F. G. F, I: 3
Kidder, A. V. F, IV: 2
Kidder, N. T. A
Kipling, R. FHM, IV: 4
Kittredge, G. L. F, IV: 3
Knudson, A. C. F, IV: 1
Köhler, W. FHM, II: 3
Kofoid, C. A. F, II: 3
Kohler, E. P. F, I: 3
Kraus, C. A. F, I: 3
Kroeber, A. L. F, IV: 2
Krogh, A. FHM, II: 3
Lake, K. F, III: 2
Laloux, V. FHM, IV: 4
Lamb, A. B. F, I: 3
Lampland, C. O. F, I: 1
Lane, A. C. F, II: 1
Lang, H. R. F, IV: 3
Langmuir, I. F, I: 3
Lanman, C. R. F, IV: 3
La Piana, G. F, IV: 2
Lapicque, L. E. FHM, II: 3
Larmor, Sir J. FHM, I: 2
Larsen, E. S. F, II: 1
Lawrence, W. F, IV: 1
Lawrence, W. H. F, I: 4
Laws, F. A. F, I: 2
Lawson, A. C. F, II: 1
Lawton, F. F, III: 1
Le Chatelier, H. L. FHM, I: 3
Lee, R. I. F, II: 4
Lefavour, H. F, I: 2
Leith, C. K. F, II: 1
Lévi, S. FHM, IV: 3
Levi-Civita, T. FHM, I: 1
Lewis, C. I. F, IV: 1
Lewis, F. T. F, II: 3
Lewis, G. N. F, I: 3
Lewis, W. K. F, I: 3
Lillie, R. S. F, II: 3
Lindgren, W. F, II: 1
Lindsay, R. B. F, I: 2
Lipman, J. G. F, II: 2
Little, A. D. F, I: 3
Livingston, B. E. F, II: 2
Locke, E. A. F, II: 4
Lodge, J. E. F, IV: 4
Loeffler, C M. T. F, IV: 4
Longcope, W. T. F, II: 4
Loomis, F. B. F, II: 1
Lowell, A. L. F, III: 2
Lowes, J. L. F, IV: 4
Lull, R. S. F, II: 3
Luyten, W. J. F, I: 1
Lyman, T. F, I: 2
Lyon, D. G. F, IV: 3
McAdie, A. G. F, II: 1
Macallum, A. B. FHM, I: 3
MacDonald, W. F, III: 2
Macdonell, A. A. FHM, IV: 3
McDougall, W. F, IV: 1
McEwen, G. F. F, II: 1
McIlwain, C. H. F, IV: 2

MacInnes, D. A. F, I: 3
McLaughlin, D. H. F, II: 1
Maginnis, C. D. F, IV: 4
Magoun, F. P., Jr. F, IV: 3
Main, C. T. F, I: 4
Mallory, F. B. F, II: 4
Manship, P. F, IV: 4
Margerie, E. de. FHM, II: 1
Mark, E. L. F, II: 3
Marks, L. S. F, I: 4
Martin, E. G. F, II: 3
Mason, D. G. F, IV: 4
Mather, F. J. F, IV: 4
Mather, K. F. F, II: 1
Mathews, E. B. F, II: 1
Matthews, A. F, IV: 3
Mayo, W. J. F, II: 4
Mead, A. D. F, II: 3
Means, J. H. F, II: 4
Meillet A. FHM, IV: 3
Melander, A. L. F, II: 3
Mendel, L. B. F, I: 3
Merriam, J. C. F, II: 1
Merrill, E. D. F, II: 2
Merriman, R. B. F, IV: 2
Merritt, E. G. F, I: 2
Miller, D. C. F, I: 2
Miller, E. F. F, I: 4
Miller, G. A. F, I: 1
Miller, G. S. F, II: 3
Miller, J. A. F, I: 1
Miller, W. J. F, II: 1
Millikan, R. A. F, I: 2
Minot, G. R. F, II: 4
Mitchell, S. A. F, I: 1
Mitchell, W. C. F, III: 3
Mitchell, W. DeW. F, III: 1
Miyajima, M. FHM, II: 4
Molengraaff, G. A. F. FHM, II: 1
Moore, E. C. F, IV: 1
Moore, E. H. F, I: 1
Moore, J. B. F, III: 2
More, P. E. F, IV: 1
Morgan, E. M. F, III: 1
Morgan, T. H. F, II: 3
Morison, S. E. F, IV: 2
Morley, F. F, I: 1
Morse, H. W. F, I: 2
Morse, M. F, I: 1
Morss, E. A
Moss, W. L. F, II: 4
Moulton, F. R. F, I: 1
Mueller, E. F, I: 3
Müller, F. von. FHM, II: 4
Mueller, J. H. F, II: 4
Mulliken, S. P. F, I: 3
Munro, W. B. F, III: 2
Munroe, C. E. F, I: 3
Murdock, H. F, IV: 4
Murdock, K. B. F, IV: 4
Murray, G. FHM, IV: 4
Neal, H. V. F, II: 3
Neilson, W. A. F, IV: 4
Newell, L. C. F, I: 3
Nichols, E. L. F, I: 2
Nitze, W. A. F, IV: 3
Norris, J. F. F, I: 3
Norton, C. L. F, I: 2
Noyes, A. A. F, I: 3
Noyes, W. A. F, I: 3
Noyes, W. A., Jr. F, I: 3
Nuttall, G. H. F. FHM, II: 3
O'Brien, R. L. A
d'Ocagne, M. FHM, I: 4
O'Connor, J. F, IV: 1
Oertel, H. FHM, IV: 3
Oldenberg, O. F, I: 2
Olmsted, F. L. F, I: 4
Osborn, H. F. F, II: 3
Osgood, R. B. F, II: 4
Osterhout, W. J. V. F, II: 2
Ostwald, W. FHM, I: 3
Page, C. H. F, IV: 4
Palache, C. F, II: 1
Palmer, G. H. F, IV: 1
Park, C. E. F, IV: 1
Park, C. F. F, I: 4
Parker, G. H. F, II: 3

Parker, H. F, III: 1
Parks, L. F, IV: 1
Parsons, W. B. F, I: 4
Paschen, F. FHM, I: 2
Patten, W. F, II: 3
Peabody, F. G. F, IV: 1
Pearl, R. F, II: 3
Pearse, L. F, I: 4
Pearson, K. FHM, IV: 2
Peers, E. A. FHM, IV: 4
Peirce, G. J. F, II: 2
Pender, H. F, I: 4
Pepper, G. W. F, III: 1
Perkins, T. N. F, III: 4
Perry, R. B. F, IV: 1
Persons, W. M. F, III: 3
Peters, A. J. A
Phelps, W. L. F, IV: 4
Phillips, H. B. F, I: 1
Phillips, J. C. F, II: 3
Philpott, A. J. A
Picard, C. E. FHM, I: 1
Pickard, G. W. F, I: 4
Pickering, W. H. F, I: 1
Pidal, R. M. FHM, IV: 3
Pierce, G. W. F, I: 2
Pigou, A. C. FHM, III: 3
Pilsbry, H. A. F, II: 3
Pirenne, H. FHM, IV: 2
Planck, M. FHM, I: 2
Plaskett, H. H. F, I: 1
Poincaré, R. FHM, III: 4
Pollock, Sir F. FHM, III: 1
Poor, C. L. F, I: 1
Porter, A. K. F, IV: 4
Post, C. R. F, IV: 4
Pound, R. F, III: 1
Powell, T. R. F, III: 1
Prandtl, L. FHM, I: 4
Pratt, F. H. F, II: 3
Pratt, J. H. F, II: 4
Prescott, S. C. F, I: 3
Probst, E. FHM, I: 4
Pupin, M. I. F, I: 2
Putnam, H. F, III: 4
Rabaud, H. FHM, IV: 4
Rand, E. K. F, IV: 3
Rand, H. W. F, II: 3
Raymond, P. E. F, II: 1
Reading, Lord. FHM, III: 4
Redfield, A. C. F, II: 3
Redlich, J. FHM, III: 1
Regan, C. T. FHM, II: 3
Rehder, A. F, II: 2
Reisner, G. A. F, IV: 2
Rendle, A. B. FHM, II: 2
Renner, O. FHM, II: 2
Richards, R. H. F, I: 3
Richardson, R. G. D. F, I: 1
Richardson, W. L. F, II: 4
Ripley, A. L. A
Ritter, W. E. F, II: 3
Roberts, O. A
Robinson, B. L. F. II: 2
Robinson, E. A. F, IV: 4
Robinson, F. N. F, IV: 3
Rogers, A. F. F, II: 1
Root, E. F, III: 1
Root, R. K. F, IV: 3
Ropes, J. H. F, IV: 1
Rosanoff, M. A. F, I: 3
Rosenau, M. J. F, II: 4
Ross, D. W. F, IV: 4
Rostovtzeff, M. I. F, IV: 2
Rowe, A. W. F, I: 3
Rugg, A. P. F, III: 1
Russell, G. E. F, I: 4
Russell, H. N. F, I: 1
Rutherford, Sir E. FHM, I: 2
Ruthven, A. G. F, II: 3
Rutledge, G. F, I: 1
Ryan, J. H. F, IV: 1
Sachs, P. J. F, IV: 4
Sanderson, G. A. F, III: 1
Sarton, G. F, IV: 2
Saunders, F. A. F, I: 2
Sauveur, A. F, I: 4
Sax, K. F, II: 2

Sayles, R. W. F, II: 1
Scatchard, G. F, I: 3
Schaller, W. T. F, II: 1
Schevill, R. F, IV: 3
Schlesinger, A. M. F, IV: 2
Schlesinger, F. F, I: 1
Schuchert, C. F, II: 1
Scott, A. W. F, III: 1
Scott, W. B. F, II: 1
Sedgwick, E. F, IV: 4
Sedgwick, H. D. F, IV: 4
Sellards, A. W. F, II: 4
Setchell, W. A. F, II: 2
Seymour, A. B. F, II: 2
Shapley, H. F, I: 1
Shaw, Sir W. N. FHM, II: 1
Shepherd, F. J. FHM, II: 4
Sherrill, H. K. F, IV: 1
Sherrill, M. S. F, I: 3
Sherrington, Sir C. S. FHM, II: 4
Shimer, H. W. F, II: 1
Sims, W. S. F, I: 4
Sitter, W. de. FHM, I: 1
Slater, J. C. F, I: 2
Slipher, V. M. F, I: 1
Slocum, F. F, I: 1
Smith, H. M. F, I: 3
Smith, J., Jr. F, III: 4
Smith, P. A
Smith, T. F, II: 4
Smyth, H. L. F, I: 4
Smyth, H. W. F, IV: 3
Snyder, V. F, I: 1
Sörensen, S. P. L. FHM, I: 3
Sollmann, T. H. F, II: 4
Sorokin, P. A. F, III: 3
Sperry, W. L. F, IV: 1
Spinden, H. J. F, IV: 2
Spofford, C. M. F, I: 4
Sprague, O. M. W. F, III: 3
Stafford, R. H. F, IV: 1
Stakman, E. C. F, II: 2
Stebbins, J. F, I: 1
Stein, Sir A. FHM, IV: 2
Stephenson, F. B. F, IV: 3
Stetson, H. T. F, I: 1
Stevens, J. F. F, I: 4
Stieglitz, J. O. F, I: 3
Stiles, C. W. F, II: 4
Stiles, P. G. F, II: 3
Stodola, A. FHM, I: 4
Stone, J. S. F, I: 2
Strong, R. P. F, II: 4
Struik, D. J. F, I: 1
Sturgis, R. C. F, IV: 4
Sturgis, W. C. F, II: 2
Sumner, J. O. F, IV: 2
Taber, H. F, I: 1
Talbot, F. B. F, II: 4
Tamarkin, J. D. F, I: 1
Tarbell, E. C. F, IV: 4
Taussig, F. W. F, III: 3
Taylor, C. H. A
Taylor, E. W. F, II: 4
Taylor, F. B. F, II: 1
Terry, C. S. FHM, IV: 2
Thaxter, R. F, II: 2
Thayer, J. E. F, II: 3
Thayer, W. S. F, II: 4
Theiler, Sir A. FHM, II: 4
Thompson, D'A. W. FHM, II: 3
Thompson, M. deK. F, I: 2
Thomson, E. F, I: 2
Thomson, Sir J. J. FHM, I: 2
Timonoff, V. E. FHM, I: 4
Tolman, R. C. F, I: 3
Torrey, C. C. F, IV: 2
Tozzer, A. M. F, IV: 2
Trelease, W. F, II: 2
Trevelyan, G. M. FHM, IV: 2
Tucker, D. S. F, III: 3
Turner, F. J. F, IV: 2
Tyler, H. W. F, I: 1
Tyzzer, E. E. F, II: 4
Unwin, W. C. FHM, I: 4
Vallarta, M. S. F, I: 2
Vallée Poussin, C. J. de la. FHM, I: 1
Vaughan, T. W. F, II: 1

Veblen, O. F, I: 1
Verhoeff, F. H. F, II: 4
Vogt, J. H. L. FHM, II: 1
Vries, H. de. FHM, II: 2
Wadsworth, E. A
Wagner, K. W. FHM, I: 4
Wait, W. C. F, III: 1
Walcott, H. P. F, II: 4
Walker, C. H. F, IV: 4
Walker, W. H. F, I: 3
Walsh, J. L. F, I: 1
Wambaugh, E. F, III: 1
Warner, E. P. F, I: 4
Warren, C. H. F, II: 1
Washburn, H. B. F, IV: 1
Watson, J. B. F, IV: 1
Wearn, J. T. F, II: 4
Weatherby, C. A. F, II: 2
Webster, D. L. F, I: 2
Webster, E. S. A
Welch, W. H. F, II: 4
Wells, A. E. F, I: 4
Weston, R. S. F, I: 4
Weston, W. H. F, II: 2
Weyl, H. FHM, I: 1
Weysse, A. W. F, II: 3
Whatmough, J. F, IV: 3
Wheeler, W. M. F, II: 3
White, B. F, II: 4
White, D. F, II: 1
Whitehead, A. N. F, I: 1
Whitlock, H. P. F, II: 1
Whitman, E. A. A
Whitney, W. R. F, I: 3
Wickersham, G. W. F, III: 1
Wieland, H. FHM, I: 3
Wilkins, E. H. F, IV: 3
Williams, F. H. F, II: 4
Williams, R. S. F, I: 3
Willis, B. F, II: 1
Willoughby, W. W. F, III: 2
Wilson, E. B. F, II: 3
Wilson, E. B. F, I: 2
Wilson, G. G. F, III: 2
Winship, G. P. F, IV: 4
Winslow, A. F, II: 1
Wissler, C. F, IV: 2
Wister, O. F, IV: 4
Wolbach, S. B. F, II: 4
Wolff, J. E. F, II: 1
Wood, R. W. F, I: 2
Woodman, A. G. F, I: 3
Woods, F. A. F, II: 3
Woods, F. S. F, I: 1
Woods, J. H. F, IV: 1
Worcester, J. R. F, I: 4
Wright, C. H. C. F, IV: 4
Wright, F. E. F, II: 1
Wright, Q. F, III: 2
Yerkes, R. M. F, IV: 1
Young, B. L. A
Zeleny, J. F, I: 2
Zinsser, H. F, II: 4

STATUTES AND STANDING VOTES.

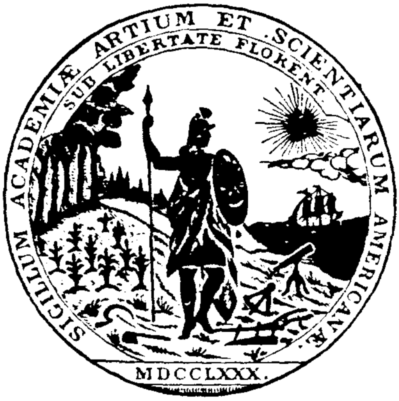
STATUTES.

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915, *April* 12, 1916, *April* 10, 1918, *May* 14, 1919, *February* 8, *April* 12, *and December* 13, 1922, *February* 14, *March* 14, *and October* 10, 1923, *March* 10, 1926, *May* 9, 1928, *April* 8, 1931, *and November* 11, 1931.

CHAPTER I.

THE CORPORATE SEAL.

ARTICLE 1. The Corporate Seal of the Academy shall be as here depicted:


SIGILLUM ACADEMIÆ ARTIUM ET SCIENTIARUM AMERICANÆ
SUB LIBERTATE FLORENT
MDCCLXXX.

ARTICLE 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v, art. 3; chap. vi, art. 2.

CHAPTER II.

FELLOWS AND FOREIGN HONORARY MEMBERS AND DUES.

ARTICLE 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in four Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

CLASS I. *The Mathematical and Physical Sciences*

Section 1. Mathematics and Astronomy
Section 2. Physics
Section 3. Chemistry
Section 4. Technology and Engineering

CLASS II. *The Natural and Physiological Sciences*

Section 1. Geology, Mineralogy, and Physics of the Globe
Section 2. Botany
Section 3. Zoölogy and Physiology
Section 4. Medicine and Surgery

CLASS III. *The Social Arts*

Section 1. Jurisprudence
Section 2. Government, International Law, and Diplomacy
Section 3. Economics and Sociology
Section 4. Administration and Affairs

CLASS IV. *The Humanities*

Section 1. Theology, Philosophy, and Psychology
Section 2. History, Archæology, and Anthropology
Section 3. Philology
Section 4. The Fine Arts and Belles Lettres

ARTICLE 2. The number of Fellows shall not exceed Eight hundred, of whom not more than Six hundred shall be residents of Massachusetts, nor shall there be more than Two hundred and ten in any one Class.

ARTICLE 3. The number of Foreign Honorary Members shall not exceed One hundred and thirty. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Thirty-five in any one Class.

ARTICLE 4. If any person, after being notified of his election as Fellow, shall neglect for six months to accept in writing, or, if a Fellow resident within fifty miles of Boston shall neglect to pay his Admission Fee, his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii, art. 2.

ARTICLE 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten dollars, unless previously as an Associate he has paid an Admission Fee of like amount.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury

Two hundred dollars in addition to his previous payments. Any Fellow shall also be exempt from Annual Dues who has paid such dues for forty years, or, having attained the age of seventy-five has paid dues for twenty-five years.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

ARTICLE 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. xi, art. 2.

ARTICLE 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to Foreign Honorary Members and to Fellows on request.

ARTICLE 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three-fourths of the Fellows present, out of a total attendance of not less than fifty at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii; chap. vi, art. 1; chap. x, art. 1, 7; chap. xi, art. 2.

CHAPTER III.

ELECTION OF FELLOWS AND FOREIGN HONORARY MEMBERS.

The procedure in the election of Fellows and Foreign Honorary Members shall be as follows:

Nominations to Fellowship or Foreign Honorary Membership in any Section must be signed by Two Fellows of that Section, or by three Fellows of any Sections, and sent to the Corresponding Secretary accompanied by a statement of the qualifications of the nominee and brief biographical data.

Notice shall be sent to every Fellow not later than the fifteenth of January in each year, reminding him that all nominations must be in the hands of the Corresponding Secretary before the fifteenth of February following.

A list of the nominees, giving a brief account of each, with the names of the nominators, shall be sent to every Fellow with a request that he return the list with such confidential comments and indications of preference as he may choose to make.

All the nominations, with any comments thereon and with expressions of preference on the part of the Fellows, shall be referred to the appropriate Class Committees, which shall canvass them, and report their recommendations in writing to the Council before the Stated Meeting of the Academy in April.

Elections of Fellows and Foreign Honorary Members shall be made by the Council before the Annual Meeting in May, and announced at that meeting.

Persons nominated in any year, but not elected, may be carried over to the list of nominees for the next year at the discretion of the Council, but shall not be further continued unless renominated.

See Chap. ii; chap. vi, art. 1; chap. x, art. 1.

CHAPTER IV.

OFFICERS.

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), four Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, a Librarian, and an Editor, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also sixteen Councillors, one from each Section of each Class. At each Annual Meeting four Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. xi, art. 1.

ARTICLE 2. If any officer be unable, through death, absence, or disability, to fulfill the duties of his office, or if he shall resign, his place may be filled by the Council in its discretion for any part or the whole of the unexpired term.

ARTICLE 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of Four Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and file it with the Recording Secretary not later than four weeks before the Annual Meeting.

See Chap. vi, art. 2.

ARTICLE 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call therefor, and shall be mailed to all the Fellows having the right to vote.

See Chap. vi, art. 2.

ARTICLE 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V.

THE PRESIDENT.

ARTICLE 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii, art. 7; chap. iv, art. 1, 3; chap. vi, art. 2; chap. vii, art. 1; chap. x, art. 6; chap. xi, art. 1, 2; chap. xii, art. 1.

CHAPTER VI.

THE SECRETARIES.

ARTICLE 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows and Foreign Honorary Members, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii, art. 7; chap. iii; chap. iv, art. 1; chap. x, art. 6; chap. xi, art. 1; chap. xii, art. 1.

ARTICLE 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow and by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

After all elections, he shall insert in the Records the names of the Fellows by whom the successful nominees were proposed.

He shall send the Report of the Nominating Committee in print to every Fellow having the right to vote at least three weeks before the Annual Meeting.

See Chap. iv, art. 3.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. 1; chap. ii, art. 7; chap. iv, art. 3, 4, 5; chap. x, art. 6; chap. xi, art. 1, 2; chap. xii, art. 1, 3.

ARTICLE 3. The Secretaries, with the Editor, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII.

THE TREASURER AND THE TREASURY.

ARTICLE 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the Appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

ARTICLE 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows who may be then delinquent in the payment of their Annual Dues.

ARTICLE 3. He shall give such security for the trust reposed in him as the Academy may require.

ARTICLE 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the Funds of the Academy.

ARTICLE 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

ARTICLE 6. The Financial Year of the Academy shall begin with the first day of April.

ARTICLE 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without previous consent and approval of the Council.

See Chap. ii, art. 4, 5; chap. vi, art. 2; chap. x, art. 6; chap. xi, art. 1, 2, 3; chap. xii, art. 1.

CHAPTER VIII.

THE LIBRARIAN AND THE LIBRARY.

ARTICLE 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

ARTICLE 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

ARTICLE 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

ARTICLE 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

ARTICLE 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow so paying, unless such remainder be valuable by reason of association.

ARTICLE 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

ARTICLE 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii, art. 6; chap. xi, art. 1, 2.

CHAPTER IX.

THE EDITOR AND THE PUBLICATIONS.

ARTICLE 1. The Editor shall have charge of the conduct through the press of the Proceedings and the Memoirs, and all correspondence relative thereto, and shall have power to fix the price at which individual numbers of the Proceedings and Memoirs are sold.

ARTICLE 2. In conjunction with the Committee of Publication, he shall have authority to expend such sums as may be appropriated by the Academy for printing the publications and for defraying other expenses therewith connected.

ARTICLE 3. All publications which are financed in whole or in part from the income of the Rumford Fund or from the income of other special funds, and all publications of work done with the aid of the Rumford Fund or other special funds, shall contain a conspicuous statement of this fact.

ARTICLE 4. Two hundred extra copies of each paper printed in the Proceedings or Memoirs shall be placed at the disposal of the author without charge.

If, on account of the number of communications offered for publication, it shall be necessary to decline for publication communications otherwise acceptable, members of the Academy shall be given preference in each of the several Classes over non-members; but whenever it shall be necessary to exercise this preference, the Editor shall inform the Council of the fact.

See Chap. iv, art. 1; chap. vi, art. 3; chap. x, art. 6; chap. xi, art. 4.

CHAPTER X.

THE COUNCIL.

ARTICLE 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and act upon them in accordance with the provisions of Chapter III.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections, reporting its action to the Academy.

See Chap. iii, art. 2, 3; chap. xi, art. 1.

Article 2. Nine members shall constitute a quorum.

Article 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

Article 4. It shall act upon all resignations of officers, and all resignations and forfeitures of Fellowship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation. They shall hold their respective positions during the pleasure of the Council.

Article 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

Article 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. xi, art. 3.

ARTICLE 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a biographical notice for publication in the Proceedings.

ARTICLE 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii, art. 4, 5, 8; chap. iv, art. 1, 2; chap. vi, art. 1; chap. vii, art. 1; chap. xii, art. 1, 4.

CHAPTER XI.

STANDING COMMITTEES.

ARTICLE 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii, art. 2.

ARTICLE 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance*, to consist of four Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and changing the investments thereof in their discretion.

See Chap. iv, art. 3; chap. vii, art. 1, 4; chap. x, art. 6.

(ii) *The Rumford Committee*, to consist of seven Fellows, who shall report to the Academy on all applications and claims for the Rumford Premium. It alone shall authorize the purchase of books, publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv, art. 3; chap. x, art. 6.

(iii) *The Cyrus Moors Warren Committee*, to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv, art. 3; chap. x, art. 6.

(iv) *The Committee of Publication*, to consist of the Editor, *ex officio*, as Chairman, and four other Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which volumes of the publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

It shall determine when the pressure of material offered for publication makes it necessary to give preference to members of the Academy as compared with non-members, or to give priority to certain members as compared with others, and to what extent this preference or priority shall be applied in each of the four Classes, to the end that a proper balance of the facilities of publication with respect to subject matter and authors may be maintained.

See Chap. iv, art. 3; chap. vi, art. 1, 3; chap. ix; chap. x, art. 6.

(v) *The Committee on the Library*, to consist of the Librarian, *ex officio*, as Chairman, and four other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv, art. 3; chap. viii, art. 1, 2; chap. x, art. 6.

(vi) *The House Committee*, to consist of four Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv, art. 1, 3; chap. x, art. 6.

(vii) *The Committee on Meetings*, to consist of the President, the Recording Secretary, and four other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv, art. 3; chap. x, art. 6.

(viii) *The Auditing Committee*, to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv, art. 3; chap. vii, art. 1; chap. x, art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii, art. 1, 7; chap. x, art. 6.

CHAPTER XII.

MEETINGS, COMMUNICATIONS, AND AMENDMENTS.

ARTICLE 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April, and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

ARTICLE 2. Twenty-five Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special Meetings. Eighteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

ARTICLE 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

ARTICLE 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow without the consent of the author; and no report shall in any case be published by any Fellow in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

ARTICLE 5. No Fellow shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after the result of the balloting upon nominations has been declared.

ARTICLE 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

ARTICLE 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two-thirds of the ballot cast, and not less than twenty-five, must be affirmative to effect enactment.

ARTICLE 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two-thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii, art. 5, 8; chap. iii; chap. iv, art. 3, 4, 5; chap. v, art. 1; chap. vi, art. 1, 2; chap. x, art. 8.

STANDING VOTES.

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. The Council, under such rules respecting nominations as it may prescribe, may elect as Associates of the Academy a limited number of men of mark in affairs or of distinguished service in the community.

Associates shall be entitled to the same privileges as Fellows, but shall not have the right to vote.

The admission fee and annual dues of Associates shall be the same as those of Fellows residing within fifty miles of Boston.

5. Communications offered for publication in the Proceedings or Memoirs of the Academy shall not be accepted for publication before the author shall have informed the Committee on Meetings of his readiness, either himself or through some agent, to use such time as the Committee may assign him at such meeting as may be convenient both to him and to the Committee, for the purpose of presenting to the Academy a general statement of the nature and significance of the results contained in his communication.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

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  K. T. Compton, 505.
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  M. T. Copeland, 506.
  S. H. Cross, 506.
  W. L. Crum, 506.
  E. F. Du Bois, 505.
  S. B. Fay, 506.
  F. T. Field, 505.
  James Ford, 505.
  Robert Frost, 506.
  W. K. Gregory, 505.
  G. R. Harrison, 505.
  R. D. Havens, 506.
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  G. L. Hosmer, 505.
  M. O. Hudson, 505.
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  T. L. Kelley, 505.
  R. B. Lindsay, 505.
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  D. G. Mason, 506.
  F. J. Mather, 506.
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  K. B. Murdock, 506.
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